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Abstract This paper explores the role of the composition of public consumption within a three sector R&D growth model. A competitive industry supplies a homogeneous good and a monopolistic sector manufactures a composite commodity differentiated in many varieties, whose size can be increased through investment in R&D. We investigate the effects of changes in the level and in the composition of public consumption on the steady state and on the economy's transitional dynamics. By varying the aggregate composition of demand, the government can effectively move resources away from traditional industry to foster innovation. Welfare effects are also evaluated. We show that the composition of government consumption affects the entire time path of utility. *JEL Classification:* J24, O31, O41

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1 Introduction

Since the late 1980's, research on the determinants of economic growth has led to a renewed attention, both theoretical and empirical, to the role of the state within the economy. To investigate how the government can affect the long run outcome of the economy, endogenous growth literature has specialized the notion of public spending, either emphasizing the role of the composition of government spending, basically distinguished between public consumption and public investment (Barro, 1990; Turnovsky and Fisher, 1995; Devarajan et al, 1996; Chen, 2006) or considering specific functional categories of expenditure, such as infrastructure (Turnovsky, 1996; Judd, 1999), education (Lucas, 1988; Glomm, 1997; Fisher and Keuschnigg, 2002), or health (Bloom et al, 2001). The emphasis on supply side effects of fiscal policy has led this literature to dismiss the view of public expenditure as a demand tool. Nevertheless, in the class of *R&D* based growth models, first proposed by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), combining increasing returns with imperfect competition, the intensity of the research is potentially responsive to changes in demand, possibly driven by an expansionary fiscal policy.

In this paper we analyze the role of demand and its composition within the mechanics of a growth model driven by research activity. In particular, we focus on the role of the composition of public consumption. Indeed, this specific notion of public expenditure composition has not been employed in long run analysis yet. To this purpose, we develop a simple model of technological change with an expanding variety of consumer products. We extend the original Grossman and Helpman (1991) model by introducing two different consumption goods: a traditional homogeous good and a composite commodity available in many varieties, whose dimension can be increased through R&D investment. In a semi-endogenous (non scale) version of the model (Jones, 1995), we investigate the effects of changes in the level and in the composition of public consumption on the steady state and on the economy's transitional dynamics. We find that, by varying the composition of the aggregate consumption spending, i.e. the relative demand conditions in the two commodity markets, the government can effectively stimulate faster innovation, by moving resources away from traditional industry. The welfare effects are also evaluated under the assumption that government expediture provides direct utility to households. The composition of government consumption affects the entire time path of utility. In particular we show that the welfare effects of fiscal policy along the transitional path can be positive or negative depending on the composition of public spending. Finally, within an endogenous growth (scale) version of the model, it is briefly discussed how the level and the composition of public consumption influence the long run rate of growth of innovation and output.

The paper is organized as follows. In section 2 household's behaviour, government, and firms activity are described. Section 3 presents the non scale version of the model and discusses the long run effect of public consumption, the transitional adjustment and the consequences on economic welfare. Section 4 briefly proposes the scale model with fiscal policy implications. Some final comments are gathered in section 5.

2 The Model

We extend the 'expanding variety ' framework of Grossman and Helpman (1991) by considering a three sector economy, producing in a competitive environment a homogeneous consumption good and a differentiated commodity available in many varieties within a market of monopolistic competition, whose dimension can be increased through investment in the R&D sector. Goods are demanded by the private sector and by the government who offers public services providing direct utility to households.

2.1 Household's Demand

We consider an economy populated by identical individuals; $L(t)$ is the size of population growing over time at the constant rate μ. Each individual is endowed with one unit of time and derives utility from consumption of a homogeneous commodity Z and a differentiated good D and from the services of a publicly provided consumption good:

$$\max U = \int_0^\infty \exp[-\rho t]\left[\alpha \ln(c_D(t)) + (1-\alpha)\ln(c_Z(t)) + \eta \ln(g(t))\right] dt \tag{1}$$

$$c_D(t) = \left[\int_0^N c_X(i,t)^\epsilon di\right]^{\frac{1}{\epsilon}} \quad , \quad 0 \leq \epsilon < 1 \tag{2}$$

subject to the constraint:

$$\dot{a}(t) = [r(t) - \mu]a(t) + w(t) - e(t) - \theta(t), \tag{3}$$

where (we drop the time index, to simplify notation) $e = P_Z c_Z + P_D c_D$ defines per capita expenditure, w is the wage rate, a net per capita asset holding, r instantaneous interest rate, θ lump sum taxation and η the elasticity of instantaneous utility with respect to g which denotes per capita public services. Lower case letters $c_j(s)$ $(j = Z, D, X)$ indicate per capita quantities. The price of Z is P_Z and given $p(i)$, the price of any single variety i, P_D is defined as follows:

$$P_D = \left[\int_0^N p(i)^{\frac{\epsilon}{\epsilon-1}} di\right]^{\frac{\epsilon-1}{\epsilon}} . \tag{4}$$

The necessary conditions for an efficient time path of consumption expenditure deliver the usual dynamic relationship

$$\gamma_e = \frac{\dot{e}}{e} = r - \rho - \mu, \tag{5}$$

and the following demand functions result from the maximization of instantaneous utility:

$$c_Z = (1-\alpha)\frac{e}{P_Z} \quad , \quad c_D = \alpha\frac{e}{P_D}, \quad c_X(i) = \frac{\alpha e}{P_D}\left(\frac{p(i)}{P_D}\right)^{\frac{1}{\epsilon-1}}, \; i \in [0,n]. \tag{6}$$

2.2 The Government

Aggregate public consumption is a composite of Z and D. To finance consumption, the government withdraws a fixed amount θ of income from every individual. Therefore $T = \theta L$ represents the overall lump sum taxation collected by the government. A fraction τ of T is allocated to the composite commodity D, and the remaining $(1-\tau)$ to the homogeneous good Z, according to the following demand functions:

$$G_Z = \frac{(1-\tau)T}{P_Z} \quad , \quad G_D = \frac{\tau T}{P_D}. \tag{7}$$

As in private consumption, G_D is composed of N different varieties with the same elasticity of substitution. Therefore the demand of the single variety i is given by

$$G_X(i) = \frac{\tau T}{P_D}\left(\frac{p(i)}{P_D}\right)^{\frac{1}{\epsilon-1}}. \tag{8}$$

>From [6], [7] and [8] we obtain total market demand for the different consumption goods, Z, D, and $X(i)$ respectively:

$$\begin{aligned} Z &= c_Z L + G_Z = \frac{1}{P_Z}\left[(1-\alpha)E + (1-\tau)T\right], && (9) \\ D &= c_D L + G_D = \frac{1}{P_D}\left[\alpha E + \tau T\right], && (10) \\ X(i) &= c_X(i)L + G_X(i) = \frac{[\alpha E + \tau T]}{P_D}\left(\frac{p(i)}{P_D}\right)^{\frac{1}{\epsilon-1}}, && (11) \end{aligned}$$

with $E = eL$ defining total expenditure of the private sector. Equations [9] and [10] highlight how the composition of government spending can affect the share of aggregate expenditure allocated between the two consumption commodities.

2.3 Production and Innovation

We consider a three sector economy. There exists a traditional industry, producing a homogeneous consumption good in a perfectly competitive environment and a monopolistic sector manufacturing, at any time, n differentiated varieties. New brands are introduced into the market through investment in the $R\&D$ sector.

The homogenous good Z is produced by a single representative firm according to the following technology: $Z = L_Z, \;\; L_Z = Ll_Z$,where l_Z represents the share of total labour force L employed in the Z industry. We assume the wage rate as the numeraire: $w = 1$. Then, profit maximization implies $P_Z = 1$ and from [9] we get the equilibrium quantity of labour employed in the production of Z:

$$L_Z = (1-\alpha)E + (1-\tau)T. \tag{12}$$

In the differentiated good sector, each firm manufactures a single brand, retaining a perpetual monopoly power over the variety it produces. Producer i maximizes profits, subject to a constant returns to scale technology, with labour as the only input:

$$X(i) = B_X L(i), \int_0^N L(i)di = L_D = Ll_D, \tag{13}$$

where $L(i)$ represents the quantity of labour employed to manufacture the variety i, l_D the fraction of time allocated to the production of D and B_X is a productivity parameter. The optimal price rule implies a constant mark-up over marginal cost: $p(i) = p = 1/B_X\epsilon$.

In the symmetric equilibrium the quantity of each of the n varieties available in the market is:

$$X(i) = X = \frac{[\alpha E + \tau T]\epsilon B_X}{N}, \tag{14}$$

and the value of per brand profits:

$$\pi = \frac{(1-\epsilon)[\alpha E + \tau T]}{N}. \tag{15}$$

Given [13] and [14] we can thus obtain the equilibrium labour requirement

$$L_D = [\alpha E + \tau T]\epsilon, \tag{16}$$

while the total output of the composite commodity D and the price index P_D in the symmetric equilibrium are the following:

$$D = N^{\frac{1}{\epsilon}} X, \tag{17}$$

$$P_D = N^{\frac{\epsilon-1}{\epsilon}} p = N^{\frac{\epsilon-1}{\epsilon}} \left(1/B_X\epsilon\right). \tag{18}$$

The innovation sector is competitive. New blueprints are produced according to the following constant returns technology:

$$\dot{N} = L_n N^{\phi}, \quad L_N = Ll_N \quad , \quad \phi \leq 1, \tag{19}$$

where l_N represents the share of total labour force L employed in the innovation sector. The average cost of inventing a new variety $(1/\ N^{\phi})$ decreases as knowledge, incorporated in N, accumulates. The parameter ϕ reflects the intensity of the externality, and crucially discriminates between two classes of growth models. With $\phi = 1$ the model shows a traditional endogenous growth setup. However, this class of *R&D* based models has been criticized due to the troublesome prediction of a growth rate proportional to the size of the economy (Jones, 1995a, 1995b). Following Jones (1999) and Eicher and Turnovsky (1999), the restriction $\phi < 1$ represents the simplest device to eliminate the scale effect.

As the industry is competitive, free entry forces profits to zero. The cost of a single blueprint must be equal to the discounted perpetual flow of profit, generated by the new variety entering the D market:

$$\frac{1}{N^{\phi}} = V \tag{20}$$

A no arbitrage condition must hold between the riskless asset yielding the interest rate r and $V = \int_t^{\infty} e^{-\int_t^{\xi} r(t')dt'} \pi(\xi) d\xi$, the asset, which entitles the individual to the flow of profits generated by the typical firm operating in the monopolistic market:

$$Vr = \dot{V} + \pi. \tag{21}$$

3 The Non Scale Model ($\phi < 1$)

The restriction $\phi < 1$ implies that the positive externality on the research activity, due to the accumulation of non rivalry knowledge will asymptotically come to an end. This assumption characterizes the *R&D* models of semi-endogenous growth [1] and leads to a long run growth rate of innovation that is proportional to the population growth rate and not to the *level* of population (scale effect). We time differentiate [19] to obtain the long run growth rate of the stock of varieties:

$$\gamma_N \equiv \frac{\dot{N}}{N} = \mu/(1 - \phi). \tag{22}$$

Two conditions are imposed to determine the dynamics of this economy: labour market clearing and equality between the rates of return to investment and saving (asset market clearing). The equilibrium dynamics of the model is

[1] See for example Jones (1995b), Kortum (1997) and Segerstrom (1998).

described by a system of two differential equations in the (e,n) space, where $n = N/L^{1/(1-\phi)}$ is the scale adjusted stock of varieties.

Using [12], [16] and [19], we can write the labour market equilibrium condition $L = L_Z + L_D + L_N$ in per capita terms as

$$\dot{n} = n^{\phi}\left[1 - e(1-\alpha+\alpha\epsilon) - \theta(1-\tau+\tau\epsilon)\right] - n\frac{\mu}{1-\phi}. \tag{23}$$

By differentiating the free entry condition [20] and by substituting [15] into [21] we solve for the equilibrium rate of interest r

$$r = n^{\phi-1}\left[e((1-\epsilon)\alpha(1-\phi)+\phi) + \theta((1-\epsilon)\tau(1-\phi)+\phi) - \phi\right]. \tag{24}$$

Using [24], the Euler condition [5] can be written as

$$\dot{e} = \left\{n^{\phi-1}[e((1-\epsilon)\alpha(1-\phi)+\phi) + \theta((1-\epsilon)\tau(1-\phi)+\phi) - \phi] - \rho - \mu\right\} e. \tag{25}$$

Equations [23] and [25] fully describe the equilibrium dynamics of the model. As shown below, the system has a unique steady state which is saddle path stable.

Steady State

The steady state is reached when $\dot{n} = \dot{e} = 0$ (star superscripts indicate steady state values):

$$n^* = \left[\frac{(1-\epsilon)(1-\phi)(\alpha+\theta(\tau-\alpha)}{\mu+(1-\alpha+\alpha\epsilon)\rho(1-\phi)}\right]^{\frac{1}{1-\phi}}, \tag{26}$$

$$e^* = \frac{\rho(1-\phi)+\mu-\theta[\mu+(1-\tau+\tau\epsilon)\rho(1-\phi)]}{\mu+(1-\alpha+\alpha\epsilon)\rho(1-\phi)}. \tag{27}$$

Given [12], [16], [19], [22], [27], [26], the steady state shares of labour in the three sectors are

$$l_N^* = \Omega\left(1+\theta\frac{\tau-\alpha}{\alpha}\right), \quad \Omega = \frac{(1-\epsilon)\alpha\mu}{\rho(1-\phi)(1-\alpha+\alpha\epsilon)+\mu} < 1 \tag{28}$$

$$l_D^* = \frac{\alpha\epsilon(1-\Omega)}{1-\alpha+\alpha\epsilon}\left(1+\theta\frac{\tau-\alpha}{\alpha}\right), \tag{29}$$

$$l_Z^* = \frac{(1-\alpha)(1-\Omega)}{1-\alpha+\alpha\epsilon} + \theta\frac{\tau-\alpha}{\alpha}\left(\frac{(1-\alpha)(1-\Omega)}{1-\alpha+\alpha\epsilon} - 1\right). \tag{30}$$

where Ω represents the labour share arising in the economy without public sector $(\theta = 0)$ [2] and the stock of varieties is simply given by $N^* = n^* L^{1/(1-\phi)}$.

[2] Notice that Ω will be higher the higher the exogenous rate of innovation γ_n, the higher the private share of expenditure allocated to the differentiated commodity α, and the higher the market power $(1/\epsilon)$ enjoyed by firms in the monopolistic industry.

In steady state price P_Z and per capita quantity of the traditional good ($z^* = l_Z^*$) are constant. Therefore, the long run dynamics of real variables are displayed by the increasing dimension of the composite good ($d^* = N^{*\frac{1-\epsilon}{\epsilon}} B_X l_D^*$), or, equivalently, by the increasing purchasing capacity of any unit of expenditure, due to the declining of the aggregate price P_D: $\gamma_d = \frac{1-\epsilon}{\epsilon}\gamma_N = -\gamma_{P_D}$.

In order to obtain a measure of per capita total output in this multisector model, we simply follow the national accounting procedure. We consider $t = 0$ as the base period. Given [22] and [18] we can write:

$$P_D(t) = \left[N(0)e^{\gamma_N t}\right]^{\frac{\epsilon-1}{\epsilon}} \frac{1}{B_X \epsilon}. \tag{31}$$

By evaluating [31] in the base period, with the simplifying assumption that $N(0) = 1$ we get:

$$P_D(0) = \frac{1}{B_X \epsilon}. \tag{32}$$

We define output as the value of aggregate expenditure and through [32] we obtain the following expression of real per capita output :

$$y = z + \frac{1}{B_X \epsilon} d = l_z + N^{\frac{1-\epsilon}{\epsilon}} \frac{l_D}{\epsilon}. \tag{33}$$

The market economy of this non scale model contains different types of distortions, which affect both labour shares allocation and the stock of product varieties. The first source of market failure are the positive external effects associated with technological knowledge, when firms do not appropriate the value of knowledge spillover from future researchers. The presence of $\phi > 0$ represents the effect associated with *intertemporal knowledge spillover.*

Second, a static distortion arises due to the monopolistic pricing of differentiated products while the homogeneous good is priced at marginal cost. Moreover, in the presence of this mixed market structure the *consumer-surplus effect* associated to the invention of a new good - the external benefit to households from increased product diversity - dominates the *profit destruction effect* - the adverse external effect on the profitability of the other firms. All these effects are associated with $\alpha < 1$.

By comparing the decentralized and socially optimal steady state equilibrium with $\theta = 0$, it can be shown that the total effect of the above market failures induces underinvestment in R&D with a suboptimal number of product varieties, overproduction of the homogenous good Z while under or overproduction of the differentiated good D can be obtained [3]

Equilibrium Dynamics

Since the dynamic system [23] and [25] is nonlinear, we consider the linearized dynamics about steady state

[3] In the symmetric case, with $\alpha = 1$, there is overproduction of the differentiated good, while with $\phi = 0$ the result is an underproduction. Indeed, with $\alpha = 1$ and $\phi = 0$ the allocative bias due to the above distortions is removed and the social optimum and the decentralized equilibrium coincide as in Grossman and Helpman (1991, chap. 3).

$$\begin{pmatrix} \dot{n} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} -\mu & -(1-\alpha+\alpha\epsilon)n^{*\phi} \\ (\phi-1)(\mu+\rho)\frac{e^*}{n^*} & e^* n^{*\phi-1}[(1-\epsilon)\alpha(1-\phi)+\phi] \end{pmatrix} \begin{pmatrix} n-n^* \\ e-e^* \end{pmatrix}. \tag{34}$$

The two eigenvalues associated to the Jacobian matrix in [34] are of opposite sign. Therefore, the transitional dynamics is characterized by a unique stable saddle path. We denote λ the stable (negative) root

$$\begin{aligned} \lambda &= -\frac{1}{2n}\left[\mu n^* - e^* n^{*\phi}(\alpha(\phi-1)(\epsilon-1)+\phi) + \sqrt{\Delta}\right], \quad (35) \\ \Delta &= \left[\mu n^* - e^* n^{*\phi}(\alpha(\phi-1)(\epsilon-1)+\phi)\right]^2 + 4e^* n^{*1+\phi}(\mu - (1-\alpha+\alpha\epsilon)(\phi-1)\rho). \end{aligned}$$

Starting from $n(0) = n_0$ the stable solution to [34] is

$$\begin{aligned} n &= n^* + (n_0 - n^*)\exp[\lambda t] \quad (36) \\ e &= e^* + (n_0 - n^*)X_{21}\exp[\lambda t] \quad (37) \end{aligned}$$

where $X_{21} = \frac{e^*(1-\phi)(\mu+\rho)}{e^* n^{*\phi}[\alpha(1-\epsilon)(1-\phi)+\phi] - n^*\lambda}$ represents the slope of the transitional path in the $\{e,\, n\}$ space, which can be shown to be strictly positive[4]

3.1 Effects of Fiscal policy

The present non-scale setting retains the standard neoclassical prediction that balanced growth rates cannot be affected by macroeconomic policy. This reflects the simple way of removing the scale effect, adopted here. The amount of labour resources devoted to research does not play any role in determining the economy's long run growth.[5] Therefore, within this framework we explore the *level* effects of fiscal policy in steady state conditional on a given composition of public consumption. Then, we describe the transitional dynamics.

Steady state effects of the composition of public consumption

The long run effects of public spending on the allocation of labour are easily derived by differentiating the steady state equations [28], [29] and [30]:

$$\frac{\partial l_N^*}{\partial \theta} = \frac{\tau - \alpha}{\alpha}\Omega\,, \quad \frac{\partial l_N^*}{\partial \theta} \gtreqless 0 \text{ if } \tau - \alpha \gtreqless 0, \tag{38}$$

$$\frac{\partial l_Z^*}{\partial \theta} = \frac{\tau - \alpha}{\alpha}\left(\frac{(1-\alpha)(1-\Omega)}{1-\alpha+\alpha\epsilon} - 1\right)\,, \quad \frac{\partial l_Z^*}{\partial \theta} \lesseqgtr 0 \text{ if } \tau - \alpha \gtreqless 0, \tag{39}$$

$$\frac{\partial l_D^*}{\partial \theta} = \frac{\tau - \alpha}{\alpha}\left(\frac{\alpha\epsilon(1-\Omega)}{1-\alpha+\alpha\epsilon}\right)\,, \quad \frac{\partial l_D^*}{\partial \theta} \gtreqless 0 \text{ if } \tau - \alpha \gtreqless 0. \tag{40}$$

[4] X_{21} is the different from unity element of the normalized eigenvector associated with the stable root λ.

[5] As in Segerstrom (1998) and Young (1998), the only line of action for enhancing growth are policies directly aimed at influencing the rate of population growth.

Assuming that the composition of public consumption differs from that of the private sector ($\tau \neq \alpha$), a rise in public spending changes the destination of productive resources among sectors. In particular, considering $\tau - \alpha > 0$, an increase in government spending raises the share of labour allocated to the production of the composite commodity and to the research sector, and decreases the labour employed in the homogeneous good industry.[6]

The effectiveness of fiscal policy crucially depends on the value of $(\tau - \alpha)$. The traditional neutrality prediction of no distortionary taxation arises in the present setting only as a particular case. When government consumption exactly tracks the composition of demand of the private sector, then changes in public spending have no real effects in the long run, except for the crowding out of private consumption.

The story behind these results is easily explained. Changes in taxation operate here through a modification of the aggregate composition of consumption spending. In particular, assuming $\tau - \alpha > 0$, an increase in public expenditure raises the demand of the composite commodity relative to the homogeneous good. The reallocation of resources among sectors results in a rise both in the stock of varieties and in the composite commodity, that crowds out the amount produced of the traditional good, resulting in an overall increase in aggregate production.

In more detail, given N, the increase of demand in manufacturing sector temporarily increases the rate of profit [15]. Any new invented variety promises a higher flow of profits, which actively stimulates a greater research effort. However, since the growth rate of innovation is given, the increased research effort converts into a permanent rise in the stock of existing varieties. The increase in the dimension of the composite commodity entails a reallocation of labour across sectors, moving resources from the competitive market to monopolistic industry, with an overall expansion of aggregate real output. The same reasoning can be put forward following a more conventional macro argument. Higher public expenditure expands national income through higher profits. However, the change in income is not entirely devoted to increasing consumption[7], and the resulting increase in national saving finances higher investment ($\frac{\partial l_N^*}{\partial \theta} > 0$) in the research sector.

Actually, there are two types of fiscal devices at government's disposal to pursue its objectives. Firstly the government can control the level of public spending, secondly it can affect the steady state allocation of resources simply by revising the composition of public consumption, for a given level of expenditure. It is easy to check that, a rise in τ expands research and output, following the same line of reasoning outlined above. Of course, the range of action is restricted by the value of α. Nonetheless, this example shows a new additional fiscal tool operating at a given level of public expenditure.

[6] Given [38], [39] and [40] the effects of fiscal policy on the other steady state levels are summarized as follows: $\frac{\partial z^*}{\partial \theta} \gtreqless 0$, $\frac{\partial n^*}{\partial \theta} \gtreqless 0$, $\frac{\partial d^*}{\partial \theta} \gtreqless 0$, $\frac{\partial y^*}{\partial \theta} \gtreqless 0$ if $\tau - \alpha \lesseqgtr 0$

[7] Given [39] and [40] it is easy to check that aggregate consumption ($P_Z z + P_D d = l_D/\epsilon + l_Z$) increases following an expansionary fiscal policy.

Transitional dynamics

The transitional effects of a permanent change in the level of per capita public spending are described in Figures 1a, 1b and 1c displaying the $\dot{e} = 0$ and $\dot{n} = 0$ curves obtained from equations [23] and [25] and the unique stable saddle path trajectory. Suppose that the economy is initially in steady state equilibrium at the point A. The impact effect of an increase in government expenditure is the crowding out of private consumption. However, the size of the initial drop in e and the transition path to the new steady state A' are crucially affected by the composition of public spending.

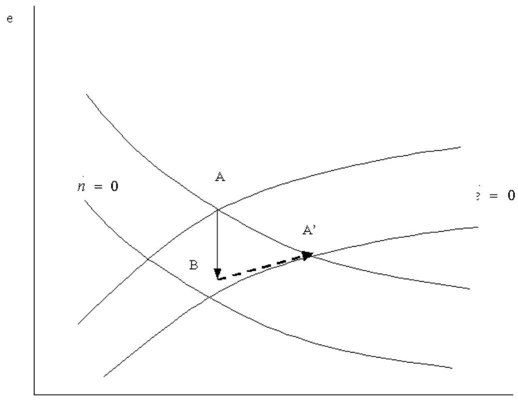

Figure 1a: $\tau > \alpha$

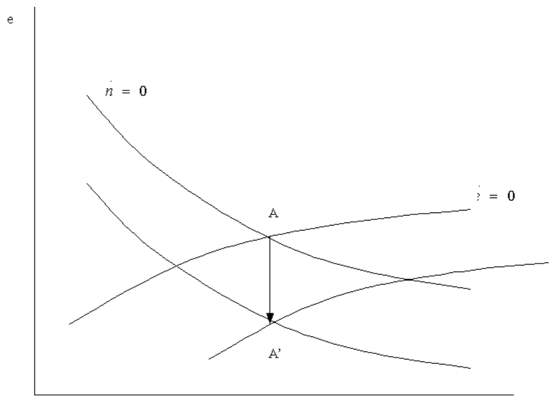

Figure 1b: $\tau = \alpha$

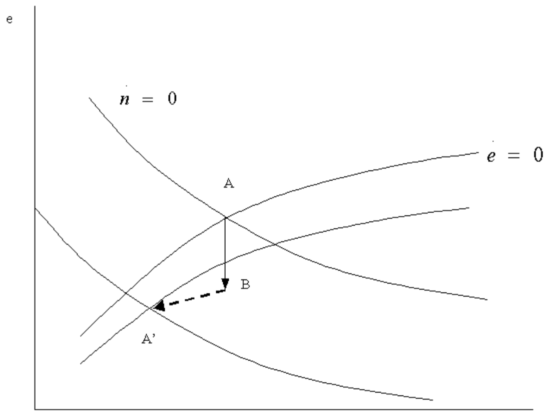

Figure 1c: $\tau < \alpha$

If $\tau = \alpha$ (Figure 1b) the economy immediately jumps from point A to the new steady state in A' with no transitional dynamics. An increase in government spending just crowds out e by the same amount, leaving the allocation of labour across sectors unchanged. If $\tau > \alpha$ (Figure 1a), then an expansionary fiscal policy directly shifts the economy from point A to point B on the new saddle path. Since $\tau > \alpha$ the rise in public spending increases the share of total demand allocated to the composite commodity. The resulting increase in profits partially offset the initial crowding out of private expenditure, so that point B lies above the new $\dot{e} = 0$ locus. Given N the higher demand in the monopolistic sector increases l_D. At the same time, the lure of higher profits stimulates $R\&D$ activity and therefore the share of labour allocated to the competitive industry falls. These initial effects start the new transition along the saddle path BA'. Approaching the new steady state, the expanding stock of varieties increases profits and thus the transition growth rate of private expenditure is positive, which implies positive growth rates of both l_Z and l_D. The reverse occurs when $\tau < \alpha$ (Figure 1c). In this case the impact of a rise in government spending moves demand from the monopolistic industry towards the competitive sector. Thus, the initial crowding out of e exceeds the downward shift of the $\dot{e} = 0$ locus (point B). The share l_Z increases while both l_N and l_D decrease. These initial conditions characterize the adjustment path towards point A', with negative growth rates of e, n and l_D, which imply a progressive reallocation of demand and labour towards the competitive sector.

For given level of per capita public expenditure, a change in the composition of government consumption affects the steady state and takes the economy on a new saddle path. In particular, an increase in τ results in higher steady state values of e and n. On impact the higher demand share for the differentiated commodity raises profits and private expenditure starting a transition with positive growth rate of real per capita output.

Moreover, in our model other interesting transitional effects of fiscal policy arise which have been usually neglected[8]. Fiscal policy influences the short run adjustment by varying the speed of convergence toward the steady state. It can be shown that the absolute value of λ in [35] is a decreasing function of θ and τ. An increase in public expenditure or a change in the composition of public consumption toward the differentiated good causes the economy to converge more slowly to the steady state with a longer transition period. This effect will be relevant in the welfare analysis developed in the next section.

3.2 Welfare Analysis

We turn now to the welfare implications of government consumption expenditure. We analyze the effects of fiscal policy on the utility level in the steady state as well as on the overall economic welfare including the transitional adjustment path.

Consider the flow of utility at time t in [1]:

$$u(t) = \alpha \ln(c_D(t)) + (1 - \alpha) \ln(c_Z(t)) + \eta \ln(g(t)).$$

By substituting [6], [18] and $P_Z = 1$ into $u(t)$ the expressions for indirect total and private utility are obtained (z and ω respectively)

[8] Remarkable exception are, for example, Yamarik (2001) and Russo (2002).

$$z(t) = \omega(t) + \psi(t),$$

where $\omega(t) = \ln(e(t)) + \frac{1-\epsilon}{\epsilon}\alpha \ln(N(t)) - \alpha \ln(B_X \epsilon)$ and $\psi(t) = \eta \ln(g(t))$
By differentiating $z(t)$ with respect to θ we get

$$z_\theta(t) = \omega_\theta(t) + \psi_\theta(t), \tag{41}$$

where $\omega_\theta(t) = \frac{1}{e(t)}\frac{de(t)}{d\theta} + \frac{1-\epsilon}{\epsilon}\alpha\frac{1}{N(t)}\frac{dN(t)}{d\theta}$ and $\psi_\theta(t)$ measures the change in utility benefit that households derive from the provision of public services.[9]

By evaluating derivatives in the steady state we get

$$z^*_\theta = \omega^*_\theta + \psi^*_\theta$$

$$\omega^*_\theta = \frac{1}{e^*}\frac{de^*}{d\theta} + \frac{1-\epsilon}{\epsilon}\alpha\frac{1}{N^*}\frac{dN^*}{d\theta} = \frac{-[\mu+\rho(1-\phi)(1-\tau+\tau\epsilon)]}{\mu+\rho(1-\phi)-g[\mu+\rho(1-\phi)(1-\tau+\tau\epsilon)]} + \frac{(1-\epsilon)\alpha(\tau-\alpha)}{\epsilon(1-\phi)[\alpha+g(\tau-\alpha)]} \tag{42}$$

where [26] and [27] give the steady state values e^* and $N^* = n^* L^{1/(1-\phi)}$.

In the determination of the optimal level of public spending, the benefit ψ^*_θ should be balanced against the cost ω^*_θ. The latter consists of two distinct components. The first term in [42] measures the direct crowding out effect. As government increases its expenditure, it takes away resources from the private sector, thereby reducing private expenditure.[10] The second term reflects the fact that a rise in θ, when $\tau \neq \alpha$, affects the number of product varieties and the price index P_D. If $\tau > \alpha$, $\frac{dN^*}{d\theta} > 0$, the utility benefit deriving from the increased purchasing capacity of any unit of expenditure due to the decline of the aggregate price.

The sign of ω^*_θ is negative if $\tau \leq \alpha$, because $\frac{dN^*}{d\theta} \leq 0$. When $\tau > \alpha$, the sign of ω^*_θ is ambiguous, since the first term in [42] is negative while the second one is positive. However only for low values of ϵ and/or high values of ϕ, $\omega^*_\theta > 0$.[11] Notice that in this case optimal public spending would be a corner solution.

The capitalized value of z^*_θ is the change in welfare which would result if the steady state were attained instantaneous, but it neglects the fact that the steady state is reached only gradually along the transitional path.

Using [41] the overall welfare effect of an increase in θ can be evaluated:

$$W_\theta = \int_0^\infty \exp[-\rho t]\,[z_\theta(t)]\,dt = \int_0^\infty \exp[-\rho t]\,[\omega_\theta(t) + \psi_\theta(t)]\,dt = \Omega_\theta + \Psi_\theta$$

[9] Since government consumption expenditure θ is measured in terms of the numeraire good, i.e. labour, and public consumption is a composite of Z and D, the measure of the change in the direct utility derived from public consumption ψ_θ takes into account also the goods' relative price behaviour.

[10] This term is increasing in ϵ. Under imperfect competition the crowding out effect is lower than under perfect competition.

[11] If $\phi = 0.6$, as in Eicher and Turnovsky (1999), $\epsilon < 0.6$ is required, corrisponding to a mark-up of about 66%. If the mark-up is around 18-20%, $\phi \approx 0.9$ is required for $\omega^*_\theta > 0$.

where $\Omega_\theta = \int_0^\infty \exp[-\rho t]\omega_\theta(t)\,dt$, $\Psi_\theta = \int_0^\infty \exp[-\rho t]\psi_\theta(t)\,dt$. In order to evaluate Ω_θ, we substitute into $\omega_\theta(t)$ a linear approximation of e and N along the equilibrium adjustment path.[12] Moreover, to catch the intuition, we approximate the exact solution of the integrals in Ω_θ obtaining the following expression (see the Appendix for technical details):

$$\begin{aligned}\Omega_\theta \;=\; & \left[\frac{\partial e^*}{\partial\theta}\frac{1}{e^*}\frac{1}{\rho} + \frac{1-\epsilon}{\epsilon}\alpha\frac{\partial N^*}{\partial\theta}\frac{1}{N^*}\frac{1}{\rho}\right] + \\ & \left[-\frac{1}{e^*}\frac{1}{(\lambda-\rho-\frac{1}{1-\phi})}\left(-X_{21}\frac{\partial N^*}{\partial\theta} + (N_0-N^*)\frac{\partial X_{21}}{\partial\theta}\right) - \frac{1-\epsilon}{\epsilon}\alpha\frac{\partial N^*}{\partial\theta}\left(-\frac{1}{N^*(\lambda-\rho)}\right)\right] + \\ & \left[(N_0-N^*)X_{21}\frac{\partial\lambda}{\partial\theta}\frac{1}{e^*}\frac{1}{(\lambda-\rho-\frac{1}{1-\phi})^2} + \frac{1-\epsilon}{\epsilon}\alpha(N_0-N^*)\frac{\partial\lambda}{\partial\theta}\frac{1}{N^*}\frac{1}{(\lambda-\rho)^2}\right]\end{aligned}$$

Notice that the first term in brackets is the capitalized value of ω_θ^* in [42]. The other two measure the welfare changes along the transitional path including the effects of θ on the speed of convergence $|\lambda|$. We have already shown that the composition of public consumption influences the size (and possibly the sign) of ω_θ^* and the welfare loss in the steady state is higher when $\tau < \alpha$. However, the composition of government expenditure affects also the sign of the welfare effects along the transitional path. With $\tau > \alpha$ the utility benefit deriving from the increase in N^* is offset not only by the lower e^* but also by the welfare loss along the transitional path. The latter is due to the lower speed of convergence and to the fact that along the adjustment path per capita expenditure is always below its steady state value [13]. With $\tau < \alpha$ $\frac{\partial N^*}{\partial\theta} < 0$ and $N_0 > N^*$, the last two terms in brackets are both positive. Therefore, the welfare loss in the steady state is reduced by the utility gain along the transitional path.

4 The Scale Model ($\phi = 1$)

This section explores the long run effects of public spending within a traditional endogenous growth setup. In this context population is assumed constant at level L. The structure of the model is unchanged, except for $\phi = 1$ in [19] and [20].

By employing [12], [16] and [19], with $\phi = 1$, the labour market equilibrium condition $L = L_Z + L_D + L_N$ can be written as:

[12] Given [36], [37] and $n = N/L^{1/(1-\phi)}$ the linearized expressions for $N(t)$ and $e(t)$ are $N(t) = N^* + (N_0 - N^*)\exp[\lambda t]$ and $e(t) = e^* + (N_0 - N^*)X_{21}\exp[(\lambda - 1/1-\phi)\,t]$.

[13] The last two terms in brackets are both negative, since with $\tau > \alpha$ $\frac{\partial N^*}{\partial\theta} > 0$ and $N_0 < N^*$. >From [35] $\lambda < 0$ and $\frac{\partial\lambda}{\partial\theta} > 0$. The slope of the transitional path is positive and $\frac{de^*}{d\theta} < 0$, then $\left(\frac{de_0}{d\theta} - \frac{de^*}{d\theta}\right) < 0$. Therefore $\left(-X_{21}\frac{\partial N^*}{\partial\theta} + (N_0-N^*)\frac{\partial X_{21}}{\partial\theta}\right) < 0$, because $sign\left(-X_{21}\frac{\partial N^*}{\partial\theta} + (N_0-N^*)\frac{\partial X_{21}}{\partial\theta}\right) = sign\left(\frac{de_0}{d\theta} - \frac{de^*}{d\theta}\right)$.

$$\gamma_N \equiv \frac{\dot{N}}{N} = L - E(1 - \alpha + \alpha\epsilon) - T(1 - \tau + \epsilon\tau), \tag{43}$$

and we manipulate the no arbitrage condition [21] to get:

$$E = \frac{1}{(1-\epsilon)\alpha}\left[\rho + \gamma_N - (1-\epsilon)\tau T\right]. \tag{44}$$

By substituting [44] in [43] , we obtain the steady state growth rate of innovation:

$$\gamma_N = L(1-\epsilon)\alpha - \rho(1 - \alpha + \alpha\epsilon) + T(\tau - \alpha)(1 - \epsilon). \tag{45}$$

Since research does not affect productivity in the competitive market, the amount of Z produced remains constant in the steady state. Consequently, the growth rate of real aggregate output coincides with the growth rate of the composite commodity D, or, equivalently, with the growing purchasing power of any unit of expenditure, owing to the decreasing of P_D[14]:

$$\gamma_Y = \frac{1-\epsilon}{\epsilon}\gamma_N \tag{46}$$

>From [45] and [46] we see that, government expenditure can actively stimulate long run growth[15]. As in the non scale exercise, the scope left for a growth enhancing fiscal policy is constrained by the composition of consumption characterizing private expenditure. Only with $\tau > \alpha$ does a rise in government spending result in a permanent increase in the growth rate. An expansionary fiscal policy changes the relative conditions of demand between the competitive and the monopolistic sector. The lure of higher profits reallocates labour towards the $R\&D$ sector, which enhances here the long run growth rate of innovation and output. Moreover, it is also possible to implement an active fiscal policy, without changing the amount of public spending or the level of taxation. Faster long run growth can be stimulated simply by reallocating a given amount of tax revenue away from the traditional good Z, to finance a higher share of consumption of the composite good D.

It can be shown that in this scale model the rate of growth in the decentralized economy is lower than the socially optimal rate of growth. As in Grossman and Helpman (1991) the market provides insufficient incentives for investment in R&D from a social point of view. However, differently from the standard Grossman and Helpman (1991) approach, by varying the composition of the aggregate consumption spending, the government can effectively stimulate faster innovation and growth.

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[14] Let $[P_Z, P_D(0)]$ the price vector in the base period $t = 0$. The level of output in real terms is then $Y(t) = P_Z Z + P_D(0)D(t)$ and the real output growth rate γ_Y can be written as

$$\gamma_Y = \left[\frac{P_D(0)D(t)}{P_Z Z + P_D(0)D(t)}\right]\frac{1-\epsilon}{\epsilon}\gamma_n$$

As the growth rate of D is positive and $P_Z Z$ is constant, the term in brackets tends to one as t becomes larger. Therefore, asymptotically we can write the expression reported in text.

[15] The constraint $T < wL$ is required to ensure positive levels of private spending.

5 Concluding Remarks

This paper explores the role of the composition of public consumption within a R&D based growth setup. The model considers a three sector economy: a competitive industry supplying a homogeneous good and a monopolistic sector manufacturing a composite commodity differentiated in many varieties, whose size increases through investment in the research sector. By varying its consumption, the government can actively stimulate research, as the higher share of demand faced by the monopolistic firms promises non negative profits to any new invented variety.

This theoretical result points to the empirical relevance of a fiscal policy carried out through a targeted composition of public consumption. On the basis of the Standard National Accounts (SNA 93) public consumption consists of wages and related payments, net purchases of goods and services and depreciation. Wages of staff in the health and education sector are typical public consumption expenditure items along with purchases of goods and services for the provision of such services. The share of public consumption, that constitutes compensation to employees, represents a fraction of aggregate demand, that reflects the private sector spending composition. It follows that changes in this component of public expenditure does not modify the aggregate composition of consumption. On the contrary, the direct purchase of goods and services on the part of the government, whose composition may discretionally differ from that of the private sector, may affect the composition of aggregate consumption. Relating to the quantitative relevance of this aggregate, in 2003 within the Euro-area (15 countries), final government consumption amounted to 20.6% as a percentage of GDP, which represents 40% of total government expenditure. 55% of total public consumption was devoted to employees compensation, while a 33% share to goods and services purchased. This latter represents 6.8% as a percentage of GDP (599 euro billions)[16], not a negligible figure, especially considering its potential impact when concentrated on a few specific markets. These considerations suggest that exploring the empirical relation between growth and composition of public consumption, might be fruitful. However, the scope of empirical analysis is limited by the scarcity/unavailability of data reporting commodity disaggregation of public purchases of goods and services. Nonetheless, as a first step in this direction, it may be of interest to reconsider existing evidence, mostly pointing to a negative/not significant relationship between government consumption spending and economic activity (Kormendi and Meguire, 1985; Barro, 1991; Easterly and Rebelo, 1993), distinguishing in regression analysis between the main components of public consumption.

Finally, from a theoretical point of view, the results obtained in this paper encourage a deeper investigation on the effect of changes in the composition of aggregate consumption expenditure within more elaborated framework, where the removal of the scale effect does not prevent endogenous growth (Dinopoulos

[16] Source: EUROSTAT, Economy and Finance 41/2004.

and Thompson, 1998; Peretto, 1998; Young, 1998).

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Appendix

By substituting $\omega_\theta(t)$ from [41] into Ω_θ:

$$\begin{aligned}\Omega_\theta &= \left[\frac{\partial e^*}{\partial\theta(t)}\int_0^\infty \frac{\exp^{-\rho t}}{e(t)}dt + \frac{1-\epsilon}{\epsilon}\alpha\frac{\partial N^*}{\partial\theta(t)}\int_0^\infty \frac{\exp^{-\rho t}}{N(t)}dt\right] + \qquad (42)\\ &\left[\int_0^\infty \frac{\exp^{(\lambda-\rho-\frac{1}{1-\phi})t}}{e(t)}dt\left(-X_{21}\frac{\partial N^*}{\partial\theta(t)} + (N_0-N^*)\frac{\partial V_{21}}{\partial\theta(t)}\right) - \frac{1-\epsilon}{\epsilon}\alpha\frac{\partial N^*}{\partial\theta(t)}\int_0^\infty \frac{\exp^{(\lambda-\rho)t}}{N(t)}dt\right] +\\ &\left[(N_0-N^*)X_{21}\frac{\partial\lambda}{\partial\theta(t)}\int_0^\infty \frac{t\exp^{(\lambda-\rho-\frac{1}{1-\phi})t}}{e(t)}dt + \frac{1-\epsilon}{\epsilon}\alpha(N_0-N^*)\frac{\partial\lambda}{\partial\theta(t)}\int_0^\infty \frac{t\exp^{(\lambda-\rho)t}}{N(t)}dt\right],\end{aligned}$$

where $e(t)$ and $N(t)$ are the linearized approximations

$$\begin{aligned}N(t) &= N^* + (N_0 - N^*)\exp[\lambda t]\\ e(t) &= e^* + (N_0 - N^*)X_{21}\exp[(\lambda - 1/1-\phi)\,t].\end{aligned}$$

To obtain the the expression for Ω_θ reported in section 3.2 we first note that the integrals in [42] are all of the type $\int \frac{\exp^{dt}}{a+b\exp^{ct}}dt$ and $\int \frac{t\exp^{dt}}{a+b\exp^{ct}}dt$, whose general solution is

$$\int \frac{\exp^{dt}}{a+b\exp^{ct}}dt = \frac{\exp^{dt}\,\Phi[-\frac{b\exp^{ct}}{a},1,\frac{d}{c}]}{ac}, \qquad (43)$$

$$\int \frac{t\exp^{dt}}{a+b\exp^{ct}}dt = \frac{\exp^{dt}\left(ct\Phi[-\frac{b\exp^{ct}}{a},1,\frac{d}{c}] - \Phi[-\frac{b\exp^{ct}}{a},2,\frac{d}{c}]\right)}{ac^2}, \qquad (44)$$

where the Lerch trascendent $\Phi[z,q,s]$ is a generalization of the zeta and polylogarithm functions defined by $\Phi[z,s,q] = \sum_{k=0}^{\infty}\frac{z^k}{(k+q)^s}$. Since $c < 0$, $z = \left|\frac{b\exp^{ct}}{a}\right|$ tends to zero as t goes to infinity. Given the properties of the Lerch functions this implies that $\lim_{t\to\infty}\Phi[z,q,s] = 1$. Since $d < 0$, evaluating of [43] and [44] between 0 and ∞ yields the following solutions of integrals in [42]

$$\int_0^\infty \frac{e^{-\rho t}}{N(t)}dt = -\frac{\Phi[\frac{N^*-N_0}{N^*},1,-\frac{\rho}{\lambda}]}{N^*\lambda} \simeq \frac{1}{N^*\rho},$$

$$\int_0^\infty \frac{e^{-\rho t}}{e(t)}dt = -\frac{\Phi[\frac{N^*-N_0}{e^*}V_{21},1,-\frac{\rho}{\lambda-\frac{1}{1-\phi}}]}{e^*(\lambda-\frac{1}{1-\phi})} \simeq \frac{1}{e^*\rho},$$

$$\int_0^\infty \frac{e^{(\lambda-\rho)t}}{N(t)}dt = -\frac{\Phi[\frac{N^*-N_0}{N^*},1,\frac{\lambda-\rho}{\lambda}]}{N^*\lambda} \simeq -\frac{1}{N^*(\lambda-\rho)},$$

$$\int_0^\infty \frac{e^{(\lambda-\rho-\frac{1}{1-\phi})t}}{e(t)}dt = -\frac{\Phi[\frac{N^*-N_0}{e^*}X_{21},1,\frac{\lambda-\rho-\frac{1}{1-\phi}}{\lambda-\frac{1}{1-\phi}}]}{e^*(\lambda-\frac{1}{1-\phi})} \simeq -\frac{1}{e^*}\frac{1}{(\lambda-\rho-\frac{1}{1-\phi})},$$

$$\int_0^\infty t\frac{e^{(\lambda-\rho-\frac{1}{1-\phi})t}}{e(t)}dt = \frac{\Phi[\frac{N^*-N_0}{e^*}X_{21},2,-\frac{\lambda-\rho-\frac{1}{1-\phi}}{\lambda-\frac{1}{1-\phi}}]}{e^*(\lambda-\frac{1}{1-\phi})^2} \simeq \frac{1}{e^*}\frac{1}{(\lambda-\rho-\frac{1}{1-\phi})^2},$$

$$\int_0^\infty t\frac{e^{(\lambda-\rho)t}}{N(t)}dt = \frac{\Phi[\frac{N^*-N_0}{N^*},2,\frac{\lambda-\rho}{\lambda}]}{N^*\lambda^2} \simeq \frac{1}{N^*(\lambda-\rho)^2},$$

where the approximations refer to the first element in the $\Phi[z,s,q] = \sum_{k=0}^{\infty} \frac{z^k}{(k+q)^s}$ sum. The above approximations can be justified on the basis that, $\Phi[z,q,s]$ converges to a real positive number as $0 < |z| < 1$, and $\lim_{z\to 0} \Phi[0,1,s] = 1/s$ and $\lim_{t\to\infty} \Phi[z,2,s] = 1/s^2$, which is precisely the first term of the sum. Threfore, the approximation error tends to vanish as variables approach their steady state values.