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Fixed Price plus Rationing: an Experiment

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Fixed Price plus Rationing: An Experiment*

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Abstract

This paper explores, theoretically and experimentally, a fixed price mechanism by which, if aggregate demand exceeds supply, bidders are proportionally rationed. If demand is uncertain, equilibrium consists in overstating true demand to alleviate the effects of being rationed. Overstating is more intense the lower the price, with bids reaching their upper limit for sufficiently low prices. In the experiment, despite of a significant proportion of equilibrium play, subjects tend to (under)overbid the equilibrium strategy when rationing is (high) low, with only this latter effect being persistent over time. We explain the experimental evidence by a simple model in which the probability of a deviation is decreasing in the expected loss associated with it.

KEYWORDS: Fixed Price Mechanism, Rationing, Experimental Economics
JEL CLASSIFICATION NUMBERS: C90, D45

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1 Introduction

Prices are not always set such that the market clears. Instead, we often observe non–price rationing of buyers, for different reasons. In initial public offerings, for example, the seller frequently sets a price at which he expects excess demand to be able to reward information revelation by large investors with some preferential treatment. In other situations, where demand is uncertain, the seller might simply not be able to set the market clearing price. In this case, two main classes of mechanisms have been proposed as solution to this problem: *auctions and fixed price mechanisms.* As for the latter, since supply is fixed (and price is chosen before actual demand reveals), the mechanism has to include a rationing device in case demand exceeds supply.

While axiomatic properties of different rationing schemes have been explored extensively by the literature, strategic behavior of buyers who expect to be rationed has up to date received little attention.[1] The few papers that explicitly analyze incentives in market games that may involve rationing of buyers find that these mechanisms are often desirable for the seller. In case a common value is sold, Bulow and Klemperer [5] show that prices which result by rationing can even be optimal. Gilbert and Klemperer [11] come to the same conclusion for situations where customers must make sunk investments to enter a market. In a private values setting, Bierbaum and Grimm [4] analyze a fixed price mechanism where buyers are proportionally rationed in case of excess demand. They find that, if total demand is uncertain, bidders overstate their true demand to alleviate the effects of being rationed in high demand scenarios. This allows the seller to set the fixed price at a rather high level which yields the surprising result that the fixed price mechanism outperforms alternative selling mechanisms (such as a uniform price auction)[2] with respect to a variety of criteria: revenue, variability of revenue in different demand scenarios, and minimum revenue that is raised if demand turns out to be low.[3]

The above findings contribute to explaining the frequent use of mecha-

[1]See, for example, Herrero and Villar [15], or Moulin [22]. Here rationing usually occurs because the allocating authority is not allowed to use prices in order to ration, e. g. in bankruptcy problems if claims are known but exceed the pie to be allocated.

[2]Since Bierbaum and Grimm consider large markets a uniform price auction is incentive compatible and therefore a very attractive mechanism, that has often been proposed as an alternative for initial public offerings but never has been widely established.

[3]Also Chun [6], Dagan *et al.* [8], Moreno-Ternero [21] and Herrero [14] look at rationing from a noncooperative perspective. Herrero *et al.* [16] provide an experimental study on the strategic behavior induced by rationing in the context of bankruptcy problems.

nisms that involve rationing of buyers. However, we know from an extensive experimental literature on market institutions that often human behavior differs substantially from theoretical predictions, which may affect the relative performance of different mechanisms. This motivated us to experimentally study bidding behavior in a fixed price mechanism with proportional rationing (FPM) quite similar to the one analyzed in Bierbaum and Grimm [4].

Our experimental design is based on a model where neither the buyers, nor the seller, know total demand due to uncertainty about the number of (identical) buyers. The seller, who is endowed with a given quantity of a divisible good, sets a fixed price, and then, buyers are asked to submit a quantity bid at this price. They are proportionally rationed in case the total quantity bid for exceeds supply, otherwise they receive their bid.[4] In the experiment, we were interested only in buyers' bidding behavior. Therefore, the seller's role was played by a computer, i.e. in each round a price was randomly chosen from the range where demand for the good was positive, which allowed us to extract complete bid functions. We also study an "incentive compatible mechanism" (ICM), which only differs from FPM with respect to the fact that buyers are never rationed. Given that the two mechanisms only differ with respect to the presence of the rationing device, we used ICM as a control treatment of the experimental results on FPM.

We shall now give a quick overview of our main results.

First, we show that Bierbaum and Grimm's [4] theoretical results on FPM are maintained in the context of small markets (i.e. a finite number of buyers). In particular, at high prices rationing never occurs and therefore bidding truthfully is optimal; at low prices bidders are always rationed and thus, in equilibrium, they demand the highest possible quantity (if any); at intermediate prices, where rationing only takes place when demand is high, bidders overstate their true demand, but only moderately.

As for the experimental evidence, subjects play extremely well ICM, where truthful bidding emerges as unanimous behavior since the very beginning. In FPM, behavior converges to equilibrium for very high and very low prices, where the equilibrium strategy is relatively easy to figure out. For intermediate prices, where the equilibrium is strategically more complex, some noise remains. As time proceeds, bidders even move away a bit (but not far) from the risk neutral equilibrium prediction in the direction

[4]This is basically the model analyzed in Bierbaum and Grimm [4]. The only differences are that Bierbaum and Grimm analyze large markets (whereas in our experiment the number of potential buyers is small). Moreover, they allow for different types of buyers.

of overbidding. Given our experimental evidence, a profit maximizing seller would then opt for FPM (i. e. commit to a fixed price), not only for the theoretical reasons highlighted by Bierbaum and Grimm [4] (and confirmed by our theoretical analysis), but also because *overbidding with respect to equilibrium takes place exactly in the price range which maximizes seller's revenues*, yielding profits even higher what seller could extract if he could be able to act as a monopolist in all demand scenarios.

Overall, the explanatory power of the theory seems impressive, especially if compared with that of standard auction theory models.[5] These considerations notwithstanding, panel data estimations yield two significant deviations from the behavior predicted by the risk neutral Nash equilibrium (RNNE) of the game: at intermediate prices, bids are at a higher level but as price sensitive as predicted. At low prices we observe — contrary to the RNNE prediction — price sensitivity of bids and underbidding.

We also find that these deviations cannot be explained by risk-attitude considerations, but are jointly consistent with the hypothesis of noisy directional learning (Anderson *et al.* [2]), where bidders adjust their actions in the direction of higher expected profits but do so subject to some exogenous noise (with the probability of an error being decreasing with the associated expected loss). In the steady state equilibrium of this process, players' behavior is given by probability distributions over the strategy space that constitute a Quantal Response Equilibrium (QRE) of the game (McKelvey and Palfrey [20]). (Maximum likelihood) estimations of the corresponding QRE for each price interval match the observed behavioral pattern: slight underbidding of RNNE together with some price sensitivity at low prices and simultaneously overbidding of RNNE at intermediate prices. They also confirm the intuition that behavior is less noisy at prices where the equilibrium is easier to figure out (i.e. high and low prices) than elsewhere. At the same time they explain why at extreme prices behavior converges to RNNE, while at intermediate prices it does not.

The remainder of the paper is arranged as follows. Theoretical properties of FPM and ICM is what we investigate first, in Section 2. Experimental conditions are described in Section 3. Section 4, devoted to experimental results, is divided in two parts. Descriptive statistics are presented first, followed by some panel data regressions in which we check the robustness of equilibrium predictions. Section 5 then checks whether risk aversion or

[5] Experimental studies of multi-unit auction formats find all kinds of out of equilibrium behavior that crucially affects the relative performance of different multi unit auction rules. See, e. g. Kagel and Levin [18], List and Lucking–Reiley [19], and Engelmann and Grimm [9].

bounded rationality (in the sense of QRE) may explain the discrepancy between theory and evidence. Conclusions and guidelines for future research are listed in Section 6, followed by an Appendix containing the proofs of the theoretical results of Section 2 and the experimental instructions.

2 The model

In section 2.1, we present the basic model and introduce the two mechanisms object of our experiment. Then, in sections 2.2 and 2.3 we characterize the equilibria of the two mechanisms.

2.1 The Model

Consider a seller who has a fixed quantity (normalized to 1) of a perfectly divisible good and does not know the number of potential buyers interested in the good. By analogy with our experimental conditions, let us assume that n, the number of buyers, is either 2 or 4, where the probability that n is 2 (4) is λ $(1-\lambda)$. Throughout the paper, we shall refer to the case of $n = 2$ $(n = 4)$ as the "low" ("high") demand scenario. We assume that all potential buyers are identical. In particular, each buyer i has decreasing linear demand for the good,

$$x_i(p) = 1 - p. \tag{1}$$

In what follows, we provide a theoretical analysis of two mechanisms: the Fixed Price Mechanism (FPM) and an Incentive Compatible Mechanism (ICM), which is identical to FPM apart from the fact that bidders are never rationed (i.e. they always get what they ask for).

2.2 FPM

We model FPM as a 3-stage, 4-player game with incomplete information. At Stage 0 Nature moves, deciding market size n. Either two or four players participate in the market. In a market with two players, players are labeled "1" and "2".[6] If $n = 4$, they are labeled "1" to "4". In what follows we look at the payoff of the representative player 1, who participates in the market, not knowing the number of his competitors.

[6]In the experiment, the other two players participated in a separate two–player market. Since we analyze the decision of the representative player 1, we ignore the existence of this parallel market.

At the remaining two stages, the seller and the buyers move in sequence. At Stage 1, the seller announces a fixed price and an upper limit on individual bids $(p, \overline{d}) \in [0,1] \times R_{+}$. At Stage 2, each participating buyer i announces the quantity he demands at the posted price, $d_i \in [0, \overline{d}]$, which we will call buyer i's bid. If aggregate bids fall short of supply, each buyer obtains his bid, otherwise buyers are proportionally rationed. Each buyer has to pay the posted price for the quantity he receives.

We formally describe proportional rationing by the following notation. Let $d \equiv \{d_i\}$ be the vector of bids and denote by $d_{-i} \equiv \{d_j\}_{j \neq i}$ the vector of bids by i's opponents. Then, aggregate bid is given by $\sum_{i=1}^{n} d_i$, $n \in \{2,4\}$. Under proportional rationing, buyer 1 demanding d_1 receives a final quantity of $d_1 Q^n(d)$, where

$$Q^n(d) = \min\{1, \frac{1}{\sum_{j=1}^{n} d_j}\}, \quad n \in \{2,4\}. \tag{2}$$

We can now specify players' expected payoffs. Let "0" index the seller's player position and recall that we only consider the representative bidder "1". Now, for a given pair $(p, \overline{d})$, let $\pi_i : [0, \overline{d}]^4 \to R$ denote player i's expected payoff, given by

$$\pi_0(d) = \lambda Q^2(d) \sum_{j=1}^{2} d_j \cdot p + (1-\lambda) Q^4(d) \sum_{j=1}^{4} d_j \cdot p \tag{3}$$

and

$$\pi_1(d_1, d_{-1}) = \lambda \int_0^{d_1 Q^2(d_1, d_{-1})} (1 - x - p) dx + (1-\lambda) \int_0^{d_1 Q^4(d_1, d_{-1})} (1 - x - p) dx. \tag{4}$$

The extension to mixed strategies of the payoff structure (4) is straightforward, once we assume that players mix independently. If $\delta_i \in \Delta([0, \overline{d}]) \equiv \Delta_i$ ($\delta_{-i} \in \Delta([0, \overline{d}]^3) \equiv \Delta_{-i}$) denotes a generic mixed strategy for player i('s opponents), with $\delta_i(d_i)$ ($\delta_{-i}(d_{-i})$) denoting the probability of bidding d_i (d_{-i}) under δ_i (δ_{-i}), then $\pi_1(\delta_1, \delta_{-1})$ defines player 1's expected profit of a generic mixed strategy profile.

2.2.1 Stage 2: the bidding stage

In the remainder of this section, we shall restrict our attention to pure strategy profiles. We begin by characterizing bidders' optimal behavior given the price p and upper limit on bids $\overline{d} \geq 1$.

Proposition 1 (Equilibria of Stage 2) *Let* $p_e = \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$ *and* $p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.

- $p \in [\frac{3}{4}, 1]$*: unique equilibrium* $d_i(p) = 1 - p$ *for all* i.
- $p \in [0, p_e)$*: unique equilibrium* $d_i(p) = \overline{d}$ *for all* i.
- $p \in [p_e, p_m)$*: two equilibria,* $d_i(p) = \overline{d}$ *for all* i *and* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *for all* i.
- $p = p_m$*: one equilibrium where* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *and a continuum of equilibria where* $d_1 + d_2 \geq 1$*: all* d *with* $d_i = d_j$*, for all* i, j.
- $p \in (p_m, \frac{3}{4})$*: unique equilibrium* $d_i(p) = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}$ *for all* i.

Proof. In the Appendix. ■

Figure 1 provides a graphical sketch of the structure of the game's equilibria, as characterized by Proposition 1.

Put Figure 1 about here

As Figure 1 shows, the interval of possible prices can be split up into three subintervals:

- *High prices:* $p \in [\frac{3}{4}, 1]$. Buyers' aggregate demand never exceeds supply. Therefore, rationing plays no role and buyers' optimal strategy is to simply bid truthfully.
- *Low prices:* $p \in [0, p_e)$. Large excess demand in the high demand scenario (and, at prices below $\frac{1}{2}$, also excess demand in the low demand scenario) yields an incentive to overstate true demand high enough to lead to rationing in both scenarios. Thus, bids explode and the only equilibrium is that every buyer bids as much as possible.
- *Intermediate prices:* $p \in [p_e, \frac{3}{4})$. Excess demand in the high demand scenario is moderate, which still yields an incentive to overstate demand. The optimal bids solve a trade-off between getting too much in the low demand scenario (where no rationing takes place) and getting

too little in the high demand scenario (where buyers are rationed). When $p = p_m$, the game has a continuum of symmetric (pure) equilibria, one for every possible bid $d_i \in [\frac{1}{2}, 1]$. For prices $p \in [p_e, p_m]$ there is also an equilibrium where demand explodes, like in the case of low prices.

2.2.2 Stage 1: price and upper-bound fixing

In Stage 1 the seller chooses the profit maximizing price anticipating buyers' behavior at Stage 2, not knowing how many of them will participate in the market. Taking into account buyers' equilibrium bids, only prices in the interval $p \in [p_e, \frac{3}{4}]$ can be rational choices of the seller: at p_e he sells the whole quantity in both demand scenarios in any equilibrium of Stage 2 and it would definitely lower his profit if he posted a lower price. Notice that $p = \frac{3}{4}$ is the linear monopoly price given high demand and thus, a higher price cannot be profit maximizing under demand uncertainty. Given these considerations, we are now in the position to solve the entire mechanism in the following

Proposition 2 (Equilibria of FPM) *An equilibrium of FPM always exists and has the following properties:*

(i) The entire quantity is sold at $p_e \leq \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$ *in every demand scenario.*

(ii) The seller optimal choice of $\bar{d}$ *is* $\bar{d}^* \geq 1$.

(iii) The seller's revenue is bounded below by p_e *and may be higher.*

Proof. In the Appendix. ■

2.3 ICM

As we already explained in the introduction, we also tested in the lab another fixed-price mechanism -we called it ICM- which only differs from FPM with respect to the fact that bidders always get what they ask for (i..e there is no rationing). In this case, player 1's payoff function (4) simplifies to

$$\pi_1(d_1, d_{-1}) = \int_0^{d_1} (1 - x - p)dx = \frac{1}{2}d_1(2 - 2p - d_1). \qquad (5)$$

The absence of rationing breaks any strategic link among the players, who basically face a simple decision problem, whose solution is truthful bidding.

Proposition 3 (Equilibria of ICM) *In ICM each bidder's optimal bid equals his true demand, i. e*

$$d_i^*(p) = 1 - p. \tag{6}$$

In our experiment, ICM mainly serves as a robustness check for our experimental design, to evaluate whether subjects bid truthfully when it is a strictly dominant strategy to do so. This replicates the situation of uniform price auctions in large markets (e. g. IPOs), where bidders cannot lower the price by reducing demand, making truthful bidding a dominant strategy. However, a crucial difference is that in large auctions bidders are well aware of the fact that they interact with other players, which may crucially influence their behavior.

3 The experimental design

In what follows, we describe the features of the experiment in detail.

Subjects. The experiment was conducted in three subsequent sessions -two sessions devoted to FPM, one to ICM- in May, 2004. A total of 72 students (24 per session) were recruited among the undergraduate student population of the Universidad de Alicante -mainly, undergraduate students from the Economics Department with no (or very little) prior exposure to auction theory. The FPM sessions lasted approximately 120' each, while the ICM session was slightly shorter (100' approx.).

Subjects were given a written copy of the instructions in Spanish, together with a table indicating their monetary payoff associated with a grid of 21×21=441 representative price-quantity pairs.[7] Instructions were read aloud and we let subjects ask about any doubt they may have had. In addition, a self-paced, interactive computer program proposed three control questions, to make sure that subjects understood the main features of game. In particular, we checked the comprehension of the rationing rule and the downward sloped demand function.

Treatment. In each session, subjects played 84 rounds of the corresponding mechanism. As for the FPM sessions, subjects were divided into three *cohorts* of 8, with subjects from different cohorts never interacting with each other throughout the session. As for the ICM session, every subject can be considered as a "cohort of size one".

[7]The complete set of instructions, translated into English, can be found in the Appendix.

Compared with the scale used in Section 2, in the experiment, all prices and quantities were multiplied by 10. We did this to mitigate "integer" frame problems.[8] Within each round $t = 1, ..., 84$, group size, composition and prices were randomly determined. Let *period* $T_k = \{t : 21(k-1) < t \leq 21k\}, k = 1, ..., 4$, be the subsequence of the $k-$th 21 rounds. Within each period T_k, subjects experienced each and every possible price $p \in P = \{0, .5, 1, ..., 10\}$, the sequence of prices randomly selected within each period being different for each cohort. After being told the current price, subjects had to determine their bid, $d_i(p) \in [0, 10]$, for that round (subjects could not bid more than the entire supply). By this design, we are able to characterize 4 complete individual bid schedules, one for each period. Moreover, in each round t, a (uniform) random draw fixed the group size $n \in \{2, 4\}$ independently for each cohort (i.e. $\lambda = \frac{1}{2}$).

Given all these design features, we shall read the data under the assumption that the history of each individual cohort (6 for FPM, 24 for ICM) corresponds to an independent observation of the corresponding mechanism.

Payoffs. Subjects participating in the FPM (ICM) sessions received 2000 (1500) ptas. (1 euro is approx. 166 ptas.) just to show up. These stakes were chosen to exclude the possibility of bankruptcy.

Ex-post information. After each round, subjects were informed on the payoff relevant information. As for FPM, this refers to group size, summary information on the aggregate behavior of their own group (both in terms of the total sum of individual bids, but also of the average bid(s) of the other member(s) of their group), the quantity of the good they actually received (FPM), together with the monetary payoff associated with it. As for ICM, subjects were simply told about the result of their individual bid. The same information was also given in the form of a *History Table*, so that subjects could easily review the results of all the rounds that they had played so far.

4 Results

In this section, we report the results of our experiment. We begin by presenting some descriptive statistics which summarize the evolution of subjects' aggregate behavior over time in ICM and FPM. We then estimate dynamic panel data regressions. As for ICM, these regressions clearly show that equilibrium analysis almost perfectly explains subjects' behavior, at least in the last repetitions of the game. This is also true in the case of FPM,

[8] Neverthelesss, in presenting the results, we shall not modify the scale to facilitate comparison with the content of Section 2.

even though our regressions unambiguously show persistent deviations from equilibrium behavior. In short, in FPM people tend to overbid (underbid) the equilibrium strategy when rationing is less (more) severe.

4.1 Descriptive statistics

Subjects played ICM extremely "well". Their behavior is close to equilibrium from the very beginning, with some initial variance quickly vanishing over time. Out of 21 prices, in period 3 (4), *all* 24 subjects *always* played their dominant strategy in 19 (17) cases. Even when equilibrium play does not correspond to subjects' unanimous decision, deviations from the dominant strategy are negligible and only observed on behalf of few subjects.[9]

Things are different when we move to FPM. Figure 4 provides a graphical sketch of the evolution of subjects' aggregate behavior, tracing the average bids in the four experimental periods. The y-axis tracks prices, while the x-axis reports average bids. The dotted line corresponds to the equilibrium strategy as given by Proposition 1; the 4 grey lines correspond to aggregate average bid functions per period, with greyscale increasing with periods.

Put Figure 4 about here

Recall from Section 2 that the structure of the equilibria of $FPM(p)$ crucially depends on the price level. Thus, we present our experimental evidence for three broad price intervals, which turn out to be crucial not only in the theoretical analysis, but also to evaluate subjects' behavior in the experiment:

At high prices ($p \geq \frac{3}{4}$), where truthful bidding corresponds to the unique equilibrium, we observe that subjects start bidding slightly more than their demand, with overbidding gradually reducing with time. At low prices ($p < p_e$), where demand explosion corresponds to the unique equilibrium, individual bids get very close to the maximum possible amount of 1. However, contrary to the theoretical prediction, average bids seem to be sensitive to prices: the lower the price, the closer average bids get to the upper limit. At intermediate prices ($\frac{3}{4} > p \geq p_e \cong 0.568$), subjects start bidding above equilibrium, with bids increasing (i.e. moving away from equilibrium) with time.

[9] Figure 2 (3) in the Appendix reports average bids and standard deviations for ICM (FPM), disaggregated for prices and periods.

We finally look at the experimental evidence from the seller's viewpoint. Figure 5 plots the evolution of expected profits (y-axis) as a function of the ruling price given the observed behavior.[10]

Put Figure 5 about here

As Figure 5 shows, at low prices ($p \leq p_m$) actual profits equal their equilibrium levels. This is basically due to the fact that, within this price range, out-of-equilibrium underbidding is not sufficient to prevent subjects to be rationed in both demand scenarios. As a consequence, the entire supply is always sold, independently on the demand scenario. At high prices ($p \geq \frac{3}{4}$) expected profits start above equilibrium (due to overbidding), but converge quickly to their equilibrium level.

At intermediate prices ($p_e < p < \frac{3}{4}$) initial overbidding raises the seller's profits above their equilibrium levels. Moreover, since overbidding within this price range increases with time, also the seller's profits increase. Recall (Proposition 2) that the profit-maximizing price always lies in the intermediate price range. Therefore, *persistent overbidding takes place exactly within the price range that would be selected by a profit maximizing seller.* In consequence, actual profits always exceed the equilibrium level and even increase with time (up to 12% above the theoretical prediction, since actual and predicted behavior lead to profits of .65 and .583 respectively).[11]

4.2 Panel-Data Regressions

In this section, our main concern is to check whether the discrepancies between observed and predicted behavior are statistically significant. To this aim, we construct a panel containing all decisions of all subjects at all times. Remember that each subject participated in 84 rounds of ICM (FPM), which

[10] Note that, in the range $[p_e, p_m]$, FPM has multiple equilibria and, therefore, also seller's profits are not uniquely determined.

[11] To illustrate the profitability of FPM, suppose that seller and buyers knew the market size, n. In such a case, the unique equilibrium would require the seller to set the linear monopoly price (i.e. either $p = \frac{1}{2}$ if $n = 2$, or $p = \frac{3}{4}$ if $n = 4$) and buyers to bid truthfully. Thus, the whole amount would be sold in both scenarios and the ex-ante expected revenue would be the expected monopoly profit $MP = \frac{1}{2}\lambda + \frac{3}{4}(1 - \lambda)$. Since both scenarios are equally likely in our experiment (i.e. $\lambda = \frac{1}{2}$), $MP = .625$. Since the theoretical expected revenue in FPM (.583) is lower than the expected linear monopoly profit theory predicts that the seller prefers a situation of full information. However, given the observed behavior the seller's profits are .65, which is higher than the expected monopoly profit.

creates a panel where subjects serve as the cross-sectional variable. The sample size is 24 (48) subjects for ICM (FPM) session(s).

As for the ICM data, we use a simple random-effect linear regression. The underlying model assumes subjects playing linear bid functions, one for each period $T_k, k = 1, ..., 4$. The model includes period as a regressor, individual (random) effects and idiosyncratic errors as follows:

$$d_{it} = \alpha + \beta p_t + \gamma T_k + \epsilon_i + \varepsilon_{it}, \tag{7}$$

where T_k denotes period as defined in Section 3; ϵ_i describes the unobserved time-invariant heterogeneity which characterizes subject i and ε_{it} is an idiosyncratic error term (we assume that $\epsilon_i \perp \varepsilon_{it}$).[12] Since, for ICM, the unique equilibrium corresponds to truthful bidding, null hypotheses for our tests are $\alpha = 1, \beta = -1$ and $\gamma = 0$.

Figure 6 reports the estimates of (7) (standard errors within brackets) for the whole ICM dataset, regression (I), and disaggregated for period, regressions (II-V).

Put Figure 6 about here

As it can be seen from the fits of regressions (I-V), bidders played closely to the assumed linear function in all periods. In regression (I), our model explains more than 92% of subjects' behavior. The R^2 jumps from .735 in (II), to .9873 in (III) and stays above .99 in (IV-V). A very low fraction of variance turns out to be due to the individual effects of the experimental subjects (measured by ρ). In other words, it seems that all subjects learned very quickly to play the equilibrium, which leads to completely homogeneous play. Consequently, ρ reaches 0 in the last two periods.

The estimates of α and β for the whole dataset (regression I) are significantly different from the theoretical prediction, as well as the estimated parameter of γ (positive γ meaning increasing bids across periods). However, if we look at regressions (II-V), we find that only parameters of period T_1 (regression II) are significantly different from their theoretical values. We cannot reject the hypothesis that the observed and the predicted behavior

[12]Notice that equation (7) implicitely assumes that each individual history correponds to an independent observation. This is certainly the case for ICM, but not for FPM, although many details of the experimental design (such as anonymous and random matching within each cohort) has been especially set to minimize "repeated game effects".

differ (neither independently, nor jointly) for regressions (III-V). This basically implies that learning mostly takes place in the first repetitions of the experiment and behavior stabilizes from T_2 on.[13]

The FPM cross-sectional time-series analysis is more complex and results are less straightforward. By analogy with regressions (I-V), Figure 7 reports estimates of a model which assumes subjects playing a 3-piecewise linear bid function, as follows:

$$d_{it} = \alpha_0 + \alpha_1 \eta_t + \alpha_2 \theta_t + \beta_0 p_t + \beta_1 p_t \eta_t + \beta_2 p_t \theta_t + \gamma T_k + \epsilon_i + \varepsilon_{it} \qquad (8)$$

where η_t and θ_t are two index functions such that $\eta_t = 1$ if $p_t \leq .55$ and $\eta_t = 0$ otherwise, whereas $\theta_t = 1$ if $p_t \in (.55; .75)$ and $\theta_t = 0$ otherwise. Observe that the dummies η_t and θ_t partition the price set into the three subintervals that emerged our theoretical analysis. In consequence, we estimate three different — but, through the individual effects ϵ_i, interdependent — linear bid functions, one for each subinterval. β_0 and $(\beta_0 + \beta_1)$ measure the sensitivity of bids on price for high and low prices, respectively, whereas α_0 and $(\alpha_0 + \alpha_1)$ determine the constant terms. By analogy, the slope $(\beta_0 + \beta_2)$ and the constant term $(\alpha_0 + \alpha_2)$ determine the estimated bid function at the intermediate subinterval.

Note that (8) can be interpreted as the natural extension of (7) to the case of FPM subject to some conditions, which we now discuss.

First, recall from Section 2.2 that there is a multiplicity of equilibria for $p \in [p_e, p_m]$. Given the price grid used in the experiment, multiplicity only occurs at $p = .6$, with equilibrium bids being 1 and .461, respectively. In order to check which of these equilibria is somehow "more consistent" with our experimental evidence, we run a Wald test with following null hypotheses: $\overline{d}(.6) = 1$ ($\overline{d}(.6) = .461$). We can (not) reject the null hypothesis which suggests that subjects bid more consistently with the equilibrium where moderate bidding prevails. Consequently, we include $p = .6$ into the intermediate price interval.[14]

Second, the equilibrium bid function of FPM is not linear (but concave) in the intermediate price interval. However, as Figure 1 shows, the demand function may well be approximated by a straight line.

[13] We also run a regression analogous to (I) excluding observations coming from T_1. As expected, the null hypotheses on α, β and γ cannot be rejected, neither independently nor jointly.

[14] P-values are 0 and .4787, respectively. In any case, we also run regressions excluding observation at $p = .6$. Results do not change (and are available on request).

Figure 7 reports the estimation results. By analogy with Figure 6, we also provide estimations disaggregated by periods (regressions (VII-X)) to allow for inter-period comparisons.

Put Figure 7 about here

Figure 8, tracing the estimated inverse bid functions disaggregated by periods.

Put Figure 8 about here

Again, subjects' behavior is close to the equilibrium bid function, although not as close as for ICM. Moreover, bidding behavior evolves quite differently for the different price intervals. Therefore, we discuss the results of the estimations separately for low, intermediate, and high prices.

- High prices ($p \geq \frac{3}{4}$). Here, the statistical analysis confirms the observations of section 4.1 that behavior converges to truthful bidding, as predicted. In T_4, the null hypothesis ($\alpha_0 = 1$ and $\beta_0 = -1$) cannot be rejected, neither jointly (p-value .2530), nor independently (p-values .1219 and .1567, respectively).

  Looking at the evolution of subjects' bidding behavior we find that both, α_0 and β_0 do not change significantly from T_1 to T_4. Moreover, initial variability of bids is moderate and decreases with time.

- Low prices ($p < p_e$). In this interval the null hypothesis corresponds to $\alpha_0 + \alpha_1 = 1$ and $\beta_0 + \beta_2 = 0$ (i.e. bids coincide with the upper bound and therefore, are independent of prices). Here, a Wald test leads to rejection of the joint and two independent hypotheses. Figure 8 indeed suggests that there exists a negative dependency of bids on prices.

  The estimated value of β_1 significantly increases from T_1 to T_4 which means that bids become less price sensitive as time proceeds. However, some price sensitivity remains. Moreover, variability of bids initially is higher than in the previous case (i. e. at $p \geq \frac{3}{4}$) and then also decreases with time.

- Intermediate prices ($\frac{3}{4} > p \geq p_e \cong 0.568$). Within this interval, the estimated bid function coincides with equilibrium if $\alpha_0 + \alpha_2 = 1.255$

and $\beta_0 + \beta_2 = -1.324$, respectively. We reject the joint test at any significance level (p-value 0). In case of two independent tests, we reject the former hypothesis, but not the latter (p-values are .0296 and 0.1106, respectively). In other words, the estimated bid function has a similar slope as the equilibrium one, while the estimated constant is bigger.

From period T_1 to T_4, α_2 increases and β_2 decreases, both significantly. As Figure 8 illustrates, overall bids move away from equilibrium. The variability of bids initially is moderate and stays basically constant over time.

To summarize, our panel-data analysis suggests that subjects behaved almost perfectly in line with the equilibrium prediction in ICM and at high prices in FPM (where the equilibria of both games coincide with truthful bidding). For the remaining prices, the observed behavior in FPM significantly differs from equilibrium. For intermediate prices, bids are as price sensitive as predicted, however, at a higher level. Bidders start overbidding insignificantly, but over time overbidding increases and becomes significant. At low prices, bids negatively depend on price level, contrary to prediction. This results in underbidding relative to equilibrium. Although underbidding tends to disappear (the estimated inverse bid function shifts to the right), some price dependence remains.

5 Out-of-equilibrium behavior: risk aversion *vs.* bounded rationality

Our experimental results show that the equilibrium analysis developed in Section 2 is an (extraordinary) good predictor of subjects behavior (as far as ICM is concerned). This consideration notwithstanding, our regressions also show that subjects consistently deviate from equilibrium play, and that these deviations (with particular reference to overbidding at intermediate prices and sensitivity of bids on prices at the low price interval) do not seem to vanish over time. To understand this empirical regularity of our experimental evidence, we resort to two "usual suspects", often invoked to explain deviation from the risk-neutral equilibrium behavior in auction experiments, that is, risk aversion and bounded rationality.

5.1 Risk aversion

Risk-aversion has proved to be an important behavioral factor in explaining subjects' behavior in auction experiments (see, for example, [7] or [12]). In the context of FPM, player 1 has to trade-off the risk of getting too much in the low demand scenario against the risk of getting too little in the high demand scenario. These risks have to be pondered by the relative likelihood of each scenario (measured by λ). Clearly, risk aversion may play a role only when these risks affect player 1's payoffs in opposite direction (i.e. in the case of rationing only in the high demand scenario).[15] Even then, it is not obvious to predict how risk aversion should modify the equilibrium behavior of Section 2. By the above considerations, it is clear that, if λ is sufficiently high (low), we expect risk-averse players to over(under)bid with respect of the equilibrium strategy, independently of what their degree of risk-aversion is. For intermediate values of λ, results will depend on how risk-aversion is formally defined.

By analogy with our experimental conditions, we shall explore equilibrium properties of FPM when (constant relative) risk aversion is taken into account and $\lambda \geq \frac{1}{2}$. To this aim, we modify the theoretical framework of Section 2 by considering preferences consistent with a (CRRA) function of expected payoffs, as follows:

$$u_i(d) = \lambda \frac{\pi_i^2(d)^{1-\rho}}{1-\rho} + (1-\lambda)\frac{\pi_i^4(d)^{1-\rho}}{1-\rho}, \tag{9}$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion. The case of $\rho = 0$ coincides with risk-neutrality (i.e. it covers the theoretical analysis we carried out in Section 2), with (CR) risk aversion increasing with ρ.

Proposition 4 (Equilibria with (CR)risk aversion) *For all $\rho > 0$, the structure of equilibria of Stage 2 of FPM when payoffs are defined by (9) is as follows:*

1. $p \in [\frac{3}{4}, 1]$*: unique equilibrium* $d_i(p) = 1 - p$ *for all* i.
2. $p \in [0, p_e)$*: unique equilibrium* $d_i(p) = 1$ *for all* i.
3. $p \in (p_m, \frac{3}{4})$*: unique equilibrium* $\check{d}_i(p)$. *If* $\lambda \geq \frac{1}{2}$, *then*

   (i) $\check{d}_i(p) > d_i^*(p)$, *for all* $\rho > 0$ *and*

[15] By the same token, risk aversion plays no role in ICM, where rationing never occurs.

(ii) $\left|\check{d}_i(p) - d_i^*(p)\right|$ *is bounded above by* $\frac{1}{40}$.

4. $p \in [p_e, p_m)$: *two equilibria,* $d_i(p) = 1$ *and* $\check{d}_i(p) < 1$.

5. $p = p_m$ *continuum of equilibria: all* d *with* $d_i = d_j$, *for all* i, j *and* $d_1 + d_2 \geq 1$

Proof. In the Appendix. ■

As Proposition 4 shows, risk aversion has the effect of "discounting" the risk of getting too much against the risk of getting too little. It is important to notice that at intermediate prices, this does not necessarily imply bidding more than $d_i^*(p)$. Overbidding only occurs when λ is sufficiently high (e.g. when $\lambda \geq \frac{1}{2}$). In any case, deviations from equilibrium due to risk-aversion are negligible (less than half a decimal point in our parameter setting). Furthermore, risk aversion has no effect on equilibria at low prices and cannot therefore explain the sensitivity observed in the lab. This is the reason why, in what follows, we shall explore an alternative justification for the discrepancy between theory and evidence in our experimental data.

5.2 Bounded rationality and Quantal Response Equilibrium

While risk-aversion explains deviation from predicted behavior as the result of a process of conscious (expected) utility maximization, in what follows, we shall assume that subjects' choices are also affected by other (unmodeled) external factors that make this process intrinsically *noisy*. This noise may be induced by the complexity of the game, limitation of subjects' computational ability, random preference shocks, etc. This kind of choice framework may be modeled by specifying the payoff associated with a choice as the sum of two terms. One term is the expected utility of a choice, given the choice probabilities of other players. The second term is a random variable that reflects idiosyncratic aspects of payoffs that are not modeled formally.

Clearly, properties of this alternative class of models crucially depend on the specific way in which the stochastic process that generates noise is formally defined. One approach that has received attention recently involves the concept of *quantal response equilibrium* (QRE), developed by McKelvey and Palfrey [20] in the context of finite games. A quantal response is, basically, a "smoothed-out best response", in the sense that agents are not assumed to select the strategy that maximizes their expected payoff with probability one. Instead, each pure strategy is selected with some positive probability, with this probability increasing in expected payoff.[16]

[16] See also Rosenthal [23].

Some recent papers (such as [2], [12]) have modified the notion of QRE to deal with games with a continuum of pure strategies, such as our ICM and FPM. A logit response function is often used to model the QRE. Formally, the standard derivation of the logit model is based on the assumption that payoffs are subject to unobserved preference shocks from a double-exponential distribution (e.g., Anderson *et al.* [1]). In this case, a (logit) QRE would be the fixed point

$$\delta_i(d_i) \equiv f_i(d_i|\delta_{-i}, \mu) = \frac{\exp\left[\pi_i(d_i, \delta_{-i})\mu\right]}{\int_0^1 (\exp\left[\pi_i(s, \delta_{-i})\mu\right])ds}, i = 1, ..., 4, \tag{10}$$

where $\pi_i(d_i, \delta_{-i})$ is the expected payoff associated with the pure strategy d_i against $\delta_{-i} \in \Delta_{-i}$, and μ is the noise parameter. As $\mu \longrightarrow \infty$, the probability of choosing an action with the highest expected payoff goes to 1. Low values of μ correspond to more noise: if $\mu \longrightarrow 0$, the density function in (10) becomes flat over the entire support and behavior becomes essentially random.

As we just noticed, a (logit) QRE is a then vector of densities that is a fixed point of (10). Continuity of the payoff function $\pi_i(.)$ ensures existence, both in the case of ICM and FPM. While Section 5.2.1 explicitly characterizes the (unique) logit equilibrium in the case of ICM, for FPM no explicit solution can be found. This is because FPM is a game with a continuum of pure strategies, for which logit equilibria can be calculated only for very special cases.[17] In this case, we are only able to evaluate a QRE numerically. This equilibrium has the property that, when $\mu \longrightarrow \infty$, it converges to the (unique) equilibrium we derived in Section 2.

5.2.1 ICM

Fix a price $p \in [0, 1]$ and consider the associated game induced by ICM. By (5), equilibrium distribution functions can be calculated as follows:

$$f(d_i|\mu) = \frac{\exp\left[\frac{\mu d_i(2-2p-d_i)}{2}\right]}{\int_0^1 \exp\left[\frac{\mu d_i(2-2p-y)}{2}\right] dy}. \tag{11}$$

In Figure 9 we use standard maximum-likelihood techniques to estimate the value of μ in each period and each treatment. The second line of Figure

[17] Such as potential games, as in Anderson *et al.* [3].

9 reports these estimations, using ICM data. The estimated noise parameter jumps dramatically between T_1 and T_2 and reaches its highest value at T_3. It then decreases at T_4, but is still significantly higher than at T_1. This confirms the finding that the learning mostly takes place in T_1.

Put Figure 9 about here

In Figure 10, we trace the equilibrium densities $f(d_i|\mu)$ for three price values: p=.2, p=.65 and p=.8. Every graph plots four curves, one for each period.

Put Figure 10 about here

Not surprisingly, these distributions are unimodal at the value $(1-p)$ -the (equilibrium) pure strategy associated with the higher expected payoff- and become flatter as μ goes to zero. We are interested in the behavior of (equilibrium) expected bids $\hat{d}_i(\mu) = \int_0^1 d_i f(d_i|\mu) dd_i$. In the following Figure 11 we trace four demand functions with the same values of μ as in Figure 10.[18]

Put Figure 11 about here

The effect of the noise (whose magnitude is measured by μ) is to create underbidding (with respect to optimal behavior) when the price is low(er than .5), and overbidding when the price is high(er than .5). This threshold value is independent of μ. To see why, notice that equilibrium distributions (11) are *symmetric with respect to the mode (i.e. wrt* $x_i(p)$). This is because the payoff function (5) is also symmetric with respect to true demand $x_i(p)$, given, by (5), $\frac{d\pi_i}{dd_i} = 1 - d_i - p$. This, in turn, implies that equilibrium average bids are biased toward the center: the cost (in terms of a payoff loss) of deviating by an ε is exactly the same whether deviation is upward or downward. What creates the bias is that deviations toward the center are more likely (since, by (10), every pure strategy belongs to the support of the logit equilibrium).

Figure 12 reports, for each period, average bids as in the experiment and in the estimated QRE (with the dotted line tracing the equilibrium strategy). T_1 graph shows that QRE predicts well the slight overbidding (underbidding) when prices are high (low). The observed threshold where the average bid

[18]The dotted line traces the predicted behavior.

switches from overbidding to underbidding (as price decreases) is situated around $p = .5$, consistently with the QRE prediction. From T_2 on, the three curves almost coincide, as we know already from Section 4.

Put Figure 12 about here

5.2.2 FPM

The last three rows of Figure 9 reports the maximum-likelihood estimates of μ in case of FPM. In each period, two sets of estimations are presented: one estimate per price interval (first column), and a unique estimate for the entire price spectrum (second column).

A first look of Figure 9 confirms the findings of Section 4. In the case of the single estimate for each period, the estimated μ raises gradually from 46 in T_1 to a final 150 in T_4. This suggests that as time proceeds, the observed behavior gets closer to the equilibrium prediction.

The per-interval estimations deserve a more detailed discussion. In T_1 the estimates of μ for the intermediate interval exceeds considerably both the low and high intervals. However, from T_2 on, it decreases slightly until it becomes the lowest in T_4. From T_3 on, μ is the highest in the high price interval.

As we previously mentioned, solutions for QRE in the case of FPM have been evaluated numerically. The corresponding distributions are plotted in Figure 13.

Put Figure 13 about here.

Equilibrium distributions are analogous to the ICM case only for very high prices. By contrast, for very low prices, distributions are unimodal at 1. This is clearly due to rationing. More importantly, for prices $.75 > p \geq p_e$, the QRE distribution is not unimodal at $(1-p)$, but has a mode at a higher level and is skewed to the right. Given we cannot provide an explicit solution for the QRE in the case of FPM, we can only search for intuitions for this (numerical) result by getting back to FOCs in the "high-demand rationing case":

$$\frac{d\pi_1}{dd_1} = \lambda(1-p-d_1) + (1-\lambda)\frac{\left(\sum_{j>1} d_j\right)^2 + pd_1 \sum_j d_j}{\left(\sum_j d_j\right)^3}. \tag{12}$$

By (12), the derivative is decreasing and convex. This, in turn, implies that *deviations in the direction of overbidding are relatively cheaper* (and, therefore, by (10), overbidding with respect to d_i^* is more likely to occur). Furthermore, the larger fraction of bidders overbids, the more attractive overbidding becomes for others. In other words, if overbidding strategies grow in probability, their payoff becomes relatively higher and this, by (10), reinforces the bias toward overbidding induced by the asymmetry in relative costs. These observations are well illustrated by Figure 14, which is the analogy with Figure 11 in case of FPM.

Put Figure 14 about here.

The qualitative features of Figure 14 reproduce our experimental evidence with remarkable accuracy, as Figure 15 shows.

Put Figure 15 about here

- $p \geq .75$. For very high prices, overbidding is basically due to the "drift effect" already discussed in Section 5.2.1.
- $p < p_e$. For very low prices, the drift effect yields underbidding (since mode correspond to the upper bound of the pure strategy space). Moreover, QRE predicts the observed sensitivity of bids on price level. This is due to the fact that the higher is the price the cheaper is to underbid the equilibrium prediction by the same amount. Therefore, it is more likely to observe such deviations the higher is the price.
- $.75 > p \geq p_e$. For intermediate prices, due to the cost asymmetries highlighted in (12), the overbidding is more likely to be observed.

6 Conclusion

Two main conclusions can be drawn from our experiment. First, equilibrium analysis provides a very good description of subjects' behavior, compared to other experimental settings. Second, there are still deviations from equilibrium, for which QRE (as opposed to risk aversion, for example) seems to produce a sufficiently consistent explanation.

We emphasize that these deviations make FPM even more attractive as a selling mechanism. Persistent overbidding of RNNE occurs exactly within the price range that would be selected by a profit maximizing seller. Revenues at this price turns out to be even higher than the expected monopoly profit.

A general and most important observation from our experimental data is that subjects were able to solve the problem well enough to achieve results closely resembling the theoretical predictions. This finding is important when it comes to the question when and where FPM should be used in practice. In this respect, two conclusions can be drawn. First, the theoretically appealing properties of FPM clearly survive (or even are improved on) in the laboratory, which suggests that FPM should be quite popular as a selling mechanism. Second, we have to keep in mind that those advantages of FPM can only be realized if the seller fixes the price correctly, anticipating buyers' bidding behavior. Thus, FPM should rather be observed in markets where sellers are experienced.

The latter observation points to a question for future research. While in our experiment we focused on buyers' behavior, the seller's decision is certainly as relevant for evaluating the attractiveness of the mechanism. Two issues are of interest here. First, does the seller anticipate bidding behavior correctly and sets the price optimally given buyers' behavior? Second, does the fact that the seller is a real player (and not imitated by the computer) change buyers' behavior at the second stage of the game?

Another natural extension of the model studied in this paper could be the replacement of proportional rationing by a different rule. Two natural candidates are *constrained equal losses* and *constrained equal awards.* The former is a rule that makes losses as equal possible, under the condition that no participant ends up with negative transfers. This rule gives priority to higher bids. Constrained equal awards is the dual rule to the constrained equal losses. In this case, supply is distributed such that each bidder receives the same amount, subject to the condition that no buyer gets more than her bid.

It is not difficult to show that the equilibria characterized in Section 2 maintain a similar feature if proportional rationing is replaced by constrained equal losses.[19] Only the intermediate price interval equilibrium may be slightly modified by the different rationing scheme. On the other hand, constrained equal awards affects the equilibrium considerably. In this case, a symmetric equilibrium is to submit the true demand for all $p > \frac{1}{2}$. For prices below this threshold, any (asymmetric) bid such that $\min\{d\} \geq \frac{1}{2}$ is

[19]FPM with constrained equal losses affects the equilibrium properties if bidders are asymmetric, since different types have very different incentives to overstate.

an equilibrium. In other words, a strong multiplicity of equilibria occurs for prices sufficiently low. How the presence of such strong strategic uncertainty may affect subjects' behavior in the lab is left for future research.

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7 Appendix

7.1 Proofs

The proof of Proposition 1 is obtained by way of four lemmas, characterizing bidders optimal behavior at all prices. Fix a pair $(p, \overline{d})$ chosen by the seller at Stage 1 and consider the corresponding vector of bids $d(p)$ at price p. Since, at a given price, the buyer's demand is just a quantity, in what follows we omit the dependency on the price to simplify notation.

Differentiating player 1's payoff function (4) with respect to d_1 yields

$$\begin{aligned} \frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} &= \lambda \left[Q^2(d_1, d_{-1}) + d_1 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} \right] (1 - d_1 Q^2(d_1, d_{-1}) - p) \qquad (13) \\ &\quad + (1 - \lambda) \left[Q^4(d_1, d_{-1}) + d_1 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} \right] (1 - d_1 Q^4(d_1, d_{-1}) - p) \end{aligned}$$

.Note that

$$\left[Q^n(d_1, d_{-1}) - d_1 \frac{\partial Q^n(d_1, d_{-1})}{\partial d_1} \right] = \begin{cases} 1 & \text{if } \sum_{j=1}^n d_j < 1, \\ \frac{\sum_{j=2}^n d_j}{(\sum_{j=1}^n d_j)^2} & \text{if } \sum_{j=1}^n d_j \geq 1. \end{cases} \qquad (14)$$

Thus, (14) is always positive whenever at least one bidder ask for a positive quantity. In the following Lemma 5, we establish that no bidder has an incentive to ask for less than her demand (independently on others' behavior):

Lemma 5 *$d_1 < 1 - p$ is strictly dominated by $d_1 = 1 - p$ for all $p \in [0, 1]$ and all $\overline{d} \in [0, R]$.*

Proof. If $d_1 < 1 - p$, then $(1 - d_1 Q^n(d) - p) > 0$, since, by (2), $Q^n(d) \leq 1$. Thus, for $d_1 \in [0, 1 - p)$, all terms in (13) are strictly positive, and thus, $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} > 0$. ■

Given Lemma 5, in what follows we shall restrict our attention to strategy profiles $d = \{d_i \geq 1 - p\}$. The remaining three lemmas establish uniqueness of the equilibrium of the bidding stage at almost all prices. We proceed by partitioning the price set [0,1] into three subintervals: *i*) prices $p \in (\frac{3}{4}, 1]$ above the market clearing price in case of 4 buyers; *ii*) prices $p \in [0, \frac{1}{2}]$ below the market clearing price in case of 2 buyers, and finally, *iii*) prices $p \in (\frac{1}{2}, \frac{3}{4}]$ in between the two market clearing prices.

Lemma 6 *Let $p \in (\frac{3}{4}, 1]$. $\pi_1(d_1, d_{-1})$ is strictly decreasing in d_1 for all d_{-1} such that $d_j > x_j$ for at least one $j \neq 1$ and $d_1 \geq \max\{d_{-1}\}$.*

Proof. If $\sum_{j=1}^4 d_j > 1$, then, by (2) it must be that $1 \geq Q^2(d) > Q^4(d)$. Thus, if $1 - d_1 Q^4 - p \leq 0$ then $1 - d_1 Q^2 - p < 0$. Since we assume $\max\{d_{-1}\} > 1 - p = x_1$, any bid $d_1 \geq \max\{d_{-1}\}$ yields a supply to bidder 1 of $d_1 Q^4 \geq \frac{1}{4} \geq 1 - p$ for all $p \in [\frac{3}{4}, 1]$ in the high demand scenario. Thus, $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 0$ and, therefore, $1 - d_1 Q^2 - p < 0$. This, in turn, implies $\frac{\partial \pi_1(d_1,d_{-1})}{\partial d_1} < 0$.

Assume instead $\sum_{j=1}^4 d_j \leq 1$. Then, it must be $1 = Q^2(d) = Q^4(d)$. Again, since $\max\{d_{-1}\} > 1 - p$ by assumption, a bid $d_1 \geq \max\{d_{-1}\}$ yields a supply to bidder 1 of $d_1 Q^n > 1-p$ in any demand scenario. Thus, $1-d_1Q^n - p < 0$ which implies $\frac{\partial \pi_1(d_1,d_{-1})}{\partial d_1} < 0$.
■

Lemma 6 implies that, at prices $p \in (\frac{3}{4}, 1]$, any buyer has a strict incentive to underbid the highest bid of his opponents if the latter exceeds true demand. Together with Lemma 5 this implies that the only equilibrium at high prices is truthful bidding.

Lemma 7 *Let $p \in [0, \frac{1}{2}]$. $\pi_1(d_1, d_{-1})$ is strictly increasing in d_1 for all d_{-1} such that $d_j \geq 1 - p$ and $d_1 \leq \min\{d_{-1}\}$.*

Proof. At prices $p \in [0, \frac{1}{2}]$, since the bidders bid at least their true demand, it holds that $Q^4 < Q^2 \leq 1$ (strict if $p < \frac{1}{2}$). Thus, if $1 - d_1 Q^2(d_1, d_{-1}) - p \geq 0$, then $1-d_1Q^4(d_1, d_{-1}) - p > 0$. If $d_1 \leq \min\{d_{-1}\}$, then $d_1 Q^2 \leq \frac{1}{2} \leq 1 - p$. Therefore, $1 - d_1 Q^2(d_1, d_{-1}) - p \geq 0$, which, in turn, implies $\frac{\partial \pi_1(d_1,d_{-1})}{\partial d_1} > 0$. ■

It follows from Lemma 7 that, at low prices, every bidder strictly wants to outbid the lowest bidder given any vector of reasonable bids of the opponents (i. e. bids above true demand). Thus, the only equilibrium at low prices is that everyone's bid equals the upper limit.

Lemma 8 *Let $p \in (\frac{1}{2}, \frac{3}{4}]$.*

1. *If there is no rationing in case $n = 2$, i. e. $d_1 + d_2 \leq 1$, then*

   (i) $\pi_1(d_1, d_{-1})$ is decreasing for all $d_1 \geq \frac{x_1}{Q^4}$.

   (ii) $\pi_1(d_1, d_{-1})$ is strictly concave in d_1 for all $d_1 \in [0, \frac{x_1}{Q^4}]$, d_{-1} such that $d_j \geq x_j$, for all $j \neq 1$.

2. *If there is rationing in case $n = 2$, i. e. $d_1 + d_2 \geq 1$, then*

   (i) $\pi_1(d_1, d_{-1}, p)$ is strictly increasing for all d_{-1} such that $d_j \geq x_j$ and $d_1 \leq \min\{d_{-1}\}$ if $p < \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.

   (ii) $\pi_1(d_1, d_{-1}, p)$ is strictly decreasing for all d_{-1} such that $d_j \geq x_j$ and $d_1 \geq \max\{d_{-1}\}$ if $p > \frac{1}{4}\frac{9-\lambda}{3+\lambda}$.

   (iii) At $p = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$, $\frac{\partial \pi_1(d_1,d_{-1})}{\partial d_1} = 0$ for any d such that $d_i = d_j$ for all i, j.

3. *Every pure strategy equilibrium of $FPM(p)$ is symmetric.*

Proof. **Part 1(i)**. Since $Q^4 < Q^2 = 1$, it holds that, if $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 0$, then $1 - d_1 Q^2(d_1, d_{-1}) - p < 0$, and thus, by (13), $\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} < 0$. $d_1 \geq \frac{x_1}{Q^4}$ yields $1 - d_1 Q^4(d_1, d_{-1}) - p \leq 1 - \frac{x_1}{Q^4} Q^4 - p = 0$, which proves the first part of the lemma.

Part 1(ii). The second derivative of π_1 with respect to d_1 is given by

$$\begin{aligned}\frac{\partial^2 \pi_1(d_1, d_{-1})}{\partial d_1^2} = & \qquad\qquad\qquad (15)\\ & -\lambda \left[Q^2(d_1, d_{-1}) + d_1 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} \right]^2 - (1-\lambda) \left[Q^4(d_1, d_{-1}) + d_1 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} \right]^2 \\ & + \lambda \left[2 \frac{\partial Q^2(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^2(d_1, d_{-1})}{\partial d_1^2} \right] (1 - d_1 Q^2(d_1, d_{-1}) - p) \\ & + (1-\lambda) \left[2 \frac{\partial Q^4(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^4(d_1, d_{-1})}{\partial d_1^2} \right] (1 - d_1 Q^4(d_1, d_{-1}) - p).\end{aligned}$$

By (14), the first two terms of the LHS of (15) must be negative. It remains to show that the sum of the last two terms is also negative. Note that

$$\left[2 \frac{\partial Q^n(d_1, d_{-1})}{\partial d_1} + d_1 \frac{\partial^2 Q^n(d_1, d_{-1})}{\partial d_1^2} \right] = \begin{cases} 0 & \text{if } \sum_{j=1}^n d_j < 1, \\ -2 \frac{\sum_{j=2}^n d_j}{(\sum_{j=1}^n d_j)^3} & \text{if } \sum_{j=1}^n d_j \geq 1. \end{cases} \qquad (16)$$

Thus, if no rationing occurs in the low demand scenario ($\sum_{j=1}^2 d_j < 1$), the third term is equal to zero. The fourth term is negative for $d_1 \in [0, \frac{x_1}{Q^4}]$, since for those bids it holds that $1 - d_1 Q^4(d_1, d_{-1}) - p \geq 0$.

Part 2(i). We substitute (14) into (13) to get

$$\begin{aligned}\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = & \lambda \frac{d_2}{(d_1 + d_2)^2} (1 - \frac{d_1}{d_1 + d_2} - p) \qquad (17)\\ & + (1-\lambda) \frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2} (1 - \frac{d_1}{\sum_{j=1}^4 d_j} - p).\end{aligned}$$

Note that, if $d_1 \leq \min\{d_j\}$, then $\frac{d_2}{(d_1+d_2)^2} \geq \frac{1}{4}$, $\frac{d_1}{d_1+d_2} \leq \frac{1}{2}$, $\frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2} \geq \frac{3}{16}$, and $\frac{d_1}{\sum_{j=1}^4 d_j} \leq \frac{1}{4}$. Substituting in yields

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} \geq \lambda \frac{1}{4} (1 - \frac{1}{2} - p) + (1-\lambda) \frac{3}{16} (1 - \frac{1}{4} - p) > 0 \quad \Leftrightarrow p < \frac{1}{4} \frac{9 - \lambda}{3 + \lambda}.$$

Part 2(ii). By (17), if $d_1 \geq \max\{d_j\}$, then $\frac{d_2}{(d_1+d_2)^2} \leq \frac{1}{4}$, $\frac{d_1}{d_1+d_2} \geq \frac{1}{2}$, $\frac{\sum_{j=2}^4 d_j}{(\sum_{j=1}^4 d_j)^2} \leq \frac{3}{16}$, and $\frac{d_1}{\sum_{j=1}^4 d_j} \geq \frac{1}{4}$. Substituting in yields

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} \leq \lambda \frac{1}{4} (1 - \frac{1}{2} - p) + (1-\lambda) \frac{3}{16} (1 - \frac{1}{4} - p) < 0 \quad \Leftrightarrow p > \frac{9 - \lambda}{4(3 + \lambda)},$$

which proves part 2(ii) of the lemma.

Part 2(iii). For any vector of equal bids such that $d_1 + d_2 \geq 1$, (13) simplifies to

$$\frac{\partial \pi_1(d_1, d_{-1})}{\partial d_1} = \frac{1}{d_1} \left[\lambda \frac{1}{4} (1 - \frac{1}{2} - p) + (1-\lambda) \frac{3}{16} (1 - \frac{1}{4} - p) \right],$$

wit $\frac{\partial \pi_1(d_1,d_{-1})}{\partial d_1} = 0$ iff $p = \frac{9-\lambda}{4(3+\lambda)}$.

Part 3. In any equilibrium, either all bidders are rationed in the low demand scenario, or none of them is rationed, because their joined quantity determines the same rationing factor for everyone. In both cases (rationing if $n = 2$, or no rationing), we have shown that there is a unique best-reply to any given strategy profile of bidder 1's opponent, d_{-1}. Since bidders' payoff functions of all bidders are symmetric, also the equilibria of the game must be symmetric. ■

>From Lemma 8, it follows that at any price but one in the interval $(\frac{1}{2}, \frac{3}{4}]$, $FPM(p)$ can have at most two equilibria in pure strategies, and all of them it must be symmetric (by part 3 of the lemma). An equilibrium where all buyers bid $d_i(p) = 1$ exists for all prices $p \leq p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$, but not for higher prices (part 2 of the lemma). At $p_m = \frac{1}{4}\frac{9-\lambda}{3+\lambda}$ also any quadruple of equal bids that leads to rationing in both scenarios is an equilibrium of the game. A symmetric equilibrium without rationing in the low demand scenario exists whenever the solution to $\max_{d_1} \pi_1(d_1, d_{-1})$ s. t.. $d_1 = d_j \; \forall \; d_j \in d_{-1}$ ensures that $d_1 + d_2 \leq 1$, which is the case for prices $p \geq p_e = \frac{1}{4}\frac{9+7\lambda}{3+5\lambda}$.

We are now in the position to prove Proposition 1.

Proof. [Proof of Proposition 1] Existence and uniqueness of equilibrium at prices $p \in [0, p_e)$ and $p \in [\frac{3}{4}, 1]$ has already been shown in lemmas 5 – 8 . Also, all remaining equilibria where $d_i = 1$, $\forall i$ and the continuum of equilibria at p_m have been derived in Lemma 8, part 2. It remains to solve for those equilibria where no rationing takes place in the low demand scenario at prices $p \in [\frac{1}{2}, \frac{3}{4}]$.

If $Q^2(d) = 1$, (13) simplifies to

$$\lambda\,[1 - d_i - p] + (1-\lambda)\left[Q^4(d_i, d_{-i}) - d_i \frac{\partial Q^4(d_i, d_{-i})}{\partial d_i}\right](1 - d_i Q^4(d_i, d_{-i}) - p) = 0. \quad (18)$$

Substituting $Q^4(d_i, d_{-i}) = \frac{1}{\sum_{j=1}^4 d_j}$ in (18), and imposing symmetry yields

$$d_i = \frac{1}{2}(1-p) + \sqrt{\frac{1-\lambda}{\lambda}(\frac{3}{4} - p)\frac{3}{16} + \frac{1}{4}(1-p)^2}. \quad (19)$$

A symmetric profile (19) can be an equilibrium only if $2d_i \leq 1$, which is the case for all $p \geq p_e$,with $p_e = \frac{9+7\lambda}{4(3+5\lambda)}$. ■

Proof. [Proof of Proposition 2] If the upper limit $\overline{d}$ is high enough not to affect revealed demand at prices in $\left[p_e, \frac{3}{4}\right]$, equilibrium demand of buyer 1 at price $p \in \left[p_e, \frac{3}{4}\right]$ is at least d_i, as given by equation (19). At price p_e the whole supply is sold in both scenarios, which implies that seller's expected revenue is safe and equal to p_e (which proves part (ii) of the proposition). Since there is only 1 unit for sale, setting a price below p_e is strictly dominated for the seller. Thus, the seller's revenue is bounded below by p_e and may be even higher (part (iii)). Since $\overline{d}$ can only reduce the demanded quantity, the seller strictly prefers a limit that does not affect revealed demand by any bidder at the posted price (part (iv)).

We have already shown in proposition 1 that for any upper bound on bids an equilibrium of the bidding stage exists at all prices $p \in [0, 1]$, and that any such equilibrium is symmetric.

Under proportional rationing any bidder who has bid the same quantity receives the same. Recall that all bidders have the same demand function. Therefore, their willingness to pay for the next unit is the same and no aftermarket trade among the bidders will occur (part (v)).

Finally, for any equilibrium played in Stage 2, there is a (not necessarily unique) profit maximizing price. Thus, an equilibrium of FPM always exists (part (i)), where the seller chooses the profit maximizing price $p^\star$ given the play at the second stage, and chooses $\overline{d}$ higher than the bidders' (unrestricted) bids at $p^\star$. ■

Proof. [Proof of Proposition 4] Assume that player 2, 3 and 4 play the equilibrium strategy, i.e. $x_i = d_i^*(p))$. If $p \geq \frac{3}{4}$, if player 1 selects a quantity sufficiently close to $d_1^*(p)$, rationing does not occur in either scenario. This simplifies the derivative of $u_i(x)$ wrt d_1 to the following:

$$\frac{du_i(x)}{dd_1} = 2^\rho(1-p-d_1)(2x_1(1-p-\frac{1}{2}d_1))^{-\rho}, \tag{20}$$

that is, FOCs equivalent to the case of risk neutrality.

Assume $p < p_e$ and $d_j(p) = 1, j \neq 1$. Then, for all $d_1 > 0$ (since player 1 is rationed in both scenarios), differentiating $u_i(x)$ wrt x_1 yields the following:

$$\frac{du_i(d)}{dd_1} = 2^\rho \left(\begin{array}{c} \frac{\lambda(1-p(1+d_1))}{(1+d_1)^3\left(d_1 Q_2^2(2+d_1-2p(1+d_1)\right)^\rho} + \\ \frac{3(1-\lambda)(3-p(3+d_1))}{(1+d_1)^3\left(d_1 Q_4^2(6+d_1-2p(3+d_1)\right)^\rho} \end{array} \right) > 0. \tag{21}$$

We know, from Proposition 1, that, when rationing takes place in the low demand scenario only, $\pi_i(d)$ is a strictly concave function. This, in turn, implies that, $u_i(d,\rho)$ is also a strictly concave function (i.e. the equilibrium is unique). In this case, first-order conditions (21) correspond to

$$\frac{du_i(d)}{dd_1} = 2^\rho \left(k(d,\rho)\frac{\lambda}{(1-\lambda)}(1-p-d1) + \frac{(\sum_{j>1} d_j)(\sum_{j>1} d_j - p\sum_j d_j)}{\left(\sum_j d_j\right)^3} \right), \tag{22}$$

with $k(d,\rho) \equiv \frac{\left(d1\left(2pd_1-d_1+2(1-p)\left(\sum_{j>1} d_j\right)\right)\right)^{-\rho}}{(d_1(2-2p-d1))^\rho\left(\sum_{j>1} d_j\right)^3}$. Condition (22) is equivalent to the FOC under risk-neutrality (13) when $k(d) = 1$. Notice that $0 < k(d) \leq 1$, and increases (decreases) with p (ρ). If $\lambda \geq \frac{1}{2}$, then (22) implies that, if a symmetric equilibrium $\frac{1}{4} \leq \check{d}_i(p,\rho) \leq \frac{1}{2}$ exists, it must be $\check{d}_i(p) \geq d_i^*(p,\rho)$. This already implies (given $p \geq p_m$), $k(d,\rho) \geq \frac{12p-11}{16d_i^*(p,\rho)(2p-2+d_i^*(p))} \geq \frac{3}{4}$.

This, in turn, implies

$$\left|\check{d}_1(p,\rho) - d_1^*(p)\right| \leq \frac{1}{8}\left(2\sqrt{7-12p+4p^2} - \sqrt{25-44p+16p^2}\right) \leq \frac{1}{8}\left(2\sqrt{2}-\sqrt{7}\right) \cong .023,$$

which proves the statement. ■

7.2 The experimental instructions

Welcome to the experiment!

This is an experiment to study how people solve decision problems.

Our unique goal is to see how people act on average; not what you, particularly, do. Do not think, then, that we expect you to take any specific behavior.

On the other hand, you should take into account that your behavior will affect the amount of money you will earn throughout the experiment. It is, therefore, your own interest to do your best.

This sheet contains the instructions explaining the way the experiment works and the way you should use your computer.

Please Do not disturb the other participants during the course experiment. If you need any help, please, raise your hand and wait in silence. You will be attended as soon as possible.

How can you earn money?

You will have to play 84 rounds of a simple game described as follows. In each round, you will be part of a group of 2 or 4 people (including you) of this room. Whether the group will be of 2 or 4 people will be decided randomly and it will change within each round.[20]

During the experiment, 50% of times you will be in a group of 4 and 50% of times in a group of 4. It is crucial to keep in mind that **the composition as well as the size of your group will change at each round!**

In each round, you and each of the other members of your group will have to make a choice. Your decision (together with the decisions of the others in your group) will determine the amount of money you will earn at the end of that round.

We will also give you a show-up fee of 2000 ptas[21]. At the end of the experiment, you will be paid the exact amount you have earned throughout its course plus the show-up fee.

How to play the game?

[20] As there were no groups in ICM session, the last two phrases of this paragraph and all the following one are omitted in the ICM insructions. All the rest of the insructions is slightly modified in parts where we talk about "the other members of your group" in order not to confuse the subjects participating to the ICM session.

[21] The show-up fee of the ICM session was 1500 pesets, since the control treament was strategically simpler than FPM.

In each round, you will participate to a market together with the other members of your group (who can be one or three). In this market, 10 units of a product are put in sale.

In each round, a price between 0 and 10 will appear in the screen of your (as well as your group members') computer. This price does not necessarily have to be an integer, and has been determined randomly. You and the other group members have to decide the amount of the product you want to bid at this price[22].

How can you get the product?[23]

You will not always get the amount of product you have bid!!!

The amount of good you will get depends on your bid and the bids of the other group members. Keep in mind that you will take part of a group that will be formed of 2 or 4 members (including you). At the moment you will have to decide your bid you will not know the size of your group!!!

In each round, we will sum the bid amounts by all your group members. Do not forget that the **maximum** amount we can distribute is 10 units.

In case that the sum of the bids of all the members of your group (including yourself) does not exceed 10, each member receives what he demanded.

Otherwise, that is, if the sum exceeds 10 units, each member receives a lower amount than what he demanded, although each member get the same percentage of his bid. This percentage is determined from the relation between the available amount and the aggregate demand of your group.

Example: Suppose that:

- the price of this round is 5.5,
- your bid was 2 units,
- each of the other members of your group demanded 6 units.

If the size of your group was 2, your group's aggregate bid would be 2+6=8. Since this amount is lower than 10 (the available amount), you will receive 2 units and the other one gets 6 units that is, what you both bid..

If the size of your group was 4, the aggregate demand would be 2+6+6+6=20. Since this amount is higher than 10 (the available amount), each member of your group receives 50% of what he has bid. This is because the available amount, 10, is 50% of the amount demanded by the whole group, 20. That is: you will get 1 unit and the others receive 3 units each.

[22] As a next paragraph the following text in Bold appears in the ICM instructions: You will always receive the amount you have bidded!!!

[23] This chapter - together with the following Summary and the Control questions 1 and 2 - does not appear in the ICM instructions, since there is no rationing.

Summary

If the aggregate bid of the group is less or equal to 10, each member gets what he has bid..

If the aggregate bid is higher than 10, each member receives the same percentage as he bid.. This percentage is determined from the relation between the available amount (10 units) and the aggregate bid (e.g. 20 units in our example).

This implies that always when a person bid more than an other one this person gets more units than the other one.

Control question # 1: If you bid 6 and each of the other members of your group bids 6,

How many units do you get if the size of your group is 4?
How many units do you get if the size of your group is 2?

Control question # 2: If you bid 8 and each of the other members of your group bids 4,

How many units do you get if the size of your group is 4?
Do you get what you bid if the size of your group is 2?

How much money you can earn?

Look at the table we give you together with these instructions. In this table, you can check how much money you earn for each quantity you get at each price. The first column of the table shows the different prices that can appear during the experiment. In the first row, you have different quantities between 0 and 10 units. In each cell , you find your profit if you get the corresponding quantity at the corresponding price.

For instance, if you like to know how much money you earn if you receive 4.5 units at price 4, have a look at the cell that corresponds to the row of price 4 and to the column corresponding to the quantity of 4.5 units. By doing so, you will see that you earn 16.88 ptas..

Control question # 3: How much money do you earn if you get 8 units at price 2.5?[24]

[24]Obviosly, this is the unique control question that appears in the ICM instructions.

In each round, you can bid any amount between 0 and 10, but it has to be a number with at most 2 decimals. You are not forced to only bid the amounts listed in the table.

It can also happen that the quantity you get corresponds to a number between two of the quantities listed in the table. In such a case, your profit will also be between the two corresponding profits.

Summary[25]

In each round, you and other members of your group will participate in a market where 10 units of product are being sold.

The size and the composition of your group will change in each round and they will, always, be determined randomly. The size of your group can be 2 or 4 (including yourself). In each round, both possibilities have the same probability (i.e.50% of times your form part of a group of 2 and 50% of a group of 4).

In each round, you and the other members of your group will face a different price.

At this price, you have to bid a quantity and you will get:

- What you have bid if the sum of bids of whole group is lower or equal to 10.
- If the aggregate bid is higher than 10, each member receives the same percentage of the total amount (10) as his bid (compared to the total sum of bids).

This implies that who bids more always receives more.

You can check your profits in the table enclosed with these instructions.

[25]Due to the simplicity of the control treatment, we have not found it necessary to place a Summary part to the instructions of the ICM session.

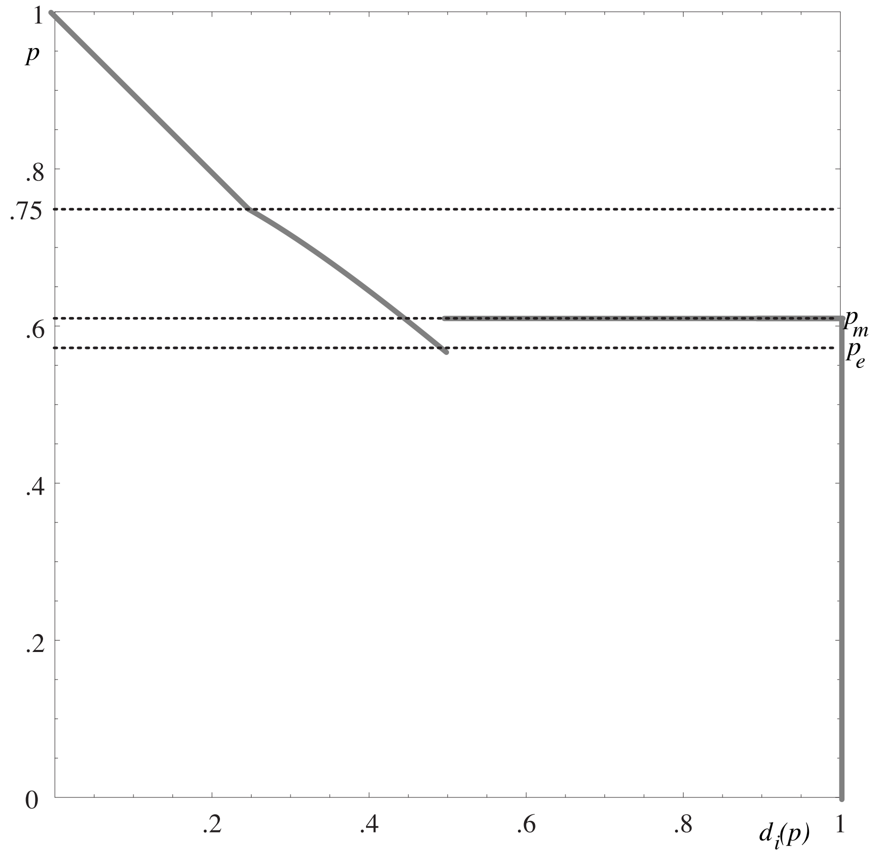

Figure 1. FPM: Equlibrium bid function(s)

p \ T_i	1	2	3	4
0	0.958 (0.204)	1 (0)	1 (0)	1 (0)
.05	0.812 (0.307)	0.952 (0.01)	0.95 (0)	0.95 (0)
.1	0.819 (0.221)	0.902 (0.01)	0.9 (0)	0.9 (0)
.15	0.781 (0.217)	0.85 (0)	0.854 (0.02)	0.85 (0)
.2	0.776 (0.122)	0.777 (0.112)	0.8 (0)	0.775 (0.122)
.25	0.704 (0.17)	0.729 (0.102)	0.75 (0)	0.75 (0)
.3	0.668 (0.111)	0.7 (0)	0.7 (0)	0.7 (0)
.35	0.584 (0.193)	0.65 (0)	0.65 (0)	0.65 (0)
.4	0.575 (0.102)	0.6 (0)	0.6 (0)	0.6 (0)
.45	0.543 (0.022)	0.542 (0.032)	0.55 (0)	0.55 (0)
.5	0.51 (0.129)	0.5 (0)	0.5 (0)	0.5 (0)
.55	0.45 (0.108)	0.45 (0.001)	0.45 (0)	0.45 (0)
.6	0.449 (0.162)	0.4 (0)	0.4 (0)	0.4 (0)
.65	0.354 (0.075)	0.349 (0.005)	0.35 (0)	0.354 (0.02)
.7	0.36 (0.168)	0.3 (0)	0.3 (0)	0.3 (0)
.75	0.249 (0.119)	0.25 (0)	0.252 (0.01)	0.248 (0.01)
.8	0.196 (0.02)	0.2 (0)	0.2 (0)	0.2 (0)
.85	0.148 (0.057)	0.15 (0)	0.15 (0)	0.15 (0)
.9	0.097 (0.016)	0.096 (0.018)	0.1 (0)	0.098 (0.01)
.95	0.114 (0.211)	0.049 (0.007)	0.05 (0)	0.05 (0)
1	0.054 (0.207)	0 (0)	0 (0)	0 (0)

Figure 2. ICM: Average aggregate bids (with standard deviation)

p \ T_i	1	2	3	4
0	0.941 (0.143)	0.948 (0.14)	0.944 (0.2)	0.977 (0.114)
.05	0.858 (0.224)	0.947 (0.154)	0.96 (0.128)	0.972 (0.105)
.1	0.871 (0.203)	0.943 (0.143)	0.945 (0.155)	0.976 (0.08)
.15	0.821 (0.202)	0.95 (0.108)	0.944 (0.163)	0.959 (0.117)
.2	0.885 (0.169)	0.906 (0.169)	0.918 (0.181)	0.96 (0.132)
.25	0.739 (0.24)	0.894 (0.184)	0.888 (0.185)	0.944 (0.123)
.3	0.804 (0.196)	0.855 (0.205)	0.863 (0.198)	0.911 (0.187)
.35	0.747 (0.195)	0.802 (0.201)	0.864 (0.186)	0.904 (0.166)
.4	0.69 (0.19)	0.776 (0.2)	0.832 (0.208)	0.84 (0.188)
.45	0.68 (0.203)	0.669 (0.19)	0.751 (0.211)	0.821 (0.209)
.5	0.632 (0.206)	0.652 (0.211)	0.692 (0.21)	0.741 (0.209)
.55	0.515 (0.177)	0.638 (0.193)	0.6523 (0.211)	0.634 (0.209)
.6	0.476 (0.131)	0.496 (0.143)	0.493 (0.141)	0.561 (0.175)
.65	0.451 (0.176)	0.458 (0.134)	0.482 (0.168)	0.513 (0.185)
.7	0.381 (0.128)	0.353 (0.105)	0.382 (0.134)	0.39 (0.126)
.75	0.316 (0.112)	0.283 (0.122)	0.267 (0.07)	0.286 (0.086)
.8	0.22 (0.086)	0.21 (0.038)	0.206 (0.028)	0.208 (0.032)
.85	0.222 (0.195)	0.156 (0.03)	0.173 (0.133)	0.163 (0.04)
.9	0.108 (0.045)	0.098 (0.018)	0.097 (0.016)	0.109 (0.074)
.95	0.107 (0.158))	0.059 (0.026)	0.052 (0.01)	0.051 (0.007)
1	0.051 (0.128)	0.066 (0.036)	0.0005 (0.003)	0 (0)

Figure 3. FPM: Average aggregate bids (with standard deviation)

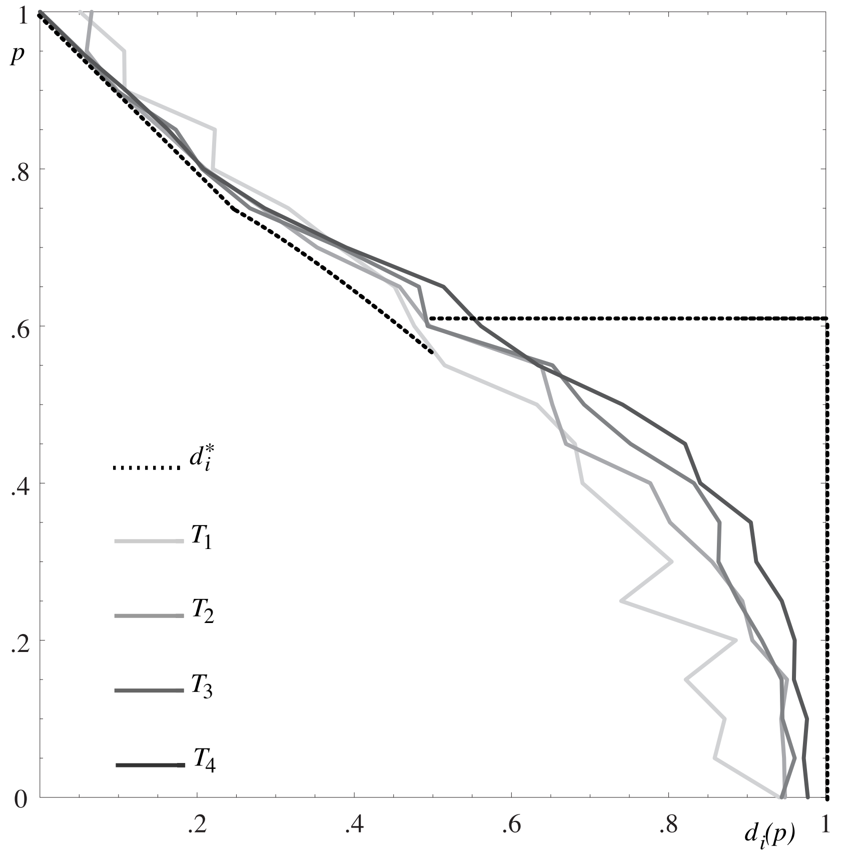

Figure 4. FPM: Evolution of aggregate bids

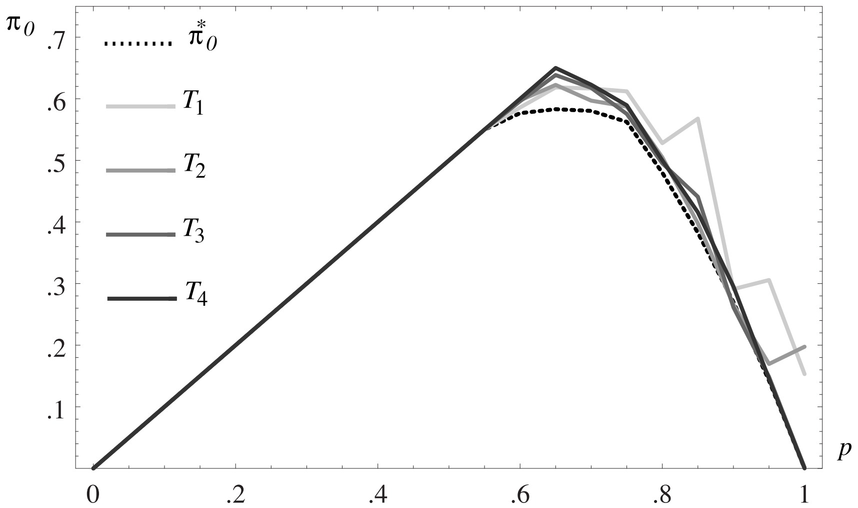

Figure 5. FPM: Evolution of the seller's profits

	T_i	α	β	γ	ρ	R^2
I	$T_1 - T_4$	.968 (.006)	$-.966$ (.006)	.004 (.001)	.037	0.9240
II	T_1	.921 (.016)	$-.871$ (.022)	-	.108	.735
III	T_2	.995 (.003)	$-.995$ (.004)	-	.044	.9873
IV	T_3	-1 (.0004)	1 (.0007)	-	0	.9997
V	T_4	.997 (.002)	$-.996$ (.004)	-	0	.9919

Figure 6. ICM: Panel Data Estimation

	T_i	α_0	α_1	α_2	β_0	β_1	β_2	γ	ρ	R^2
VI	$T_1 - T_4$	.993 (.044)	−.049 (.042)	.252 (.101)	−1.028 (.048)	.439 (.050)	−.273 (.150)	.021 (.001)	.248	.798
VII	T_1	1.059 (.094)	−.119 (.094)	−.003 (.225)	−1.015 (.106)	.382 (.112)	.061 (.332)	-	.232	.7194
VIII	T_2	.943 (.082)	.065 (.082)	.420 (.195)	−.912 (.092)	.268 (.098)	−.517 (.288)	-	.267	.8081
IX	T_3	1.068 (.079)	−.065 (.079)	.105 (.188)	−1.069 (.089)	.530 (.094)	−.039 (.278)	-	.333	.8162
X	T_4	1.113 (.073)	−.078 (.073)	.488 (.175)	−1.117 (.082)	.576 (.087)	−.597 (.258)	-	.288	.8553

Figure 7. FPM: Panel Data Estimations.

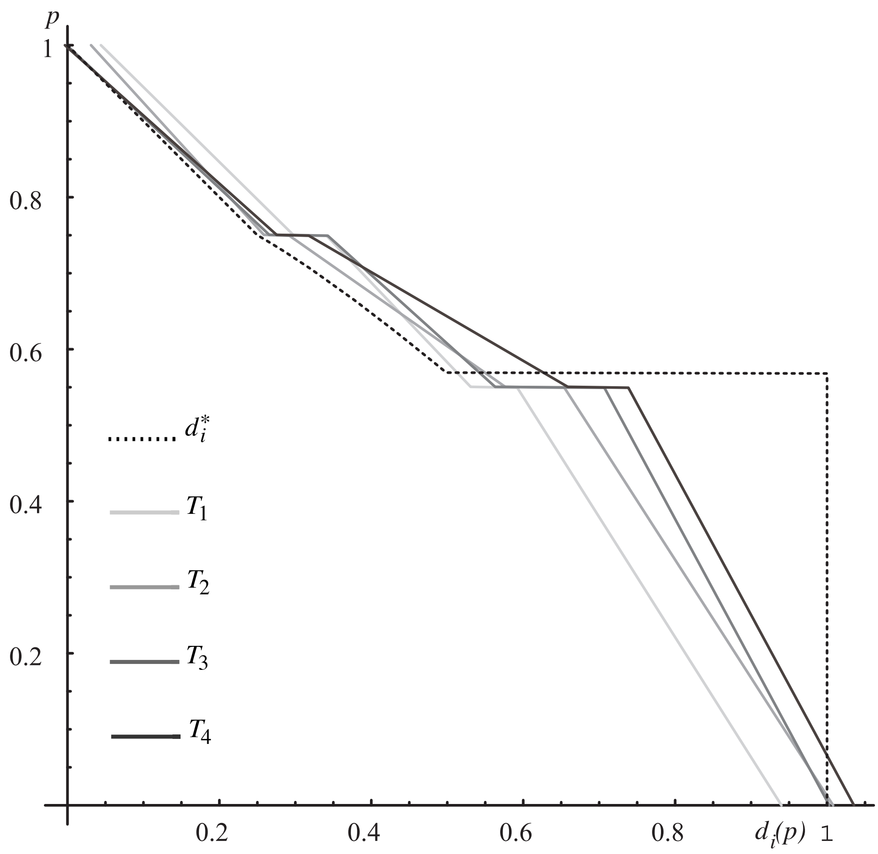

Figure 8. FPM: estimated bid functions

T_k		1		2		3		4	
ICM		31.3		818.7		201200		1325.4	
	$p \geq 0.75$	61		81		358		401	
FPM	$p \in [p_e, 0.75)$	148.2	46	182	79	146.5	98	126	150
	$p < p_e$	37.5		70.6		76		129	

Figure 9. Maximum-Likelihood estimations of μ

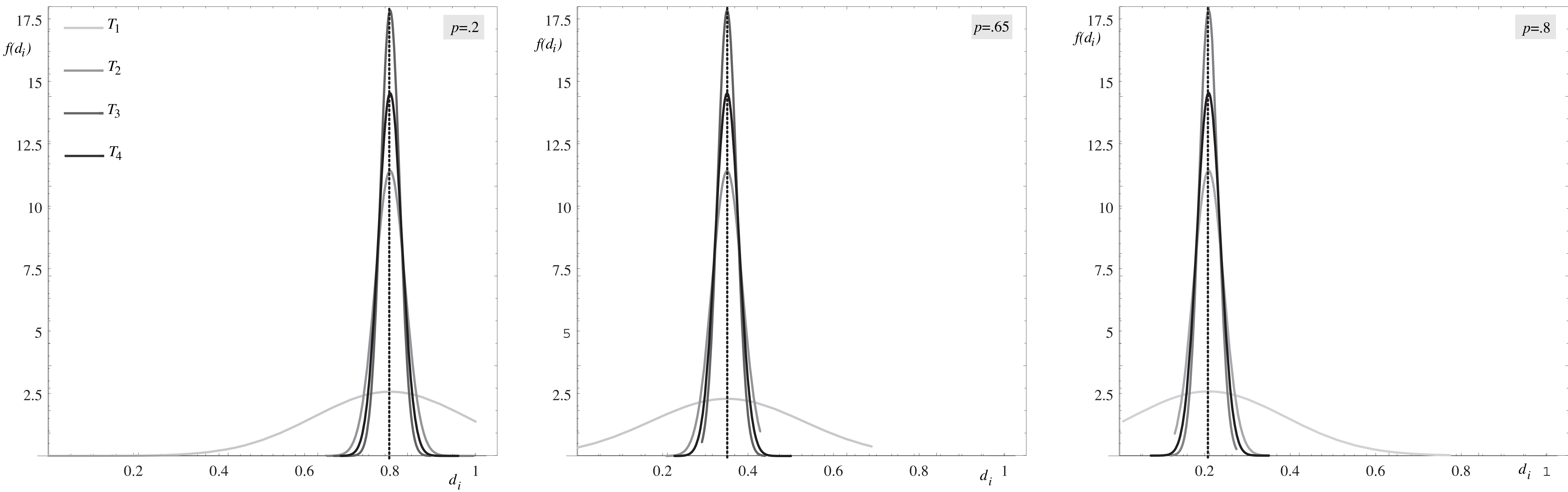

Figure 10. ICM: estimated QRE distributions

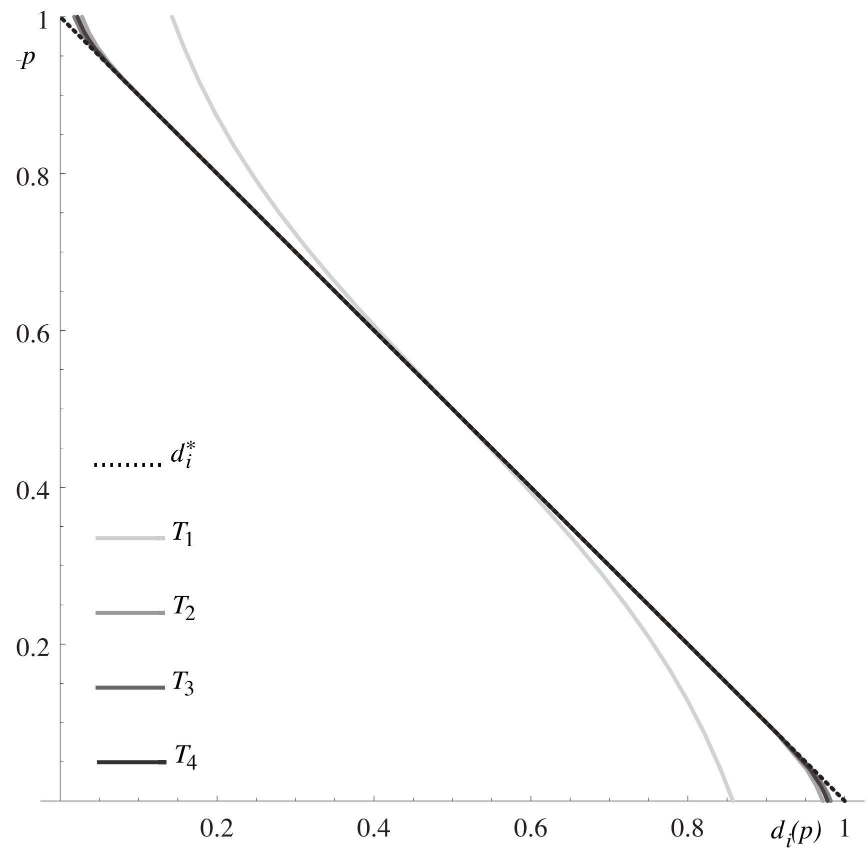

Figure 11. ICM: estimated QRE average bids

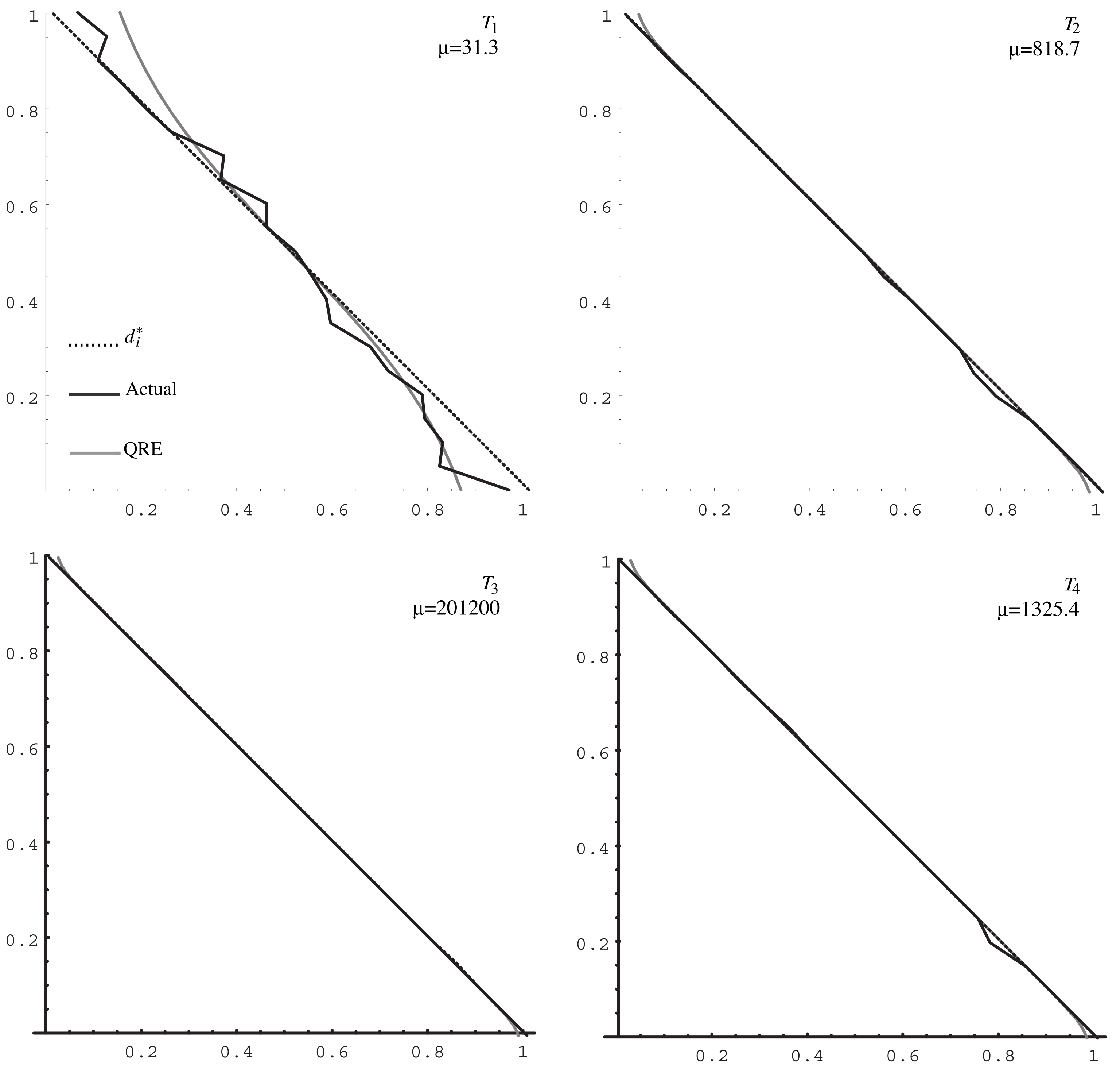

Figure 12. FPM: evolution of estimated QRE average bid functions

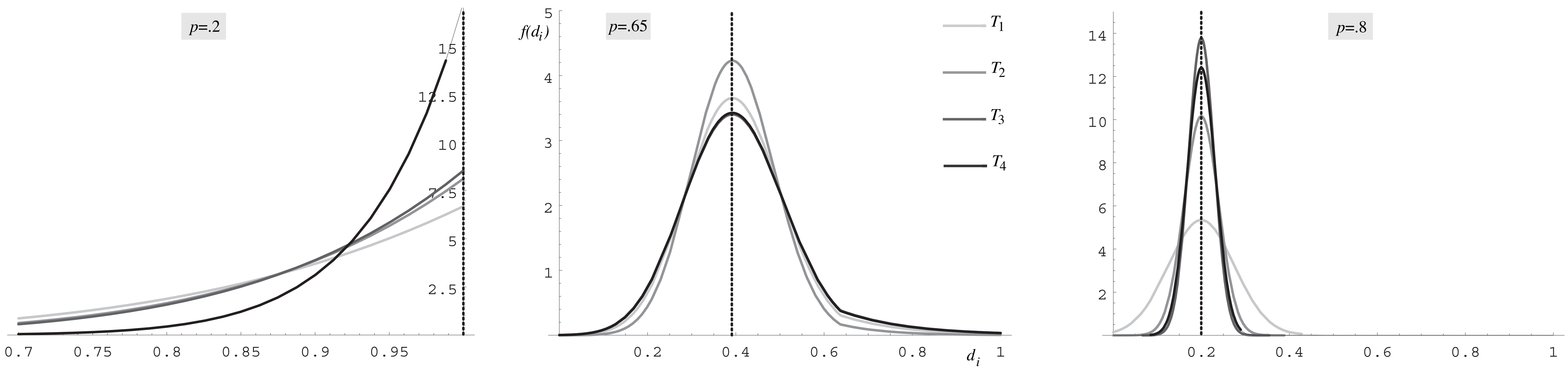

Figure 13. FPM: QRE distributions

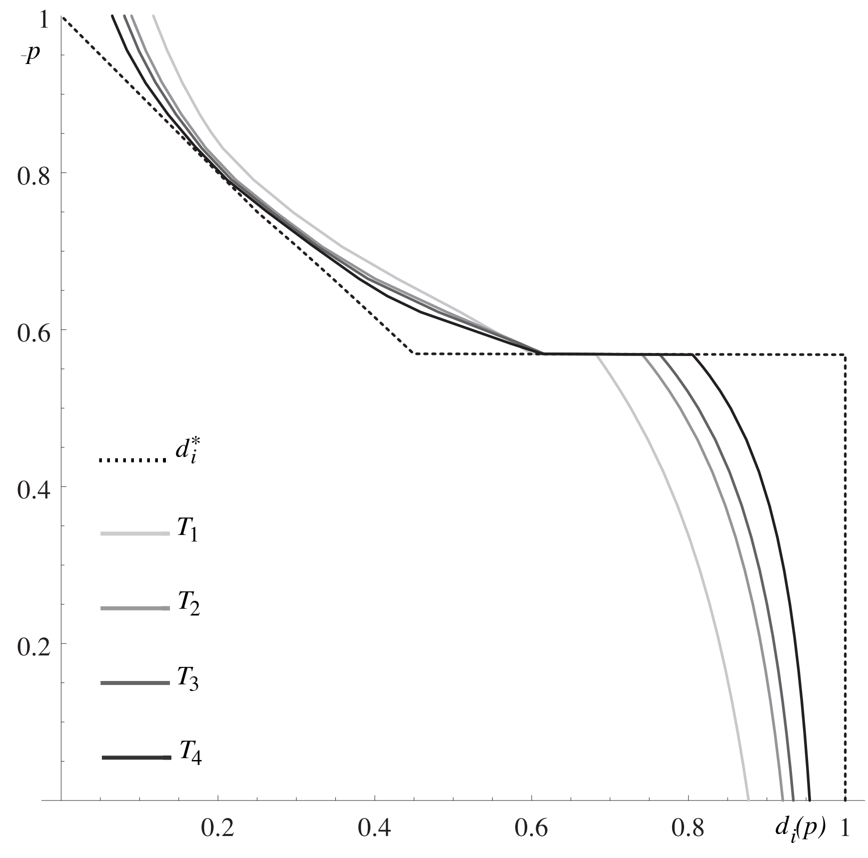

Figure 14. FPM: estimated QRE average bids

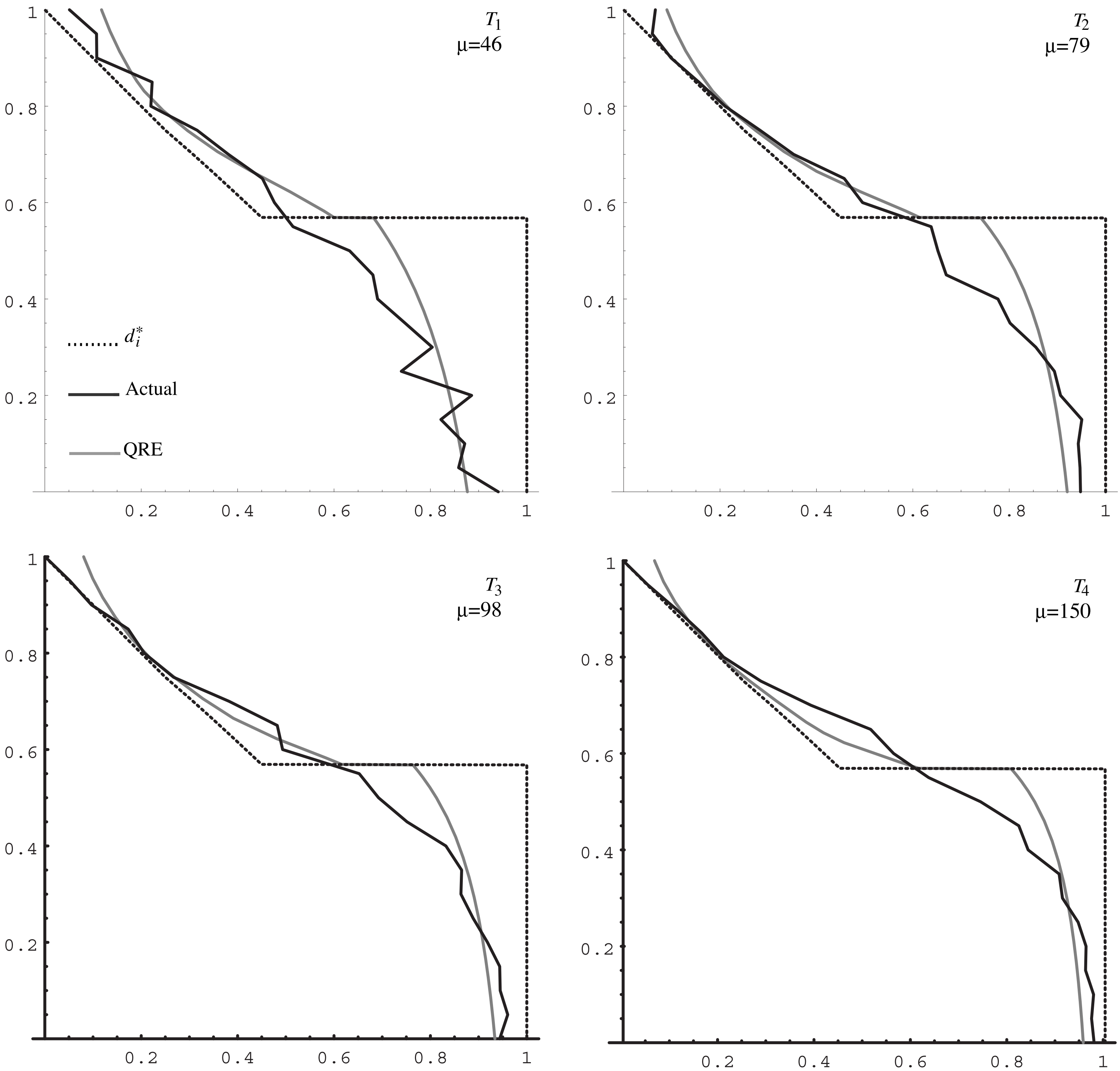

Figure 15. FPM: evolution of estimated QRE average bid functions