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Abstract

This paper explores the role of social integration on altruistic behavior. To this aim, we develop a two-stage experimental protocol based on the classic *Dictator Game*. In the first stage, we ask a group of 77 undergraduate students in Economics to elicit their social network; in the second stage, each of them has to unilaterally decide over the division of a fixed amount of money to be shared with another anonymous member in the group. Our experimental design allows to control for other variables known to be relevant for altruistic behavior: framing and friendship/acquaintance relations. Consistently with previous research, we find that subjects favor their friends and that framing enhances altruistic behavior. Once we control for these effects, social integration (measured by *betweenness*, a standard *centrality* measure in network theory) has a positive effect on giving: the larger social isolation within the group, the more likely it is the emergence of selfish behavior. These results suggest that information on the network structure in which subjects are embedded is crucial to account for their behavior.

Keywords: social networks, altruistic behavior, Experimental Economics

Jel Classification: C93, D85, Z13

INTRODUCTION. The so-called "Dictator Game" is a classic experimental protocol in which a subject (the "Dictator") is invited to decide unilaterally over the division of a fixed amount of money with another -usually anonymous- subject (the "Recipient").*(1-5)* The anonymity of the protocol may suggest that dictators may well keep all the money for themselves. However, this kind of selfish behaviour is rarely observed. In contrast, around 20% of the money is given to the recipient (this result being robust to a wide variety of experimental conditions) and such positive transfers are usually interpreted as an instance of *altruistic behavior*. This outcome is just an example of a more general observation: humans tend to behave cooperatively towards other unknown individuals. Some of the factors affecting the level of altruism (repeated interaction and reputation, among others) are now well understood.*(6-9)* However, it remains a level of cooperation that apparently cannot be explained by any of those variables.

This paper adds a new dimension to this discussion by exploring the role of *social integration* (measured by the degree of social proximity between subjects) on giving, that is, the *effect of socialization on altruism*. In recent years, a large number of Dictator Game experiments have focused on several social aspects, such as the degree of anonymity in the Dictator-Recipient and the Dictator-Experimenter relationships, induced by the experimental conditions, that may play a role as determinants of giving.*(10-13)* This literature unambiguously shows that the smaller the (social) distance between the parties involved, the larger the giving.

This evidence notwithstanding, to the best of our knowledge little attention has been paid to social network relationships to explain giving.*(14-15)* Our paper opens a novel line of research by exploring how the social network in which experimental subjects are embedded is related with their attitude to give. To this aim, we design a two-stage experiment in which, before being exposed to a standard Dictator Game, we ask subjects to elicit the social network characterizing their undergraduate class. Results are conclusive: on average, more *central* (i.e. highly integrated within the network) players give more, whereas the number of selfish "key players" is marginal. By the same token, "isolated" players behave selfishly, and subjects with a low degree of integration rarely show altruistic behavior.

METHODS. The experiment was conducted at the University of Granada (Spain) in January 2006. Subjects were first-year undergraduate students in Economics. Participation was voluntary. Given the high sensitivity to framing effects in this kind of experiments, we were extremely careful in preserving subjects-experimenter distance. To this aim, the experiment was conducted by assistants who did not have any previous contact with the students (see supplementary materials for a detailed description of the experimental design and additional material).

STAGE 1: *A "benefit-your-friend" Incentive Device for Network Elicitation.* The protocol for network elicitation was extremely simple: subjects were asked to write down on a piece of paper the name of their classmate friends who "*may have the chance to be benefited later in the experiment*". At this stage, no information was provided about the type of decisions they would make afterwards, or what the possible benefit would be. However, since we were interested in subjects revealing the identity of *their closest friends*, the instructions clearly stated that they might be given the chance to benefit *only one* of the friends, randomly chosen within their list, so that the more friends they had listed, the lower the chance of benefiting any particular individual. Despite its simplicity, we obtained an average of 50.5% of corresponded links, that is, an extremely accurate mapping of social correspondences, when compared with more sophisticated protocols used for analogous purposes. *(14)*

STAGE 2: *Dictator Game.* Subjects received two 11.5x22 cm. (4.5x8.8 inch.) envelopes in their hand-out package. One envelope was empty, the other contained ten 50 eurocent coins. Subjects were informed that their task was to divide this endowment of 10 coins between them and another subject in whatever way they wished. Then, depending on the treatment, they knew from the instructions whether the recipient would be a friend taken from their own list (treatment "Friends" in Box 1) or someone from their class *with the exception* of their named friends (treatment "No Friends").

	Frame	No Frame
Friends		27
No Friends	25	25

Box 1. Observations per treatment

We also controlled for another experimental condition, *framing*, which has been the object of interest in experimental research.[10-13] Again, the protocol was remarkably simple: one half of the subjects who were facing a no-friend as Recipient, had an additional sentence framing the Dictator Game by stating that the Recipient "…*would rely on them...*".

RESULTS. To simplify the exposition and the comparison with other related papers, our results are presented in terms of the number of coins (from 0 to 10) chosen by the Dictator in favor of the Recipient. Fig. 1 reports the giving distributions in the three treatments: Baseline (i.e. "No Friends-No Framing"), Framing and Friends. While average behavior is more selfish in the baseline, the other two treatments exhibit a (significant) increase in altruism, with behavior more disperse in the Friend treatment.

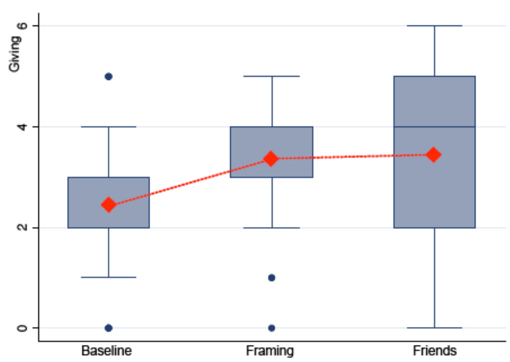
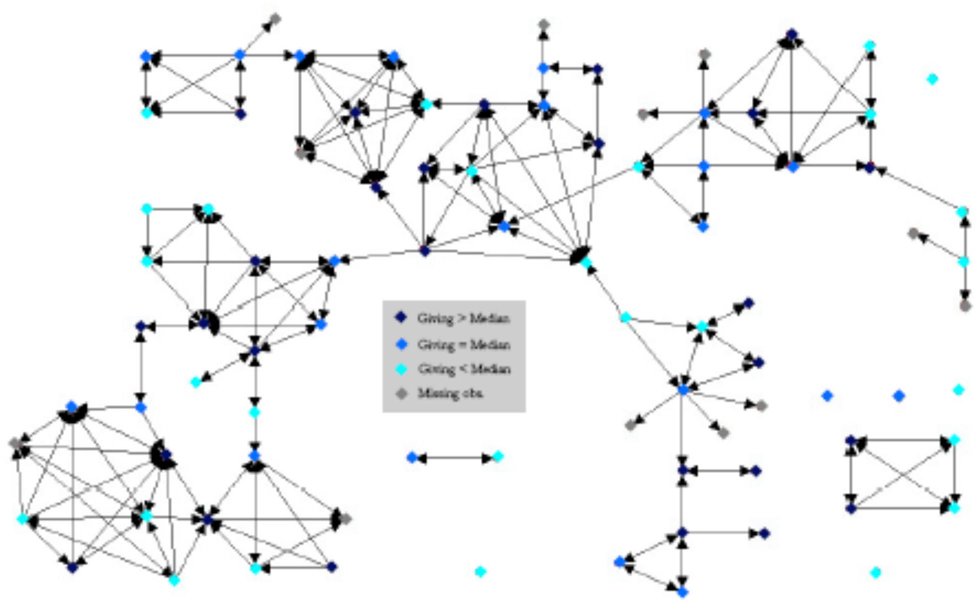


Fig 1. *Distribution of giving in the three experimental conditions.* "Box plot" graphic analysis. Within the box lie 50% of total observations (from the 25% to the 75% percentile). Adjacent lines trace the first upper and lower adjacent values; points denote outliers. Line within the box highlights the median. The broken red line connects the means of the three distributions. The first box shows observations on the Dictator Game from our Baseline (i.e. *No Friend-No Frame*) treatment. In this case, the Dictator keeps, on average, 8 coins for her and passes out only 2 to the Recipient. This result is in line with analogous experiments.*(2-5)* Very few subjects (3 out of 25) shared equally the €5; only 4 individuals behaved completely selfish, that is, kept all the money for themselves. The central box illustrates the *Framing* case. As Fig. 1 shows, the use of sentences framing the game enhances altruistic behavior, as the whole distribution "shifts" up, compared with the Baseline treatment. As a consequence, equal splitting becomes much more frequent (20% of total observations). The third box shows the giving distribution for the *Friend* treatment. In this case, average offers further increase, together with their variability: 25% of subjects pass (at least) half of the endowment, another 25% behave completely selfishly. Summing up, our two experimental treatments significantly enhance giving compared to the control condition, while we cannot reject the null hypothesis of no difference in giving between our "Framing" and "Friends" treatments (standard *t*-statistics, p=.85).

Stage 1 provides us with a mapping of the *directed* network (i.e. a network in which each link has a direction, from the eliciting to the elicited subject) of our experimental subject pool. The use of the "benefit-you-friend"

incentive device yields a network of *close-friends,* to the extent to which subjects were aware of the potential drawbacks of naming too many friends. As a result, on average subjects named 2.7 friends (mode=3), which is lower than the values obtained in related works. (14) Five individuals did not name anybody, but other subjects named up to 11 individuals; some of these individuals did not come to play the Dictator Game in Stage 2, but they are included in the dataset because they are a relevant to define the network.
In what follows, we shall apply a standard measure in network theory to test our working hypothesis that *altruism is (positively) correlated with social integration.* Among the classic measures provided by the literature, we focus on the so-called *betweenness centrality.(15-16)* Loosely speaking, betweenness centrality measures, for each individual, the relative frequency, among all shortest paths which connect any pair of subjects in the network, of paths which pass through that particular individual. In other words, betweenness is an index which measures how *central* is the individual within the network (or, the impact on the connection structure were that particular person be removed from the network –see the webpage for technical details).
Fig. 2 reports, together with the mapping of social relations derived from Stage 1, information about individual contribution with respect to the treatment median. A close inspection of Fig. 2 provides some intuition of our main result: *a*) more isolated subjects are more likely to give less; or *b*) more integrated subjects are more likely to be on or over the median of their corresponding treatment.

Fig 2. *Contributing within the social network*. Fig. 2 combines information from both stages of the experiment.

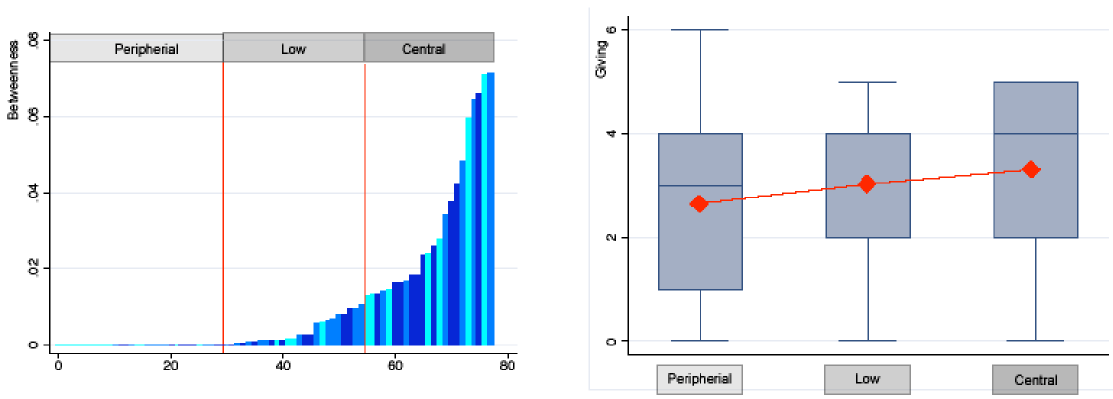


Elicited social links connect individuals in the network with an arrow (in the direction of the elicited friend). Each individual participating in Stage 1 is a diamond, whose color highlights whether that individual gave more or less than the corresponding treatment median. This figure maps into the social network subjects giving more than the treatment median, that is "altruists" (27 subjects), and subjects giving less that the median, that is "selfish" (26 individuals). Those subjects who were named by others but did not participate in Stage 2 are colored in grey.

Fig. 3 explores the effect of betweenness on the willingness to give. As the box-plots show, the higher the group level of betweenness, the more giving distribution shifts up.
Clearly, the analysis in Fig. 3 does not properly account for treatment conditions. To fix this problem, in Tab. 1 we estimate a standard order Probit regression, in which the probability of any possible level of giving (from 0 to 10) is regressed against individual betweenness *and* treatment conditions. While the estimated coefficient associated to the "Low" reference group is positive, but not significant (indicating that there is no significant difference in average contribution between Peripheral and Low subjects), the coefficient for the Central group is statistically significant (coeff. 0.51, p-value=.076), which means that *centrality matters*. Not surprisingly, both treatment conditions (with particular reference with Friends) are positive and significant. In sum, the regression analysis provides evidence that individuals who are central in their social network have a more altruistic behavior even after controlling for framing and friendship effects.

There is a caveat here. Our statistical exercise of Tab. 1 fixes a causal relation between altruism and network centrality, insofar we use betweenness within the list of regressors. This is consistent with the fact that the decision of giving is made *after* the network elicitation Stage 1 and, most likely, we can consider the social network depicted in Fig. 2 already well established at the time subjects had to make their contribution decision.

Fig. 3. *Network integration and giving.* Figure *3a)* partitions our subject pool in three subsamples, depending on their level of individual betweenness. The first 29 subjects are not integrated at all (*Peripheral*), as their betweenness is equal to zero. The remaining 48 subjects are partitioned in two groups of equal size (24 subjects each) *low*-integrated and *central*-players. In Fig. 3*a)* each subject is represented by a bar, whose height equals the corresponding level of betweenness centrality. By analogy with Fig. 2, the color of each bar identifies the level of each individual contribution with respect to the treatment median.



a) *b)*

Figure *3b*) shows, again with the aid of box plots, the distribution of giving in the three subsamples. As the diagram shows, giving increases with integration (all mean differences are significantly different).

On the other hand, the basic question remains: *are subjects (on average) more altruistic because they are pivotal in their social network, or are they central exactly because they show (for whatever reason) a more altruistic attitude toward the rest of the group?* To answer this question, a more detailed investigation on which (demographic, social or psychological) characteristics are correlated with our network centrality measure, something we cannot do with our limited experimental evidence. What is clear is that our results unambiguously show that the social network architecture has to be included within the list of determinants of our subjects' behavior.

Tab. 1. *Ordered Probit analysis.* Estimated probability of any particular contribution as a function of betweenness reference group (Low, Central) and treatment conditions (Friend, Framing). All regressors are dummies. A dummy variable for individuals belonging to the "Peripheral" reference group is omitted, to avoid perfect collinearity in the data. In this respect, estimated coefficients have to be interpreted as difference in the probability of giving with respect to the benchmark (i.e. Peripheral) reference group. We do not need to take into account for treatment interactions, as we get data out of a block design (with three treatment conditions out of four, see Box 1).

Variable	Coeff.	Std. Err.	z	P>z
Low	.3309	.3003	1.10	0.270
Central	.5190	.2929	1.77	0.076
Friends	.8424	.3065	2.75	0.006
Frame	.6399	.2993	2.14	0.033

Number of subjects/observations=77. Log likelihood=-130.732. Wald χ^2_4 =10.22. Prob.> χ^2 =.0368.

DISCUSSION. This paper explores a new aspect on the determinants of altruistic behaviour: *social integration*. Most of previous experimental literature has focused on economic incentives, reputation effects, framing or between-subject relations to explain altruism. Altruism may be driven by self-interest and it has been shown that reputation, or the promise of future rewards, do indeed affect generosity. Framing effects and between-subject relations are also important factors for altruism (as our experiment confirms). However, the puzzle is that, even when all these factors have been accounted for, there is still some human cooperation that remains unexplained. Some work has pointed to socialization and cultural transmission of social values as a solution to this puzzle. There is experimental evidence with children showing that younger children are less generous/cooperatives in the dictator/public good games.*(18-19)* These results could be due to the fact that older children are more advanced in their socialization process and, in consequence, show a more altruistic behaviour. Along these lines, our research explores the idea that social integration may also be an important determinant for altruism. Note that this hypothesis is consistent with the evidence on children playing the dictator game since older children also have, on average, a higher level of social capital. However, since socialization and transmission of values is more complex process, rather than just social capital accumulation, we test our hypothesis with subjects at the same stage of the socialization process but, nonetheless, with different centrality positions in their social network. The results are clear: social integration is related to altruistic behaviour. Our results do not exclude other preference-based explanations, like inequity aversion, which are well supported by the experimental evidence. Nevertheless, as we previously argued, there is no obvious link between preferences for fairness or inequity, and centrality within a social network so that social integration appears as an independent factor explaining altruism. Social integration, measured as centrality within the social network, is a feature distinctly human which could explain why humans show a cooperative behaviour towards other individuals with no genetic relation to them, while this behaviour is not observed in other animals.*(11-13)*

1. Hoffman, E., McCabe, K., Shachat, K. & Smith, V. L. Preferences, Property Rights, and Anonymity in Bargaining Games. *Games and Economic Behavior* **7**: 346-380 (1994)
2. Hoffman, E., McCabe, K. & Smith, V. L. Social Distance and Other-Regarding Behavior in Dictator Games. *American Economic Review* **86**(3): 653-660 (1996).
3. Eckel, C. & Grossman, P. J. Altruism in Anonymous Dictator Games. *Games and Economic Behaviour* **16**, 181-191 (1996).
4. Bolton, G. E., Katok, E. & Zwick, R. Dictator Game Giving: Rules of Fairness versus Acts of Kindness. *International Journal of Game Theory* **27**: 269-299 (1998)
5. Frohlich, N., Oppenheimer, J. & Moore, J. B. Some Doubts about Measuring Self-Interest Using Dictator Experiments: The Costs of Anonymity. *Journal of Economic Behavior and Organization* **46**: 271-290 (2001).
6. Rabin, M. Incorporating Fairness into Game Theory and Economics, *American Economic Review* **83**:1281-1302 (1993).
7. Fehr, E. & Fischbacher, U. The Nature of Human Altruism, *NATURE* **425**:785-791 (2003).
8. Fehr, E. & Gächter, S. Altruistic Punishment in Humans, *NATURE* **415**:137-140 (2002).
9. Fehr, E. & Camerer, C. When Does 'Economic Man' Dominate Social Behavior?, *SCIENCE* **311**:47-52 (2006)
10. Charness, G. & Gneezy, U. What's in a Name? Anonymity and Social Distance in Dictator and Ultimatum Games. *Journal of Economic Behavior and Organization* (forthcoming).
11. Bohnet, I. & Frey, B. S. Social Distance and Other-Regarding Behavior in Dictator Games: Comment. *American Economic Review* **89**(1): 335-339 (1999)
12. Brañas-Garza, P. Poverty in Dictator Games: Awakening Solidarity, *Journal of Economic Behavior & Organization* **60**(3):306-320 (2006).
13. Burnham, T. C. Engineering altruism: a theoretical and experimental investigation of anonymity and gift giving. *Journal of Economic Behavior & Organization* **50**: 133-144 (2003)
14. Mobius, M., Quoc-Anh, D. & Rosenblat, T. Social Capital in Social Networks. *Harvard* Manuscript in preparation, (2004).
15. Brañas-Garza, P. & Espinosa, M. P. Being generous within social networks, The Papers 06/05, Universidad de Granada (2006).
16. Jackson, M. O. The Economics of Social Networks. In: *Proceedings of the 9th World Congress of the Econometric Society* (Eds.: R. Blundell, W. Newey and T. Persson) Cambridge University Press (forthcoming).
17. Vega-Redondo, F. *Diffusion, search and play in complex social networks*. Econometric Society Monograph Series (forthcoming).
18. Krause, K. & Harbaugh, W. T. Children's Contributions in Public Good Experiments: The Development of Altruistic and Free-riding Behaviors, *Economic Inquiry* **38**(10):95-109 (2000)
19. Harbaugh, W.T., Krause, K. & Liday, S. Children Bargaining, *University of Oregon* working paper, (2003).

Supplementary Information is linked at http://www.ugr.es/~pbg/material/network.htm

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Appendix: Experimental instructions[1]

You are going to participate in an Economics Experiment. For your participation you get the 2 euros that have been placed on your desk. This work is coordinated by a researcher at the University of the Basque Country. She asks you for your cooperation during the experiment. The aim of this experiment is to study how individuals take decisions in certain contexts. Instructions are simple. If you carefully follow them, you could earn an additional amount of money at the end of the experiment; this amount will be paid in private (no one will know how much you have earned). You are allowed to ask questions to the monitors at any moment during the experiment. To call monitors, raise your hand. Any communication between participants is strictly forbidden and would imply the exclusion from the experiment.

Three qualifications are in order before proceeding.

1. First, the money you will be using DOES NOT belong to the researchers, it has been provided by the Spanish government for scientific purposes.
2. Second, not all of the participants face the same decision problem; three different experiments are been conducted in this room.
3. Last, you can check that your name will not be asked –exception made of the consent form, that is a legal requirement. Everything is confidential.

The experiment is divided in different phases that are explained below in detail.

Phase 1: Group of friends: We need you to give us a list of classmate friends. There is a chance that if you list them here they will participate in the experiment with you and could benefit from your decisions. 'Your list' will be the basis to randomly select those individuals.

"Please, write down your list of friends below. If you do not list any friend, then, we will randomly select any classmate and you will not have the chance to benefit a friend. The number of marks that appear below does not determine the number of friends you have to list. Feel free to name as many friends as you wish. Remember though that *the higher the number of friends you list, the lower the chances of beneffiting a particular friend of yours are.*

YOUR LIST of FRIENDS:

*
*
*
*
*
*
*

[1] Original instructions were given in Spanish.

Phase 2: The decision problem: One of the envelopes on your desk contains ten 0.5 euro coins (real money). This money has been provisionally allocated to you and another subject. You have to decide how to divide those 10 coins between the other subject and you. Any allocation is possible, even that in which you keep everything for yourself and the other subject gets nothing. Your task is to take the decision you wish.

Who are you going to be matched with? Your partner will be RANDOMLY selected from the class list, excluding your friends (those included in YOUR list).

> —Treatment variation— [*friends*] Your partner will be RANDOMLY selected the from friends you have included in your list (YOUR list)].

You will divide the money in the following way. Since you have two envelopes, put in one of them the amount of money you want to leave for your partner, and in the other one, the amount you want to take with you. You just have to leave on the desk the first envelope. You take with you the other one. Whatever amount of money there is in it, it is something you will be the only one to know.

PLEASE, do not forget to fill in, sign and leave on the table the consent form. It is a legal requirement. Thank you for participating.

> —Treatment variation— [*frame*]:
>
> Note that your recipient relies on you.

Networks Characteristics

June 17, 2006

1 Network

A *network* $\Gamma = (N, L)$ is given by a set of nodes $N = \{1, 2, ..., n\}$ and a set of links $L = \{(i, j) \mid i, j \in N,\ i \neq j\ \} \subset N \times N$. We will assume that there are no reflexive links, that is $(i, i) \in N,\ \forall i \in N$. Networks can be directed or undirected.

1.1 Directed

A network Γ is *directed* if the existence of a link (i, j) from i to j does not imply that the converse link (j, i) should necessarily exist.

1.2 Undirected

A network Γ is *undirected* if $(i, j) \in L \iff (j, i) \in L$, $\forall i, j \in N$. In this kind of network the notation of a link connecting i and j may be simplified to ij instead of (i, j).

2 Degree

The set of nodes that are connected to any given $i \in N$ define its neighborhood $N_i = \{j \in N \mid (i, j) \in L\}$[1]. The *degree* or *connectivity* of any given $i \in N$ is the cardinality of its neighborhood, $z_i = |N_i|$.

In a directed network there may be defined two types of degree: degree-in and degree-out.

2.1 Degree-in

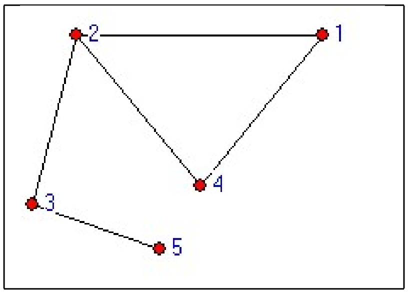
The *degree-in* of any given $i \in N$ is the cardinality of the set of nodes connected and directed *to* i $N_i^{in} = \{j \in N \mid (j, i) \in L\}$, that is $z_i^{in} = \left|N_i^{in}\right|$. The rank of the degree is $[0, n-1]$.

[1] It is clear that a network is characterized for the set of all its neighborhoods, that is $\{N_i, i \in N\}$.

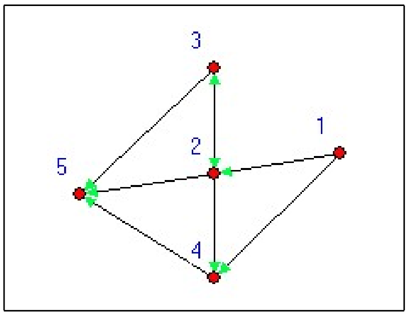
2.2 Degree-out

The *degree-out* of any given $i \in N$ is the cardinality of the set of nodes connected to i and directed *from* i $N_i^{out} = \{j \in N \mid (i, j) \in L\}$, that is $z_i^{out} = |N_i^{out}|$.

Example 1 *The following figure represents an undirected network where the degrees of its nodes are* $z_1 = 2,\ z_2 = 3,\ z_3 = 2,\ z_4 = 2,\ z_5 = 1$ *respectively.*



Example 2 *The following figure represents a directed network where the degrees-in of its nodes are* $z_1^{in} = 0,\ z_2^{in} = 2,\ z_3^{in} = 1,\ z_4^{in} = 2,\ z_5^{in} = 3$ *respectively, whereas the degrees-out are* $z_1^{out} = 2,\ z_2^{out} = 3,\ z_3^{out} = 2,\ z_4^{out} = 1,\ z_5^{out} = 0$.



Remark: The concept of *degree* is related in social networks to the "popularity" of an individual within a population. In a directed network, the *degree-out* of a given subject i may be interpreted as how this individual perceives her/his "popularity" in the group she/he belongs to, while her/his *degree-in* may be interpreted as her/his real "popularity".

3 Clustering

The *clustering* of a given node i is defined as the fraction of pairs of neighbors of i that are themselves neighbors. The rank of clustering is $[0,1]$.

This concept may be formally stated different for undirected and directed networks.

In a **undirected network**, the *clustering* of a given node i ,c_i^d is given by[2]

$$c_i^d = \frac{|\{jk \in L: \ j,k \in N_i\}|}{|\{jk \in L\}|} = \frac{2\,|\{jk \in L: \ j,k \in N_i\}|}{z_i\,(z_i-1)}$$

In a **directed network**, the *clustering* of a given node i ,c_i^u is given by[3]

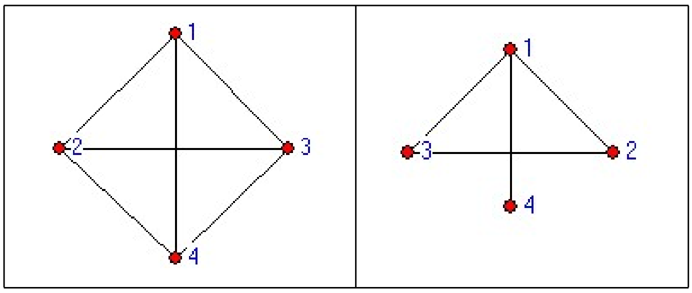
$$c_i^u = \frac{|\{(j,k) \in L: \ j,k \in N_i\}|}{|\{(j,k) \in L\}|} = \frac{|\{(j,k) \in L: \ j,k \in N_i\}|}{z_i\,(z_i-1)}$$

Example 3 *Next figure represents two undirected networks. In the left side network the clustering of node 1 is*[4]

$$c_1^u = \frac{2\,|\{jk \in L: \ j,k \in N_i\}|}{3\,(3-1)} = \frac{2\,|\{23,24,34\}|}{6} = 1$$

In the right side network, the clustering of node 1 is

$$c_1^u = \frac{2\,|\{(j,k) \in L: \ j,k \in N_i\}|}{3\,(3-1)} = \frac{2\,|\{23\}|}{6} = \frac{1}{3}$$



[2]Note that the number of possible neighbor pairs of a node i is simply $\begin{pmatrix} z_i \\ 2 \end{pmatrix} = \frac{z_i\,(z_i-1)}{2}$, where recall that z_i is its degree (i.e. the number of neighbors).

[3]In this case, the number of possible neighbor pairs is twice the number in a undirected network given that there are two possibilities of connecting any pair of node.

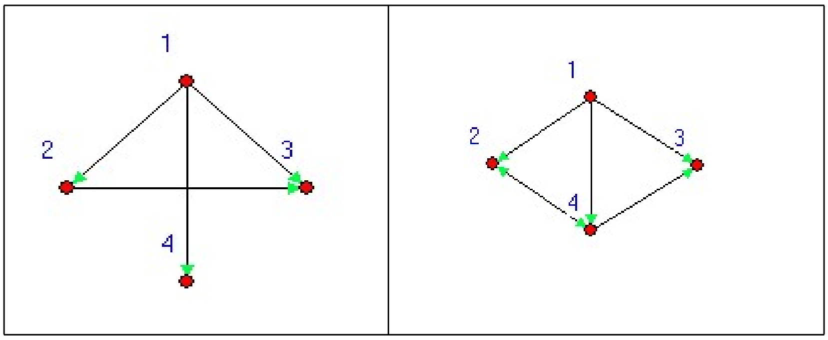
[4]Recall that in undirected networks the link $ij \in L$ denotes exactly the same as the link $ji \in L$.

Example 4 *Next figure represents two directed networks. In the left side network the clustering of node 1 is*

$$c_1^d = \frac{|\{(j,k) \in L: \ j,k \in N_i\}|}{3\,(3-1)} = \frac{|\{(2,3)\}|}{6} = \frac{1}{6}$$

In the right side network, the clustering of node 1 is

$$c_1^d = \frac{|\{(j,k) \in L: \ j,k \in N_i\}|}{3\,(3-1)} = \frac{|\{(2,4),(4,2),(4,3)\}|}{6} = \frac{1}{2}$$



Heuristically, in a social network clustering captures the idea of "how close" is the society , in other words this measure accounts for the level of "compactness" or "cohesiveness" of a network. Moreover, the implications of a high clustered network in a social context are connected to the ability of social pressure to impose a desired behavior on members of the network[5].

4 Betweenness Centrality

This concept is a characteristic of a node's position in the network, i.e. its centrality.

Intuitively, the centrality of a node measures the importance of this node in connecting other nodes.

In order to define properly the betweenness, assume that the network is connected[6]. The *betweenness centrality* of a node i can be defined as the fraction of all **shortest** paths connecting any two nodes j and k which **also** pass through

[5] In addition, if the aim of the network is the transmission of information, high clustering is detrimental through local search because there may be loops when sending the information to a neighbor node.

[6] A connected network is a network such that there exists a path between all pairs of nodes. A path in a network between any pair of nodes i and j is a finite sequence of alternating links $(i,k),(k,l),(l,m)...(n,p),(p,j)$ such that every consecutive pair of nodes are adjacent and the first link of the sequence starts in node i and the last link ends in node j.

i from the set of all shortest paths connecting any two nodes j and k. The normalized version divides this value by the maximum possible betweenness centrality, that is all possible shortest paths in a completely connected network. Let denote $s(j,k)$ the total number of all possible shortest paths connecting j and k, and let $s_i(j,k)$ stand for the cardinality of the subset of those that not only connect j and k but also go through a particular node i $(j \in N \diagdown \{i\}, k \in N \diagdown \{i,j\})$. Then, the betweenness of node i is given by

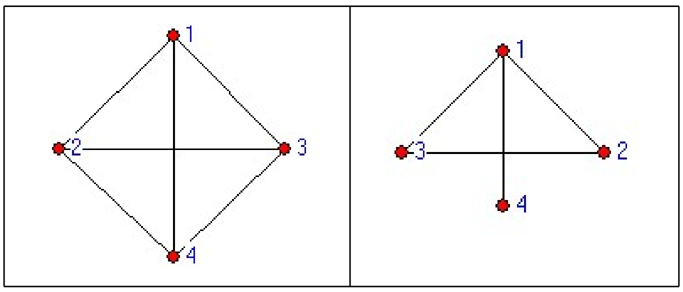
$$b_i = \sum_{j \neq i} \sum_{k \in N \setminus \{i,j\}} \frac{s_i(j,k)}{s(j,k)}$$

Example 5 *Next figure represents two undirected networks. In the left side network the* betweenness *of node 1 is* 0[7]*:*

$$b_1 = \sum_{j \neq i} \sum_{k \in N \setminus \{i,j\}} \frac{s_1(j,k)}{s(j,k)} = \frac{s_1(2,3) + s_1(2,4) + s_1(3,4)}{s(2,3) + s(2,4) + s(3,4)} = \frac{0+0+0}{1+1+1} = 0$$

In the right side network, the betweenness *of node 1 is* $\frac{2}{3}$ *:*

$$b_1 = \sum_{j \neq i} \sum_{k \in N \setminus \{i,j\}} \frac{s_1(j,k)}{s(j,k)} = \frac{s_1(2,3) + s_1(2,4) + s_1(3,4)}{s(2,3) + s(2,4) + s(3,4)} = \frac{0+1+1}{1+1+1} = \frac{2}{3}$$



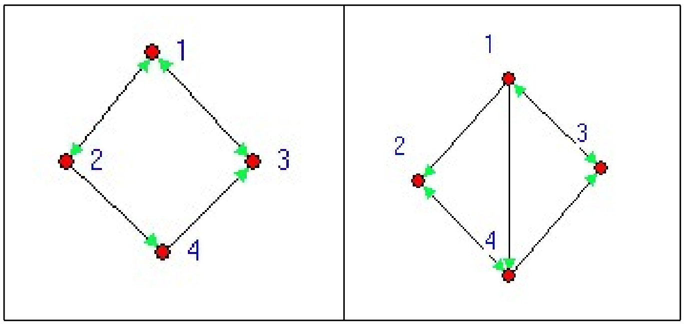
[7] Note that this network coincides with the left side network of Example 3, so the clustering of subject 1 is 1, in this sense those two measure, clustering and betweenness may be seen as negatively correlated, but this is not satisfied in general.

Example 6 *Next figure represents two directed and connected networks. The* betweenness *of node 1 in the left side network is computed as follows:*

$$\begin{aligned} b_1 &= \sum_{j \neq i} \sum_{k \in N \setminus \{i,j\}} \frac{s_1(j,k)}{s(j,k)} = \frac{s_1(2,3)+s_1(3,2)+s_1(2,4)+s_1(4,2)+s_1(3,4)+s_1(4,3)}{s(2,3)+s(3,2)+s(2,4)+s(4,2)+s(3,4)+s(4,3)} = \\ &= \frac{1+1+0+1+1+0}{2+1+1+1+1+1} = \frac{4}{7} \end{aligned}$$

The betweenness *of node 1 in the right side network is computed as follows:*

$$\begin{aligned} b_1 &= \sum_{j \neq i} \sum_{k \in N \setminus \{i,j\}} \frac{s_1(j,k)}{s(j,k)} = \frac{s_1(2,3)+s_1(3,2)+s_1(2,4)+s_1(4,2)+s_1(3,4)+s_1(4,3)}{s(2,3)+s(3,2)+s(2,4)+s(4,2)+s(3,4)+s(4,3)} = \\ &= \frac{0+1+0+0+1+0}{1+1+1+1+1+1} = \frac{1}{3} \end{aligned}$$



This definition of centrality explores a node's ability to be "irreplaceable" in the communications of two random nodes[8].Then, in the context of social networks an individual with a high centrality may be interpreted as a subject who has a high pressure on her/his reputation. The main disadvantage of betweenness is that the summation operator practically means that it needs global information about the network, in order to compute a single node's betweenness, and that is simply not possible in many contexts such as criminal networks.

5 Diameter

The *diameter* of a connected network is the largest distance that occurs in it, where distance, between two nodes, refers to the length of the shortest path between them.

[8]It is of particular interest in the study of network attacks, because at any given time the removal of the maximum betweenness vertex seems to cause maximum damage in terms of connectivity and mean distance in the network.

In disconnected graphs the diameter is undefined (usually it is used the convention which defines the diameter in this case as infinite). In other words, a network diameter is the largest number of nodes which must be traversed in order to travel from one node to another when paths with backtracks, detours, or loops are excluded from consideration.

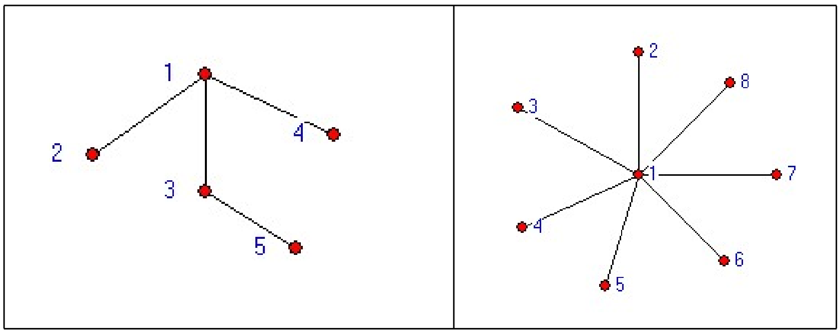
Let denote $\delta(s)$ as the cardinality of node pairs which are at distance s, where $d(i, j)$ denotes the geodesic distance between any pair of nodes i and j. That is,

$$\delta(s) = |\{(i, j) \in N \times N : d(i, j) = s\}|$$

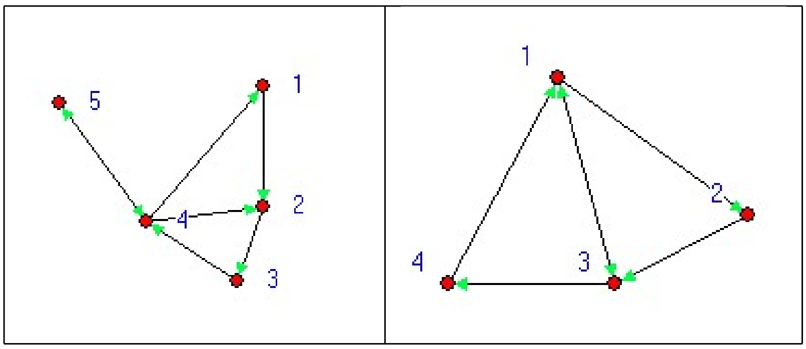
Then, the diameter is formally defined as

$$d = \max\{s \in \mathbb{Z}_{++} : \delta(s) > 0 \wedge s < \infty\}$$

Example 7 *Next figure represents two undirected networks which* diameters *are 3 (coincides with the length of shortest distance between nodes* 4 *and* 5*) and 2 (coincides with the length of shortest distance between any nodes* i *and* j*,* $i, j \neq 1$*) respectively.*



Example 8 *Next figure represents two directed networks which* diameters *are 4 (coincides with the geodesic distance between nodes* 1 *and* 5*) and 2 respectively.*



References

[1] Vega-Redondo F. “Complex Networks: Lecture notes” *mimeo,* 2004.

[2] PlanetMath.org. http://planetmath.org/encyclopedia.html

[3] Wikipedia. http://en.wikipedia.org/wiki/Main_Page