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Monetary Policy, Financial Stability and Interest Rate Rules

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Abstract

This paper examines the interaction between monetary policy and financial stability and provides an assessment of the implications of banks' risk management practices for monetary policy. We derive inertial interest rate rules - characterized by backward and forward interest rate smoothing - by explicitly modeling the desire of the central bank to stabilize different definitions of the basis risk. The paper shows that, contrary to what found in the literature, smoothing interest rates does not in general alleviate problems of indeterminacy of the economy's rational expectations equilibrium. However, from an empirical point of view, monetary policy rules that embed backward and forward interest rate smoothing seem to perform quite well.

JEL classification: E44, E52, E58, G12, G13.

Keywords: Central Bank, Interest Rate Rule, Monetary Policy, Financial Stability, Asset Prices, Futures Market, Hedging, Basis Risk, Federal Reserve.

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1. Introduction

Central banks around the world have recently started to devote increasing attention to the objective of financial stability. This move has been associated, on one side, with the impressive progresses made in the fight against inflation; on the other side, with the many episodes of financial and currency crises that have continued to challenge the international financial system.

After two decades of large and volatile swings in prices (during the 70s and the 80s), in most industrial countries the inflation rate has been reduced and maintained in a quite narrow and low range. This has occurred in a context of gradual but intense structural reforms leading to enhance central bank independency as an essential tool to separate monetary policy from excessive political interference. The consensus view regarded price stability as a primary public good and an essential ingredient of welfare increasing policies.

However, while inflation was widely brought under control, financial crises and severe episodes of instability in the banking and financial systems have continued to hit different countries around the world, from the North of Europe (Scandinavia) to the South East of Asia (Indonesia, Thayland, and Malaysia) from Central and Latin America (Mexico and Brazil, then Argentina) to Russia. Among G7 countries, Japan experienced a long lasting crisis in the financial sector that asked for an enormous restructuring of banks and domestic financial companies: the country had to cope with stagnation and deflation in the 90s and early 2000s. Financial stability was severely challenged also in the US: first, with hedge funds such as LTCM (and, more recently, Amaranth) blamed to have put under stress the safety walls built by financial supervisory rules and authorities, then as a result of the excessive euphoria and consequent sharp adjustment of equity prices; finally, as a result of huge corporate and financial scandals.

It is then somehow natural that central banks have shifted to devote increasing attention to how to prevent or reduce the risk of financial crisis and of contagion waves. This has also been considered a plausible rationale in support of the stylized fact that interest rates move gradually in response to changes in macroeconomic conditions (notably output gap and inflation). It is usually argued that by making interest rate changes smaller and more predictable, Central Banks reduce the volatility of commercial banks' profits and reduce the risk of bank insolvencies.

In this paper, we ask whether and how financial stability considerations interact with the mandate of central banks to pursue and maintain price stability. In the literature, this topic is currently at the centre of policy and academic debates. Investigations have followed different paths, going from institutional analysis of whether responsibilities for supervising banks and other financial institutions would be a natural assignment for central banks to theoretical and empirical studies of different kinds of central banks' reaction functions.

Some research focused its attention on central banks' practice of smoothing interest rate movements by showing the optimality of such behaviour. In particular, Woodford (1999) showed that interest rate smoothing is an essential ingredient of optimal monetary policy under commitment, and Woodford (2003b) showed that it is optimal to delegate (under discretion) monetary policy to a central bank with an interest rate smoothing term in the objective function. Moreover, Woodford (2003a) and Bullard and Mitra (2005) showed that monetary inertia can help alleviate problems of indeterminacy (and learning) of stationary rational expectations' equilibria.

In the literature, interest rate smoothing is simply assumed without any formal link to interest rate risk management by banks. However, given the hedging practices related to interest rate risk followed by banks and other financial institutions, it is by no means obvious why central banks should smooth interest rates. We offer a new rationale for such behaviour based on stabilizing "basis" risk, i.e. the residual risk that remains after all imperfect hedging opportunities have been exploited (see Hull, 2000), as a contribution to financial stability.

We show that the desire of central banks to stabilize such basis risk leads to inertial interest rate rules characterized by either backward or forward interest rate smoothing. We find that, contrary to the results in Woodford (2003a) and Bullard and Mitra (2005), backward interest-rate smoothing does not in general alleviate indeterminacy problems of rational expectations equilibria. In our analysis, it only makes determinacy easier to be reached in the limiting case of an aggressive response to inflation and output, when the policy rate reacts to expected future inflation. Moreover, we also show that under forward interest rate smoothing, an aggressive response to the future expected interest rate may lead to indeterminacy and enhance the trade off between price and financial stability.

Finally, we estimate a set of interest rate rules that embed backward and forward interest rate smoothing based on US data. Our empirical investigation seems to support the importance of the future expected interest rate as an additional argument in the interest rate rules of the Federal Reserve.

The paper is organized as follows. Section 2 discusses risk management practices used by banks and argues that these lower (although do not eliminate) the necessity for the central banks to smooth interest rates as a contribution to financial stability. Section 3 presents a formal analysis of equilibrium determinacy based on the explicit inclusion of an additional term in the interest rate rule reflecting the desire of the central bank to stabilize basis risk. In section 4 we present the empirical exercise based on US data. Section 5 summarizes and concludes.

2. Risk management and the contribution of monetary policy to financial stability

In the literature, the definition of financial stability has been neither unique nor homogeneous[1]. However, safeguarding the stability of the financial system has always been considered a proper function of the central bank, although in different institutional arrangements this responsibility has been assigned either to the central bank alone or shared it with other financial regulatory and supervisory agencies or government bodies (Di Giorgio and Di Noia, 2005). Safeguarding financial stability clearly requires macroeconomic controls over financial exchanges, clearing houses, payment and securities settlement systems. These controls may benefit from information and knowledge acquired by the central bank from activities more oriented to the microeconomic stability of banking and other financial intermediaries, such as rules on the required amount of capital, borrowing limits and integrity requirements as well as rules on risk based capital ratios, limits to portfolio investments and the regulation of off-balance sheet activities. However, it has also been argued that the assignment to the central bank of prudential supervisory tasks on banks and other financial intermediaries may pose at risk the proper conduct and the outcome of monetary policy. That is why in many countries, taking for granted that the fundamental task of the central bank is to preserve the value of the currency, it is becoming quite common to separate responsibility for banking and financial supervision from the central bank (Di Noia and Di Giorgio, 1999). A major concern is whether the combination of different responsibilities and objectives in only one agency may result in weak banking supervision and negatively affect monetary policy and/or financial stability. Most of the existing studies on the topic have been limited to providing arguments in favour of institutional merger or separation of monetary policy and financial supervisory duties, or have compared the macroeconomic performance of countries where monetary policy and banking supervision are combined with that of countries where the two functions are formally separated.[2]

The only robust result is that no regulatory arrangement has been shown to be clearly superior. However, the idea of assigning the two functions to different agencies is becoming more popular in practice. A plausible rationale for this is based on the evolution of financial intermediaries, markets and instruments, which blur the borders among banking, securities and insurance activities (Allen and Santomero, 1997). As the specificity of banks becomes less relevant, the solution of establishing common rules and having homogenous prudential supervision for all intermediaries gains ground (Di Giorgio and Di Noia, 2005). According to this view, and in order to avoid the risk of excessive concentration of powers, the task of regulating and supervising the activities of all

[1] See for example Mishkin (1999) and Crockett (1997).

[2] See Goodhart and Schoenmaker (1992, 1995), Haubrich (1996), Heller, (1991).

financial intermediaries could hardly be assigned to the same agency that is already responsible for macroeconomic stability. However, no matter whether in charge of prudential supervision of banks or not, central banks should certainly give proper attention to the objective of financial stability and should carefully consider its interaction with traditional monetary policy targets.

In this perspective, many authors have interpreted the observed practice of smoothing interest rates undertaken by central banks as directed to preserve the stability of financial markets.[3] By responding slowly over a period of several months to changes in macroeconomic conditions, the central bank reduces the size of unanticipated changes in short-term interest rates that commercial banks and other participants in financial markets have to face. This reduces the chance that a bank's profits from its loan portfolio will be put under pressure or that its balance sheet will be weakened.

The main reason why sharp changes in short-term policy rates may damage banks' profits is that banks tend to borrow short and lend long. The consequences of such a maturity mismatch could in principle be serious. However, maturity transformation has been the stock-in-trade of commercial banks for centuries, and they employ well-known methods for dealing with the risks posed by interest rate movements. Overdraft facilities or lines of credit are often made at variable interest rates, while fixed term loans are generally made at rates that allow for default risk and usually account only for a relatively small fraction of banks' portfolios. More importantly, financial institutions have increasingly used derivatives as part of their strategy for managing exposure to interest rate risk. Banks can use interest-rate related derivatives to hedge maturity mismatch and the variety of these products and the liquidity of the markets in which they are traded have indeed recently increased. However, interest rate futures and swaps may not remove all the risk arising from maturity mismatch. The hedging instruments available do not permit banks to insure against fluctuations in the rate of interest they pay on short-term deposits and reserves, which is closely related but not identical to the overnight rate. As an example, banks may use futures markets to switch interest payments based on the 3-month Libor rate with those based on the average overnight rate over the same three-month period. But they remain exposed to the risk of fluctuation in the cost of their deposits with respect to the overnight rate: this residual risk is known as basis risk (Hull, 2000).

These arguments strengthen the view that the risks to banking and financial stability posed by movements in short-term interest rates are quite small, and they should therefore have only a limited influence on monetary policy decisions. Nevertheless, the residual basis risk may still induce some caution on the part of the central bank, as banks cannot insure against it. However, while in the short term cautious interest-rate changes may enhance financial stability, in the longer term they

[3] See the empirical evidence reviewed in Clarida *et al.* (1998, 2000).

may even contribute to undermine it. The achievement of low and stable inflation by the Fed, for example, has been achieved by vigorous use of monetary policy in the early 80s. It may sometimes be necessary for central banks to make substantial changes in policy in order to maintain stability in the medium term. If central banks were to limit changes in interest rates to protect banks' balance sheets, banks may feel they have some implicit insurance, and this may induce a moral hazard problem. Banks may assume to be free to operate on thinner margins and with riskier portfolios of assets and liabilities. It may then be argued that proper regulation of financial markets and supervision of financial institutions is the appropriate policy response, rather than smoothing of interest rate changes, to ensure that banks operate with sufficient margins of capital and liquid reserves to withstand the consequences of fluctuations in financial market conditions (Smith and van Egteren 2005). In any case, it is somehow peculiar that considerations about the risk management of interest rate risk by banks are totally absent in the literature that analyses the nexus between monetary policy and the financial conditions of banks and other financial institutions exposed to such risk.

In the following, we analyze how central banks that care about financial stability but are aware of the instruments that banks may use to hedge against the risk of sharper policy decisions conduct monetary policy. We will focus on determinacy issues and on the potential risks to macroeconomic stability stemming from the response of monetary policy to futures prices movements, given the presence of a particular type of financial stability objective in the central bank reaction function.

3. Financial stability and determinacy of interest rate rules

The link between monetary policy and asset price movements has been long investigated and several channels by which asset prices affect real activity have been identified.[4] It is less clear, however, whether such transmission channels provide a strong argument for basing monetary policy decisions on asset prices movements.[5] Bullard and Schaling (2002) and Driffill *et al.* (2006) already showed that introducing asset prices in the central bank's interest rate rule may weaken the requirement for determinacy of the rational expectations equilibrium and potentially lead to macroeconomic instability. In an open economy model, this has been confirmed by Di Giorgio and

[4] The main channels are: a) a wealth effect on consumption expenditure, b) a Tobin's Q effect on investment, and c) a financial accelerator effect on investment (see Bernanke and Gertler, 1989).

[5] In the literature, it has been argued that the gain of including asset prices in monetary policy rules in practice adds little to stabilizing output and inflation (Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002)), although no consensus has been reached (see Cecchetti et al. 2000, 2002). Borio and Lowe (2002) argue that what really matters for monetary policy is not to respond to asset price bubbles *per se*, but rather to reduce the risk of financial distress resulting from the occurrence of financial imbalances.

Nisticò (2005), who show how reacting more intensively to stock price misalignments may ask for central banks' stronger response to inflation in order to avoid indeterminacy of the rational expectations equilibria.

Here, in order to explore further the issue of including explicitly asset prices in the central bank's reaction function, we focus on the analysis of determinacy of rational expectations equilibrium provided by Woodford (2003a) and Bullard and Mitra (2005), under the assumption of interest rate smoothing. However, we do not simply assume, but explicitly derive interest rate smoothing by introducing in the central bank's rule a reaction to basis risk, assuming that banks hedge risks of interest rates changes by actively using futures contracts. As discussed above, this hedging behavior is relevant for financial institutions, but may still justify some limited financial stability concerns from the point of view of monetary authorities. In our framework, interest rate smoothing is showed to be induced by this particular modeling of a financial stability concern for the central bank. We will argue that this alternative formalization of monetary inertia considerably weakens the benefits of interest-rate smoothing found in Bullard and Mitra (2005) and Woodford (2003a). In particular, we will show that the Bullard-Mitra-Woodford findings are based on the restrictive assumption that the introduction of monetary inertia in the policy rule leaves unchanged the response to inflation and output. We provide an alternative set up in which this assumption is not plausible given a more practical concern for financial stability by the central bank. We also highlight the possibility of an implicit trade off between the financial stability objective and traditional macroeconomic targets in terms of price and output stabilization. Finally, we will show that the inclusion of a basis risk stabilization argument in the interest rate rule may lead the central bank, when adjusting the current level of the policy rate, to take into account future expected levels of the policy rate. This may even lead, in the case of high interest rate inertia, to new indeterminacy problems.

3.1The model

We follow the standard literature on New Keynesian microfounded dynamic general equilibrium models and assume:

- a New Keynesian Phillips curve relating inflation positively to the output gap and to future expected inflation:

$$\pi_t = ky_t + \beta E_t \pi_{t+1}, \tag{1}$$

with $0<\beta<1$ and $k>0$;

- a IS curve relating the output gap positively to its future expected value and negatively to the current real interest rate:

$$y_t = E_t y_{t+1} - \sigma(r_t - r_t^n - E_t \pi_{t+1}), \tag{2}$$

with $\sigma>0$.

The model represents a log-linear approximation of the equilibrium conditions under the assumption of a deterministic steady state. Hence, all variables are expressed in log-deviation from their long run level. The nominal short-term interest rate r_t is the instantaneous interest rate or continuously compounded interest rate and empirically could be approximated by the overnight interest rate (in the US, the Fed funds rate). Thus, if R_t is the gross nominal interest rate on a risk-free one-period bond, then $r_t = \log R_t$, given the assumption of no arbitrage opportunities and complete financial markets.

Following Bullard and Mitra (2002), we assume that the natural rate of interest r_t^n is an exogenous stochastic term that follows an AR(1) process given by

$$r_t^n = \omega r_{t-1}^n + \varepsilon_t, \tag{3}$$

where $0<\omega<1$ and ε_t is an *iid* disturbance with variance σ_ε^2 and mean zero.

We close the model by assuming that monetary policy is formulated in terms of a feedback rule for setting the nominal short-term interest rate:

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_{BR}[(\log P_t^A - \log F_t) - (\log P_{t-1}{}^A - \log F_{t-1})], \tag{4}$$

where the last term captures the intention of the central bank to stabilize “basis” risk because of the contribution that this policy might give to banking and financial stability. In the equation above, we assume that banks and other financial institutions manage risk by using futures: F_t is the price of a one-period eurodollar future contract and P_t^A is the price of the asset underlying such future, i.e. a one-period eurodollar deposit. Again, these variables are expressed in log-deviation from their long run level. In order to simplify the analysis, without affecting the results, we assume that the central bank smoothes the ratio of P_t^A over F_t, instead of the spread.

In the interbank market, banks have the possibility of switching between a one-period Eurodollar deposit - i.e. lending to another bank for a one-period horizon - and a strategy of rolling over loans in the overnight market. When banks take a long position in the interbank market they might decide whether to hedge or not their investment. If we consider a hedge put in place at time *t-1*, the hedging risk is the uncertainty associated with the spread realized at time *t* and is termed as basis risk. When the price of the asset increases by more (less) than the futures price, the basis increases (decreases). This is referred to as a strengthening (weakening) of the basis. Moreover the Libor rate is strongly influenced by the (average) overnight rate expected to prevail over one-period ahead.[6] According to the policy rule (4), the central bank is concerned about the deviation of the spread between the price ratio of the future and of the underlying asset from its past level. This concern reflects the idea that, for banks being locked in hedging positions, failure to adjust reserves in response to unexpected rate changes would have direct impact on their balance sheet and profitability. It is important to observe that the spread considered above is an ex post measure related to basis risk in a hedging situation. Hence, equation (4) explicitly assumes a central bank concern with stabilizing basis risk as a contribution to financial stability.

Let us consider a one-period futures contract. It is possible to show that the central bank by setting the short-term interest rate according to (4) affects the basis risk by smoothing the basis over time. In order to see this we introduce the (quite common) assumption that futures and forward prices are perfect substitute.[7] This implies that

$$F_t = P_t^A e^{\log R_t}. \tag{5}$$

From (5), it also follows that

$$\begin{aligned} \log P_t^A - \log F_t &= -\log R_t; \\ \log P_{t-1}^A - \log F_{t-1} &= -\log R_{t-1}. \end{aligned} \tag{6}$$

Substituting (6) back into expression (4) and using the definition of the instantaneous rate we get

$$r_t = \phi_\pi \pi_t + \phi_y y_t - \phi_{BR}(r_t - r_{t-1}). \tag{7}$$

From (7) we obtain the following policy rule

[6] Notice that the relationship between the Libor rate and the overnight rate is not exact and the differences that might arise reflect another type of basis risk, that we analyze below (see Section 3.4).

[7] See for instance Hull (2000) for a discussion on the validity of this assumption.

$$r_t = \rho r_{t-1} + \Phi_\pi \pi_t + \Phi_y y_t; \tag{8}$$

with

$$\begin{aligned} \rho &= \frac{\phi_{BR}}{1+\phi_{BR}}; \\ \Phi_\pi &= \frac{\phi_\pi}{1+\phi_{BR}}; \\ \Phi_y &= \frac{\phi_y}{1+\phi_{BR}}; \end{aligned} \tag{9}$$

where the coefficient ρ, with $0 \le \rho < 1$, measures the degree of inertia in the central bank's response to macroeconomic shocks. Notice that the rule (8) can be also rewritten as

$$r_t = \rho r_{t-1} + (1-\rho)\cdot(\phi_\pi \pi_t + \phi_y y_t),$$

where we have a partial adjustment mechanism with a convex combination between the operating target, which specifies the reaction of monetary policy to changes in macroeconomic conditions, and the lagged interest rate. This formalization represents the standard specification used in the empirical literature on inertial interest rate rules. It also makes explicit the existence of a trade off between the objective of financial stability and the one of macroeconomic stabilization.

Summing up, our analytical framework derives the partial adjustment mechanism implied by interest-rate smoothing from a Taylor-type rule augmented with a reaction to the change of the basis. From (8)-(9) it is possible to see that as $\phi_{BR} \to +\infty$ the current interest rate tends to the previous period level, and the change of the basis tends to zero.[8] Accordingly, rational agents expecting this behavior from the central bank will find the basis risk reduced to zero. Clearly, $\phi_{BR} \to +\infty$ implies monetary policy following a superinertial interest rate rule, without reaction to deviations of inflation or output from their trend level.

3.2 Determinacy of equilibrium, case I

Following Woodford (2003a), the determinacy conditions for the model constituted by (1), (2), (3), (8) and (9) should be derived from the following system

[8] Recall that all variables are expressed as log-deviations from their trend level and constants are omitted for simplicity.

$$E_t z_{t+1} = Az_t + ar_t^n , \tag{10}$$

where $z_t = [\pi_t, y_t, r_{t-1}]'$, and

$$A \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1}k & 0 \\ \sigma(\Phi_\pi - \beta^{-1}) & 1+\sigma\left(\Phi_y + \beta^{-1}k\right) & \sigma\rho \\ \Phi_\pi & \Phi_y & \rho \end{bmatrix}, \qquad a \equiv \begin{bmatrix} 0 \\ -\sigma \\ 0 \end{bmatrix}. \tag{11}$$

In (10) there is a single predetermined state variable, r_{t-1}, and two nonpredetermined state variables. Hence the equilibrium is determinate if and only if A has exactly one eigenvalue inside the unit circle and the other two eigenvalues outside the unit circle. As shown by Woodford (2003a), the necessary and sufficient condition for rational expectations equilibrium to be unique is:

$$\Phi_\pi + \frac{1-\beta}{k}\Phi_y > 1-\rho . \tag{12}$$

The condition (12) is the generalization of the basic 'Taylor principle' appropriate for the case at hand.[9] After substituting (9) in (12) we get

$$\frac{\phi_\pi}{1+\phi_{BR}} + \frac{\phi_y(1-\beta)}{(1+\phi_{BR})k} > 1 - \frac{\phi_{BR}}{\left(1+\phi_{BR}\right)}, \tag{13}$$

We can then prove the following:

Proposition 1 – *Assume that monetary policy is conducted by the central bank so as to ensure that the short-term interest rate follows a rule of the form of (4), for any fixed values* $\phi_\pi, \phi_y > 0$ *and* $\phi_{BR}, \rho = 0$. *The introduction of monetary inertia does not alleviate indeterminacy problems.*

[9] The principle that interest rate rules should respond more than one for one to changes in (expected) inflation is called 'Taylor principle'. However, Bullard and Mitra (2002) and Woodford (2003a) have shown that in general the necessary and sufficient condition required for stability may have a more complex form than that expressed by the Taylor principle. In particular it is possible to show that $\Phi_\pi > 1$ is only a sufficient condition for the determinacy of the rational expectations equilibrium, but that also values of $0 < \Phi_\pi < 1$ can be consistent with stability. However, as argued by Woodford (2003a, p. 254) the Taylor principle continues to be a crucial condition for determinacy if it is reformulated as: "[...] *At least in the long run, nominal interest rates should rise by more than the increase in the inflation rate*".

PROOF. Consider fixed values of ϕ_π and ϕ_y and assume that the central bank starts to include a reaction to futures price movements leading to interest-rate smoothing in its policy rule. Then, by multiplying both sides of inequality (13) by $(1+\phi_{BR})$ and rearranging we get

$$\phi_\pi + \frac{\phi_y(1-\beta)}{k} > 1, \tag{14}$$

which, as shown by Woodford (2003a), is the same condition for determinacy when $\phi_{BR} = 0$ *and* $\rho = 0$. QED.

Woodford (2003a) and Bullard and Mitra (2005) have shown that monetary inertia may help alleviate indeterminacy problems. In our setting this result does not hold. The rationale for the difference is related to our choice of modelling explicitly the financial-stability determinants of the interest-rate smoothing term, while in the existing literature this term is simply added to the central bank's reaction function. Our modelling choice implies that smoothing the policy rate reduces at the same time the response to inflation and output, as it is possible to see from expressions (9), thereby offsetting the improvement in the requirement (12) due to the introduction of monetary inertia.

3.3Determinacy of equilibrium, case II

Let's assume that in the central bank's interest rate rule we have a response to expected future inflation instead of current inflation. Hence we replace equation (4) with:

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_y y_t + \phi_{BR}[(\log P_t^A - \log F_t) - (\log P_{t-1}{}^A - \log F_{t-1})]. \tag{15}$$

Now the matrix A is given by

$$A \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1}k & 0 \\ \sigma\beta^{-1}(\Phi_\pi - 1) & 1+\sigma[\Phi_y - \beta^{-1}k(\Phi_\pi - 1)] & \sigma\rho \\ \beta^{-1}\Phi_\pi & \Phi_y - \beta^{-1}k\Phi_\pi & \rho \end{bmatrix}, \qquad a \equiv \begin{bmatrix} 0 \\ -\sigma \\ 0 \end{bmatrix}. \tag{16}$$

As shown by Woodford (2003a), in this case the necessary and sufficient conditions for rational expectations equilibrium to be unique are (12) and

$$\Phi_{\pi} < 1+\rho+\frac{1+\beta}{k}[\Phi_{y}+2\sigma^{-1}(1+\rho)]. \tag{17}$$

After substituting (9) in conditions (12) and (17) we get (13) and

$$\frac{\phi_{\pi}}{1+\phi_{BR}} < 1+\frac{\phi_{BR}}{(1+\phi_{BR})}+\frac{1+\beta}{k}\left[\frac{\phi_{y}}{1+\phi_{BR}}+2\sigma^{-1}\left(1+\frac{\phi_{BR}}{(1+\phi_{BR})}\right)\right]. \tag{18}$$

We can then prove the following:

Proposition 2 – *Assume that monetary policy is conducted by the central bank so as to ensure that the short-term interest rate follows a rule of the form of (15), for any fixed values* $\phi_{\pi}, \phi_{y} > 0$ *and* $\phi_{BR}, \rho = 0$. *The introduction of monetary inertia does not alleviate indeterminacy problems, but for the special case of an excessively aggressive response to inflation and output.*

PROOF: Consider fixed values of ϕ_{π} and ϕ_{y} and assume that the central bank starts to include a reaction to futures price movements leading to interest-rate smoothing in its policy rule. Then, multiplying both sides of equation (12) by $(1+\phi_{BR})$ you still get equation (14) (which is the same with $\phi_{BR}=0$ *and* $\rho=0$), while multiplying both sides of inequality (18) by $(1+\phi_{BR})$ and rearranging we get

$$\phi_{\pi} < 1+2\phi_{BR}+\frac{1+\beta}{k}\left[\phi_{y}+2\sigma^{-1}(1+2\phi_{BR})\right] \tag{19}$$

This clearly shows that condition (18) for determinacy of equilibrium is less binding as $\phi_{BR} \to +\infty$ and $\rho \to 1$. QED

While Bullard and Mitra (2002) have shown that an excessively aggressive response of monetary policy to inflation and output leads to indeterminacy, Proposition 2 implies that, in such cases, if the central bank displays sufficient inertia this may actually help alleviate problems of indeterminacy.

3.4 An alternative definition of basis risk

Thus far, we have assumed that the interest rate rule is augmented with a term that captures the intention of the central bank to stabilize one possible source of basis risk, namely the differences that might occur between the price of the underlying asset to be hedged – e.g. the eurodollar deposit - and the price of the future contract used – i.e. a eurodollar future contract. Here we consider a second type of basis risk (see Hull, 2000), which is related to the differences that might arise between the Libor rate and the average overnight rate in a hedging situation.

The one-period eurodollar future rate, R_t^{EF}, can be expressed as the sum of the expected future level of the underlying interest rate, i.e. the one-period eurodollar Libor rate R_{t+1}^E, and a risk premium as follows

$$R_t^{EF} = E_t R_{t+1}^E + \theta_t , \tag{20}$$

where θ_t is the risk premium. As shown in Sack (2004) definition (20) can be modified to express the expectations in terms of the federal funds rate rather than the Libor rate as follows

$$R_t^{EF} = E_t \bar{r}_{t,t+1} + E_t \left(R_{t+1}^E - \bar{r}_{t,t+1} \right) + \theta_t , \tag{21}$$

where $\bar{r}_{t,t+1}$ is the average of the daily Fed funds rates from t to t+1, when the futures contract is expiring.[10] As the relationship between the Libor rate and the overnight rate is not exact, the term $E_t \left(R_{t+1}^E - \bar{r}_{t,t+1} \right)$ reflects another type of basis risk. The excess expected return of the Libor rate over the average overnight rate will typically be positive, reflecting that for a bank lending to another bank for a one-period horizon (1 month or 3 months) implies a greater credit risk than lending on an overnight basis. Using the Expectations Hypothesis we can rewrite the expression for the basis risk as

$$E_t \left(R_{t+1}^E - R_t^E \right). \tag{22}$$

[10] Here for convenience we assume that the average is over the entire horizon of the futures contract. However, more correctly, the average should be referred only over the delivery period of the futures contract (i.e. near the expiration: for example for a three-month eurodollar futures contract we should take the average of the daily Fed funds rates over the delivery month).

Now again we can think of a central bank trying to stabilise this type of basis risk and insert the term (22) in the interest rate rule as follows

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$$r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_{BR} E_t (\log R_{t+1}^E - \log R_t^E) \quad (23)$$

Again, these variables are expressed in log-deviation from their long run level and. We continue to assume that the central bank stabilises the ratio of R_{t+1}^E over R_t^E, instead of the spread. It is important to observe that the spread considered above is an ex ante measure related to basis risk in a hedging situation. After substituting with the Fed fund rate and collecting terms we get

$$r_t = \rho E_t r_{t+1} + \Phi_\pi \pi_t + \Phi_y y_t , \quad (24)$$

with expressions (9) holding also in this case.

3.5 Determinacy of Equilibrium with forward interest rate smoothing.

The determinacy conditions for the model constituted by (1), (2), (3), (24) and (9) should be derived from the following system

$$E_t z_{t+1} = A z_t + a r_t^n ,$$

where $z_t = [\pi_t , y_t , r_t]'$, and

$$A \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1} k & 0 \\ -\sigma\beta^{-1} & 1+\sigma\beta^{-1} k & \sigma \\ -\rho^{-1}\Phi_\pi & -\rho^{-1}\Phi_y & \rho^{-1} \end{bmatrix}, \qquad a \equiv \begin{bmatrix} 0 \\ -\sigma \\ 0 \end{bmatrix}. \quad (25)$$

Notice that we now have three nonpredetermined state variables. Hence the equilibrium is determinate if and only if *A* has exactly three eigenvalues outside the unit circle. In Appendix A, we derive the necessary and sufficient conditions for the rational expectations equilibrium to be unique under such a circumstance. In the same Appendix, we also show that in our system, such conditions are the following:

$$\phi_\pi + \frac{\phi_y(1-\beta)}{k} > 1, \tag{26}$$

$$\frac{-\sigma k(1-\beta)\phi_{BR}^2 - \left(\Delta + \Lambda\sigma k\phi_\pi + \Omega\sigma\phi_y\right)\phi_{BR} + \Psi(\Psi-\beta)}{\phi_{BR}^2\beta^2} > 0, \tag{27}$$

with

$$\begin{aligned}
&\Lambda \equiv \sigma k - 1 + 2\beta;\\
&\Omega \equiv \Lambda - \beta^2;\\
&\Delta \equiv (1-\beta)\cdot(\Lambda-\beta);\\
&\Psi \equiv \sigma\phi_y + \sigma k\phi_\pi + 1.
\end{aligned} \tag{28}$$

We can then prove the following:

Proposition 3 – *Assume that monetary policy is conducted by the central bank so as to ensure that the short-term interest rate follows a rule of the form of (23), for any fixed values* $\phi_\pi, \phi_y > 0$ *and* $\phi_{BR}, \rho = 0$. *The introduction of a large degree of monetary inertia, with* $\phi_{BR} \to +\infty$ *and* $\rho \to 1$, *leads to indeterminacy.*

PROOF. Consider fixed values of ϕ_π and ϕ_y and assume that the central bank starts to include a reaction to the expected future interest rate. Condition (26) is not affected. While it is possible to see that as $\phi_{BR} \to +\infty$ and $\rho \to 1$ condition (27) tends to $-\sigma k(1-\beta)/\beta^2 < 0$. QED.

Proposition 3 implies that for monetary policy there may exist a trade-off between macroeconomic stability and financial stability. Forward-looking interest-rate smoothing, reflecting the introduction of a basis risk stabilization motive linked to maintain financial stability of the banking sector, can reduce the ability of achieving macroeconomic stability. In particular, a large value of ϕ_{BR} can compromise the achievement of macroeconomic stability by creating indeterminacy of the equilibrium, when such indeterminacy was not otherwise at stake. Clearly, the existence of this possible trade off between macroeconomic stability and financial stability calls for caution, but does not necessarily imply that the central bank cannot pursue the stabilization of the basis risk.[11]

[11] Determinacy conditions for interest rate rules responding to expected future inflation could also be obtained. Although more cumbersome to derive, they imply the same results of Proposition 3.

3.6 A Remark on Optimality of backward and forward interest rate smoothing.

In this section we have provided a financial stability – or more exactly a basis risk stabilization – motivation for including backward or forward interest-rate smoothing terms in the central bank's interest rate rule. Under some restrictive assumptions which are necessary to solve analytically the model, however, it is also possible to show that an interest rate rule with both backward and forward smoothing can be optimally derived in the framework used by Woodford (2003b) for examining monetary policy under a discretionary regime when the central bank has a (backward) interest-rate smoothing objective. In Appendix B, we explicitly derive such interest rate rule.

4. Empirical evidence

In this section, we make an empirical assessment of the relevance of the expected future rate in the policy rule of the Federal Reserve.

We start by estimating standard interest rate rules with inertia (backward smoothing) for the period from 1987-Q4 to 2005-Q3. In particular, we estimate the following baseline interest rate rules:

$$r_t = \rho_1 r_{t-1} + (1 - \rho_1 - \rho_2)\bar{r}_t + \rho_2 r_{t-2} + \theta_t, \tag{29}$$

and

$$\bar{r}_t = \phi + \phi_\pi E_t \pi_t + \phi_y E_t y_t, \tag{30}$$

or

$$\bar{r}_t = \phi + \phi_\pi E_t \pi_{t+1} + \phi_y E_t y_t. \tag{31}$$

Where $\bar{r}_t$ is an operational target. The estimation approach used is based on the Generalized Method of Moments (GMM) and is the same as that of Clarida, Galì and Gertler (2000) for the case of the Federal Reserve.[12] The second-order partial adjustment mechanism modeled in the specification is intended to capture the degree of monetary inertia by the Fed.

[12] The econometric approach used relies on the assumption that, within our short sample, short term interest rates, inflation and output gap are I(0). However, standard Dickey-Fuller test of the null that the above series are I(1) is not rejected for the US. Nevertheless, as argued for instance by Clarida, Galì and Gertler (1998), standard Dickey-Fuller test has lower power against the alternative of stationarity for short samples. For this reason the assumption of stationary series is standard in the empirical literature of interest rate rules, as this literature is in general based on short samples with a stable monetary regime like in our case.

The data used are the Federal funds interest rate, defined as the average effective Federal funds rate over the quarter, the output gap, defined as percent deviation of actual real GDP from the potential output estimated by the Congressional Budget Office, and inflation, measured as four-quarter change in the GDP deflator.[13] We have used a correction for heteroskedasticity and autocorrelation of unknown form with a Newey-West fixed bandwidth, and chosen Bartlett weights to ensure positive definiteness of the estimated variance-covariance matrix.[14] The instrument set includes the constant and 1-4 lagged values of output gap, inflation and the federal funds rate.[15]

In columns 1 and 3 of table 1 we report the estimates obtained respectively for the backward-looking specification (29) and (30) and the forward looking specification (29) and (31).

Next, consistently with the analysis developed in section 3.4, we include the expected future rate in the interest rate rules considered (backward and forward interest rate smoothing). We then estimate:

$$r_t = \rho_1 r_{t-1} + (1 - \rho_1 - \rho_2)\bar{r}_t + \rho_2 r_{t-2} + \mu E_t r_{t+i} + \theta_t, \qquad (32)$$

where either (30) or (31) holds. In the present analysis we have considered the expected future interest rate two quarters ahead.[16] Rudd and Whelan (2005) show that in tests of the new-Keynesian Phillips curve GMM implies biased estimates when the instruments used belong to the true model of inflation.[17] Thus similarly to the case of the New Keynesian Phillips curve we need to provide additional instruments that can be assumed to be not included in the interest rate specification.[18] We choose as instruments for the expected future interest rate 1-4 lagged values of the rate on a eurodollar futures contract that settles three months ahead, taken as average rate over the quarter.

As it is possible to see from the estimations reported in columns 2 and 4 of table 1, the introduction of the expected future interest rate improves substantially the goodness of fit of the estimated policy

[13] Data on the Fed funds rate, output gap and inflation are taken from FRED, of the Federal Reserve Bank of St. Louis.

[14] The optimal weighting matrix is obtained from first-step Two-Stage Least Squares (2SLS) parameter estimates.

[15] The J-test reported in the tables is the test for the validity of the instruments used. The associated statistic is distributed as a χ^2.

[16] We have considered also other forecasting horizons for the future interest rate, but for simplicity we have reported only the case with the best fitting and diagnostic outcomes. In particular, if we include the expected future interest rate one quarter ahead (not reported in the table) the goodness of fit improves compared to the baseline case without the future interest rate, but the response to inflation and output becomes not significant. While with forecasting horizons greater than two quarters ahead the goodness of fit worsens.

[17] See also the debate on the remedies proposed for solving this problem: Lindé (2005) and Rudd and Whelan (2007).

[18] As discussed by Rudd and Whelan (2005) the chosen instruments should also not be correlated with erroneously omitted variables in the interest rate specification.

rules. The coefficient of the expected future interest rate is statistically significant (at the 1 percent level) and positive, as expected. As it is possible to observe, the introduction of the expected future interest rate reduces the response to inflation and output.

Overall, our empirical evidence supports the presence of the expected future interest rate as an additional argument of the Fed's interest rate rule.

5. Conclusions and future research

This paper provides a first attempt to link banks' risk management practices and interest rate policy decisions by central banks who care about monetary and financial stability. We assume that the central bank is aware that financial institutions may hedge against the risk of sharp interest rate movements by using derivatives. However, part of this risk, namely "basis" risk, cannot be hedged and hence provides a limited motivation for central banks to stabilize it as a contribution to financial stability. We show that the desire to stabilize basis risk leads central banks to smooth interest rates, either backward or forward. Backward interest rate smoothing does not in general alleviate, as previously believed, indeterminacy problems. Moreover, under forward smoothing, it is possible that new indeterminacy problems are induced by large degrees of monetary inertia. Hence, a trade off between monetary and financial stability emerges and may suggest to assign to central banks a preferred target in terms of price stability, while mainly pursuing financial stability through proper regulation and supervision of financial markets and intermediaries.

Our estimates of different interest rate rules suggest that embedding backward and forward interest rate smoothing allows to improve the econometric specification and provides a better explanation of the conduct of the Federal Reserve in our sample.

Our results are derived under particular but plausible assumptions about banks' hedging behaviour and an alternative modelling of the reaction function of the central bank. We are currently investigating whether such results may hold more in general and under different definitions and operating specifications of the central bank concern for financial stability.

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Appendix A

In this Appendix, we derive the necessary and sufficient conditions for uniqueness of the rational expectations equilibrium in the case in which there are three non predetermined state variables. To our knowledge, these conditions have not been explicitly derived elsewhere. We follow the same approach used by Woodford (2003a) in Appendix C and consider a linear rational-expectations model of the general form

$$E_t z_{t+1} = Az_t + as_t, \tag{A1}$$

where z_t is a 3-vector of nonpredetermined endogenous state variables, s_t is a vector of exogenous disturbance terms, and A is a 3×3 matrix. Rational expectations equilibrium is determinate if and only if the matrix A has exactly three eigenvalues outside the unit circle (i.e. with modulus $|\gamma| > 1$).[19] The characteristic polynomial is given by

$$P(\gamma) \equiv \gamma^3 + A_2\gamma^2 + A_1\gamma + A_0 = 0. \tag{A2}$$

We will now prove the following:

Proposition A1 - *The equation (A2) has three roots outside the unit circle if and only if:*

either (Case I)

$$P(1) = 1 + A_2 + A_1 + A_0 > 0; \tag{A3}$$

$$P(-1) = -1 + A_2 - A_1 + A_0 > 0; \tag{A4}$$

$$A_0 > 0; \tag{A5}$$

$$\left|\frac{1}{A_0}\right| < 1; \tag{A6}$$

$$A_0^2 - A_0 A_2 + A_1 - 1 > 0; \tag{A7}$$

or (Case II)

$$P(1) = 1 + A_2 + A_1 + A_0 < 0; \tag{A8}$$

$$P(-1) = -1 + A_2 - A_1 + A_0 < 0; \tag{A9}$$

$$A_0 < 0; \tag{A10}$$

and (A6) and (A7) hold in addition.

[19] We follow the conventional analysis of determinacy provided by Blanchard and Kahn (1980).

PROOF. We start by showing that the two cases listed are sufficient conditions for determinacy. It is well known that the following relationships between the coefficients of the characteristic polynomial and the eigenvalues hold:

$$\begin{aligned} A_0 &= -\gamma_1\gamma_2\gamma_3; \\ A_1 &= \gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_2\gamma_3; \\ A_2 &= -\gamma_1 - \gamma_2 - \gamma_3. \end{aligned} \tag{A11}$$

Notice that $P(\gamma) > 0$ for sufficiently large, positive γ and $P(\gamma) < 0$ for sufficiently large, negative γ, as the γ^3 term dominates the other terms. Conditions (A3)-(A4) in Case I, or conditions (A8)-(A9) in Case II, imply that when all three roots are real there is an even number of roots between –1 and 1, i.e. inside the unit circle (this even number may be zero) and an odd number of roots outside the unit circle. Requirements (A5), in case I, or (A10), in case II, implies that when there are two roots inside the unit circle these roots have the same sign, with either $-1 < \gamma_2, \gamma_3 < 0$ or $0 < \gamma_2, \gamma_3 < 1$. While, when one root is real and two roots are a complex pair, conditions (A3)-(A4), in Case I, or conditions (A8)-(A9), in Case II, imply that the real root is outside the unit circle.

From the definition of A_0 in (A11) we can see that condition (A6), in both Case I and Case II, is always satisfied if the three roots have modulus greater than one. Condition (A6) can be satisfied also in the case of one real root outside the unit circle, say with $|\gamma_1| > 1$, and two real roots inside the unit circle. However, in this latter case condition (A6) holds if and only if $|\gamma_1 \cdot \gamma_2| > 1$ and $|\gamma_1 \cdot \gamma_3| > 1$.

After substituting expressions (A11) in condition (A7) we get (see Woodford 2003a):

$$(\gamma_1\gamma_2 - 1)\cdot(\gamma_1\gamma_3 - 1)\cdot(\gamma_2\gamma_3 - 1) > 0\,. \tag{A12}$$

This condition implies that an odd number of the three products of roots are real numbers greater than one. It also implies, in the case of real roots that all roots lie outside the unit circle. In fact, one can show that if we have two real roots γ_2 and γ_3 inside the unit circle, in which case their signs are the same (as ensured by requirement (A5) or (A10)) and we have that $|\gamma_1 \cdot \gamma_2| > 1$ and $|\gamma_1 \cdot \gamma_3| > 1$ (as ensured by requirement (A6)), condition (A12) is violated. In the case of two roots that are complex conjugates, say γ_2 and γ_3, then only the product $\gamma_2 \cdot \gamma_3$ is a real number; while the products $\gamma_1 \cdot \gamma_2$ and $\gamma_1 \cdot \gamma_3$ are complex numbers. This implies that $(\gamma_1\gamma_2 - 1)\cdot(\gamma_1\gamma_3 - 1) > 0$, so that condition (A12) holds if and only if $\gamma_2 \cdot \gamma_3 > 1$.

All these conditions are sufficient for determinacy. In the case there exist one real root and a complex pair, conditions (A3)-(A4), in Case I, or conditions (A8)-(A9), in Case II, imply that the real root is outside the unit circle, with $|\gamma_1| > 1$; while condition (A7) implies that the complex pair must satisfy $|\gamma_2| = |\gamma_3| > 1$. Hence we have determinacy. Alternatively, if there exist three real roots conditions (A3)-(A4), in Case I, and conditions (A8)-(A9), in Case II, imply that when all three roots are real there is an even number of roots inside the unit circle and an odd number of roots outside the unit circle. If conditions (A5) in Case I [or (A10), in Case II], and (A6)-(A7) are satisfied the even number of roots inside the unit circle is zero and the odd number of roots outside the unit circle is three. Hence we again have determinacy.

Now we show that the two cases listed are necessary conditions for determinacy as well. Let's consider first the possibility that there exist three real roots. As for determinacy we must have that all roots lie outside the unit circle, the roots must satisfy one of the following conditions:

(a) $\gamma_1, \gamma_2 < -1 < 0 < 1 < \gamma_3,$

(b) $\gamma_1, \gamma_2, \gamma_3 < -1,$

(c) $\gamma_1 < -1 < 0 < 1 < \gamma_2, \gamma_3,$

(d) $1 < \gamma_1, \gamma_2, \gamma_3.$

Cases (a) and (d) require that *P(-1)* < 0, *P(1)* < 0 and $A_0 < 0$, so that conditions (A8)-(A10) required in Case II are satisfied. Cases (b) and (c) imply instead that *P(-1)* > 0, *P(1)* > 0 and $A_0 > 0$, so that conditions (A3)-(A5) required in Case I are satisfied.

It is possible to see that there are other conditions for the roots that satisfy conditions (A8)-(A10) or (A3)-(A5) as well:

(e) $\gamma_1 < -1 < \gamma_2, \gamma_3 < 0 < 1,$

(f) $\gamma_1 < -1 < 0 < \gamma_2, \gamma_3 < 1,$

(g) $-1 < \gamma_1, \gamma_2 < 0 < 1 < \gamma_3,$

(h) $-1 < 0 < \gamma_1, \gamma_2 < 1 < \gamma_3.$

Where cases (e) and (f) satisfy requirements (A3)-(A5), while cases (g) and (h) satisfy requirements (A8)-(A10).

Moreover it is possible to see that the following conditions for the roots satisfy requirements (A8)-(A7) or (A3)-(A4) as well, but without satisfying requirement (A5) or (A10):

(i) $$-1<\gamma_1<0<\gamma_2<1<\gamma_3,$$

(j) $$\gamma_1<-1<\gamma_2<0<\gamma_3<1.$$

Where case (i) requires that *P(-1)* < 0, *P(1)* < 0 and $A_0>0$, and case (j) implies instead that *P(-1)* > 0, *P(1)* > 0 and $A_0<0$.

Hence we need to show that requirement (A6) is necessary for eliminating cases (e)-(h), while requirements (A5) and (A10) are necessary for eliminating cases (i) and (j). In cases (a)-(d) we have the following conditions for the products of the roots:

(a1) $$\gamma_1\cdot\gamma_3,\gamma_2\cdot\gamma_3<-1<0<1<\gamma_1\gamma_2,$$

(b1) $$1<\gamma_1\cdot\gamma_2,\gamma_2\cdot\gamma_3,\gamma_1\cdot\gamma_3,$$

(c1) $$\gamma_1\cdot\gamma_2,\gamma_1\cdot\gamma_3<-1<0<1<\gamma_2\cdot\gamma_3,$$

(d1) $$1<\gamma_1\cdot\gamma_2,\gamma_2\cdot\gamma_3,\gamma_1\cdot\gamma_3.$$

Observe that in cases (a)-(d) we have an odd number of products that are greater than one and the inequality (A12) is satisfied. Thus these cases satisfy (A7).

In cases (e)-(h) we have the following conditions for the products of the roots:

(e1) $$0<\gamma_2\cdot\gamma_3,\gamma_1\cdot\gamma_2,\gamma_1\cdot\gamma_3<1,$$

(e2) $$0<\gamma_2\cdot\gamma_3,\gamma_1\cdot\gamma_3<1<\gamma_1\cdot\gamma_2,$$

(e3) $$0<\gamma_2\cdot\gamma_3,\gamma_1\cdot\gamma_2<1<\gamma_1\cdot\gamma_3,$$

(e4) $$0<\gamma_2\cdot\gamma_3<1<\gamma_1\cdot\gamma_2,\gamma_1\cdot\gamma_3,$$

(f1) $$\gamma_1\cdot\gamma_2,\gamma_1\cdot\gamma_3<-1<0<\gamma_2\cdot\gamma_3<1,$$

(f2) $$\gamma_1\cdot\gamma_2<-1<\gamma_1\cdot\gamma_3<0<\gamma_2\cdot\gamma_3<1,$$

(f3) $$\gamma_1\cdot\gamma_3<-1<\gamma_1\cdot\gamma_2<0<\gamma_2\cdot\gamma_3<1,$$

(f4) $$-1<\gamma_1\cdot\gamma_2,\gamma_1\cdot\gamma_3<0<\gamma_2\cdot\gamma_3<1,$$

(g1) $$-1<\gamma_3\cdot\gamma_1,\gamma_3\cdot\gamma_2<0<\gamma_1\cdot\gamma_2<1,$$

(g2) $$\gamma_3\cdot\gamma_1<-1<\gamma_3\cdot\gamma_2<0<\gamma_1\cdot\gamma_2<1,$$

(g3) $\gamma_3 \cdot \gamma_2 < -1 < \gamma_3 \cdot \gamma_1 < 0 < \gamma_1 \cdot \gamma_2 < 1,$

(g4) $\gamma_3 \cdot \gamma_1, \gamma_3 \cdot \gamma_2 < -1 < 0 < \gamma_1 \cdot \gamma_2 < 1,$

(h1) $0 < \gamma_1 \cdot \gamma_2, \gamma_1 \cdot \gamma_3, \gamma_2 \cdot \gamma_3 < 1,$

(h2) $0 < \gamma_1 \cdot \gamma_2, \gamma_1 \cdot \gamma_3 < 1 < \gamma_2 \cdot \gamma_3,$

(h3) $0 < \gamma_1 \cdot \gamma_2, \gamma_2 \cdot \gamma_3 < 1 < \gamma_1 \cdot \gamma_3,$

(h4) $0 < \gamma_1 \cdot \gamma_2 < 1 < \gamma_2 \cdot \gamma_3, \gamma_1 \cdot \gamma_3.$

As it is possible to see in cases (e2), (e3), (h2) and (h3) we have an odd number of products that are greater than one and the inequality (A12) is satisfied. Thus these cases satisfy (A7) as well.

In order to eliminate cases (e2), (e3), (h2) and (h3) we impose the requirement (A6), which is always satisfied in cases (a)-(d). It is straightforward to show that in case (e) condition (A6) implies that we can have only condition (e4) for the products of the roots, while in case (h) condition (A6) implies that we can have only condition (h4) for the products of the roots. Thus the requirement (A6) rules out the cases (e2), (e3), (h2) and (h3).

In cases (i)-(j) we have the following conditions for the products of the roots:

(i1) $-1 < \gamma_1 \cdot \gamma_2, \gamma_1 \cdot \gamma_3 < 0 < \gamma_2 \cdot \gamma_3 < 1,$

(i2) $\gamma_1 \cdot \gamma_3 < -1 < \gamma_1 \cdot \gamma_2 < 0 < 1 < \gamma_2 \cdot \gamma_3,$

(i3) $-1 < \gamma_1 \cdot \gamma_2, \gamma_1 \cdot \gamma_3 < 0 < 1 < \gamma_2 \cdot \gamma_3,$

(i4) $\gamma_1 \cdot \gamma_3 < -1 < \gamma_1 \cdot \gamma_2 < 0 < \gamma_2 \cdot \gamma_3 < 1,$

(j1) $-1 < \gamma_2 \cdot \gamma_3, \gamma_1 \cdot \gamma_3 < 0 < \gamma_1, \gamma_2 < 1,$

(j2) $\gamma_1 \cdot \gamma_3 < -1 < \gamma_2 \cdot \gamma_3 < 0 < 1 < \gamma_1, \gamma_2,$

(j3) $-1 < \gamma_2 \cdot \gamma_3, \gamma_1 \cdot \gamma_3 < 0 < 1 < \gamma_1, \gamma_2,$

(j4) $\gamma_1 \cdot \gamma_3 < -1 < \gamma_2 \cdot \gamma_3 < 0 < \gamma_1, \gamma_2 < 1.$

As it is possible to see in cases (i2), (i3), (j2) and (j3) we have an odd number of products that are greater than one and the inequality (A12) is satisfied. Thus these cases satisfy (A7) as well. Unfortunately, here we cannot use requirement (A6) in order to rule out cases (i2), (i3), (j2) and (j3). In fact in case (i) condition (A6) implies that we can have only condition (i2) for the products of the roots, while in case (j) condition (A6) implies that we can have only condition (j2) for the products of the roots. In order to eliminate cases (i2), (i3), (j2) and (j3) we impose the requirements (A5) and (A10) for Case I and Case II respectively, which are always satisfied in cases (a)-(d). It is straightforward to see that requirements (A5) – in Case I - and (A10) – in Case II - ensure that in the case of two real roots γ_2 and γ_3 inside the unit circle their signs are the same. Thus we have eliminated the possibility of having cases (i) and (j).

We conclude that if there are three real roots, any determinate case must satisfy one of the two sets of conditions stated in the proposition A1.

Finally consider the possibility that there exist one real root and a complex pair. In this case determinacy requires that the real root lies outside the unit circle, while the complex pair has a modulus greater than one. In this case we must have either *P(-1)* > 0 and *P(1)* > 0 or *P(-1)* < 0 and *P(1)* < 0. Thus requirements (A3)-(A4) or (A8)-(A9) are satisfied. Moreover, as discussed in Woodford (2003a), condition (A7) ensures that the complex pair has a modulus greater than one. Finally, observe that requirements (A6) and (A5) in Case I [or (A6) and (A10), in Case II], are not necessary but are nevertheless satisfied when the real root lies outside the unit circle and the complex pair has a modulus greater than one.

In conclusion we have shown that every determinate case satisfies one of the sets of conditions stated in the proposition A1. QED.

Now, let's consider the case of the alternative definition of basis risk, studied in the text. The characteristic polynomial of matrix A, given by expression (25), is

$$P(\gamma) \equiv \gamma^3 + A_2\gamma^2 + A_1\gamma + A_0 = 0. \tag{A13}$$

where

$$\begin{aligned} A_0 &= -\frac{1+\sigma\Phi_y + \sigma k\Phi_\pi}{\beta\rho} < 0, \\ A_1 &= \frac{1+\sigma k + \rho + \beta + \sigma\beta\Phi_y}{\beta\rho} > 0, \\ A_2 &= -\frac{\rho + \beta + \beta\rho + \sigma k\rho}{\beta\rho} < 0. \end{aligned} \tag{A14}$$

Given the signs of the coefficients of the characteristic polynomial, one can immediately see that we can exclude Case I of Proposition A1. Thus in the present case the requirements for determinacy are given by (A6), (A7), (A8), (A9) and (A10). As it is possible to observe conditions (A9) and (A10) are satisfied, as we have $A_0 < 0$ and *P(-1)* < 0. Condition (A8) implies that

$$\Phi_\pi + \frac{1-\beta}{k}\Phi_y > 1 - \rho\,. \tag{A15}$$

After substitution of

$$\rho = \frac{\phi_{BR}}{1 + \phi_{BR}};$$
$$\Phi_{\pi} = \frac{\phi_{\pi}}{1 + \phi_{BR}}; \qquad \text{(A16)}$$
$$\Phi_{y} = \frac{\phi_{y}}{1 + \phi_{BR}};$$

in expression (B3) we get

$$\phi_{\pi} + \frac{\phi_y(1-\beta)}{k} > 1, \qquad \text{(A17)}$$

which is expression (26) reported in the text. Now as concerns requirement (A6) we can see that it is always satisfied for $0<\beta<1$ and $0<\rho<1$. Finally, from requirement (A7), after substituting for the definitions (A16), we get expression (27) reported in the text.

Appendix B

Consider equations (1) and (2) in the text. Woodford (2003b) adds two additional elements to the macroeconomic model, the loss functions of the society and of the central bank. The society's loss function is given by

$$L_t = \pi_t^2 + \lambda_y y_t^2 + \lambda_r r_t^2, \tag{B1}$$

while that of the central bank is given by

$$L_t^{cb} = \pi_t^2 + \hat{\lambda}_y y_t^2 + \hat{\lambda}_r r_t^2 + \lambda_\Delta (r_t - r_{t-1})^2, \tag{B2}$$

where there is a term that penalizes interest-rate changes, not present in the true social loss function. Now, consider the limiting case of $k = 0$ in equation (1) and set $\hat{\lambda}_y = \lambda_y$.

Woodford derives the following first-order condition:

$$\lambda_y (y - \sigma) y_t + \hat{\lambda}_r r_t + \lambda_\Delta (r_t - r_{t-1}) + \beta \lambda_\Delta (r_t - E_t r_{t+1}) = 0 \tag{B3}$$

with

$$y = -\frac{(\beta^{-1} - \mu_1)\lambda_r}{\sigma \lambda_y} < 0, \tag{B4}$$

where μ_1, with $0 < \mu_1 < 1 < \beta^{-1} < \mu_2$, is one of the two solutions of the following quadratic equation

$$\mu^2 - \left(1 + \beta^{-1} + \frac{\sigma^2 \lambda_y}{\lambda_r}\right)\mu + \beta^{-1} = 0 .$$

Woodford also shows that the optimal values of $\hat{\lambda}_r$ and λ_Δ are given by

$$\begin{aligned} \hat{\lambda}_r &= -(1 - \beta\omega) \cdot (1 - \beta\mu_1)\lambda_\Delta < 0; \\ \lambda_\Delta &= \lambda_r \frac{\lambda_r (\beta^{-1} - \mu_1) + \lambda_y \sigma^2}{(1 - \beta\omega\mu_1)\beta\lambda_y\sigma^2} > 0. \end{aligned} \tag{B5}$$

It is now straightforward to solve for the current interest rate in expression (B3) in order to get

$$r_t = \frac{\lambda_\Delta}{(1+\beta)\lambda_\Delta + \hat{\lambda}_r} r_{t-1} + \frac{\beta\lambda_\Delta}{(1+\beta)\lambda_\Delta + \hat{\lambda}_r} E_t r_{t+1} + \frac{\lambda_y(\sigma - y)}{(1+\beta)\lambda_\Delta + \hat{\lambda}_r} y_t . \tag{B6}$$

Expression (B6) is an optimal interest rate rule for the case at hand, which features backward and forward interest-rate smoothing.[20]

[20] Notice that the fraction terms are all positive if $\left[(1+\beta)\lambda_\Delta + \hat{\lambda}_r\right] > 0$, which holds under the calibrated parameter values considered in Woodford (2003b).

Table 1 - Estimation of the Federal Reserve's interest rate rule

	Backward-looking specification		Forward-looking specification	
	Standard inertial rule	*With expected future rate*	*Standard inertial rule*	*With expected future rate*
	(1)	(2)	(3)	(4)
ρ_1	1.40	0.92	1.41	0.95
	20.59	*15.27*	*20.74*	*17.16*
Φ	1.77	-0.55	1.06	-0.72
	1.50	*-1.98*	*0.63*	*-1.82*
Φ_π	1.70	0.36	1.81	0.34
	3.21	*3.85*	*2.59*	*2.65*
Φ_y	1.29	0.25	1.30	0.21
	4.87	*3.29*	*4.39*	*2.45*
ρ_2	-0.47	-0.19	-0.48	-0.20
	-7.69	*-4.44*	*-7.37*	*-4.82*
μ		0.26		0.25
		8.51		*8.80*
Adj. R^2	0.98	0.99	0.98	0.99
S.E.	0.34	0.25	0.33	0.26
J-statistic	0.62	0.91	0.69	0.84

Notes : GMM estimation for the period 1987Q4 - 2005Q3. Adjusted R squared, regression standard error and the p-value of the J-statistic for overidentifying restrictions are reported at the bottom of the table. Robust T-statistic in italics.