UNIVERSITÀ DEGLI STUDI DI FERRARA
DIPARTIMENTO DI ECONOMIA ISTITUZIONI TERRITORIO
Corso Ercole I d'Este, 44 – 44100 Ferrara

Quaderno n. 30/2005

Dicember 2005

Is Prevention Better than Cure?
Framing Effects in Public Good Provision

Iñigo Iturbe   Giovanni Ponti   Josefa Tomás   Luis Ubeda

Quadeni deit

Editor: Giovanni Ponti (ponti@economia.unife.it)
Managing Editor: Marisa Sciutti (sciutti@economia.unife.it)
Editorial Board: Giovanni Masino
Simonetta Renga
http://deit.economia.unife.it/collana_quaderno.php

Is Prevention Better than Cure? Framing Effects in Public Good Provision*

Iñigo Iturbe-Ormaetxe†
Universidad de Alicante

Giovanni Ponti
Universidad de Alicante and
Università di Ferrara

Josefa Tomás
Universidad de Alicante

Luis Ubeda
Universidad de Alicante

Abstract

This paper studies, both theoretically and experimentally, frame effects in the context of public good provision. We apply Prospect Theory to read the experimental evidence of a Voluntary Contribution Threshold Game which, depending on the sessions, was framed as a situation in which a) subjects had to make a costly contribution to gain a common prize or b) they had to make a costly contribution to avoid to loose it. By contrast with standard expected utility theory, by which frames have no impact on equilibrium behavior, Prospect Theory predicts more contribution in the latter situation. Our experimental evidence backs up Prospect Theory when the contribution threshold is high (i.e. the public good is relatively more difficult to achieve). When contribution threshold is low, expected utility seems more consistent with the experimental evidence.

JEL Clasification: C92, D81, H40

Keywords: Public Goods Provision, Framing, Prospect Theory

*We are grateful to Jaromir Kovarik, Ricardo Martinez and Borja Montano for their stimulating suggestions and their valuable help to run the experiment. Usual disclaimers apply. Financial support from Spanish Ministry of Education and Science (SEJ2004-08011ECON -cofinanced by FEDER- and BEC2001-0980), Instituto Valenciano de Investigaciones Económicas (IVIE), and Generalitat Valenciana (Research Project No. Gruposo3/086) is gratefully acknowledged.

†Corresponding author. Departamento de Fundamentos del Análisis Económico - Universidad de Alicante - 03071 Alicante - SPAIN - e-mail:iturbe@merlin.fae.ua.es.

1 Introduction

Free-riding is a pervasive problem in situations where societies have to decide the level of provision of some public good. This is so because public goods have the feature of being non-excludable. In particular, when we talk about pure public goods, it is assumed that excludability is only feasible at an infinite cost. Then governments cannot use rationing by price which implies that a competitive market cannot generate a Pareto efficient amount of the public good. This is the reason of the so-called *free rider problem.* Since any individual perceives that she will benefit from the public good irrespective of her contribution to finance it, she will have no incentives to contribute voluntarily. If the public good is to be financed by voluntary contributions, its level will fall short its efficient level. As an extreme case, consider a situation in which consumers' utility functions take a quasilinear form with respect to a numeraire commodity, and they can be ranked according to the marginal benefits they get from consuming the public good. Assume that the public good is provided by private purchases of the consumers. Then, every individual must decide her own contribution taken the contributions of the other agents as given. Nash equilibrium entails, in this extreme situation, that only the individual with the highest marginal benefit will provide the public good.[1] The free-rider problem here appears in a very stark way, since all individuals but the one with the highest valuation will free-ride. Obviously, this implies that the resulting amount of the public good will be far below its inefficient level. This conclusion is somehow mitigated by the extensive, and extremely robust across a wide variety of treatment conditions, experimental evidence on the classic Voluntary Contribution Mechanism (VCM) protocol. Here we find that subjects usually set initially a contribution which is halfway between the Pareto-efficient level and the free-riding level. If the same protocol is repeated for a finite number of times, average contribution declines over time, but stays always above the Nash equilibrium level. More efficient results are usually obtained in experiments which the VCM is modified by introducing a threshold in the total contribution, below which the public good is not provided.[2] These experimental protocols -usually termed as Voluntary Contribution Threshold Games (VCTG)- have, usually, multiple equilibria.

[1]See the details in Mas-Colell, Whinston and J. Green [?], Section 11.C.
[2]See Ledyard [7].

Precisely, all strategy profiles in which a player is *pivotal* in reaching the threshold are equilibria of the underlying game.[3]

Consider now a (slightly) different set-up, in which there is a set of individuals *who are already enjoying some public good.* However, they realize that at some point in the future the existing public good can deteriorate, or even disappear. To prevent this possibility, they need, somehow, to cooperate. Think of a situation where a group of neighbors use a bridge to cross the river and go hiking to a nearby forest. The base of the bridge is damaged and, unless a major reparation is done, it eventually collapse. Another example is saving the whales. We shall refer to this set-up as *prevention of public good deterioration* (PPGD), in contrast to the most classical case of *public good provision* (PGP). The crucial difference between the two cases of PGP and PPGD is just whether individuals have initially the public good or not.

The aim of the paper is to answer, both theoretically and experimentally, to this very simple question:

Do people contribute more in PGP, rather than in PPGD?

To this aim, the remainder of the paper is arranged as follows. In Section 2, we set up the basic setup in which the public good problem is presented as a VCTG under two different frames: PGP and PPGD. As we show in Proposition 1, the game has a unique (symmetric) Bayes Nash equilibrium, which uniquely define the same cutoff value c^* in the individual cost of contributing, below (above) which players do (not) contribute. As Proposition 1 shows, this cutoff value is constant across players and frames.

We use Proposition 1 as our benchmark for subsequent analysis. In Section 2.1 we follow Andreoni and Miller [1] by assuming that players' preferences not only depend upon their individual monetary rewards in the game but also on two individual parameters which measure players' satisfaction when i) they simply contribute (independently on the outcome) and ii) when the public good is provided (independently on whether they have contributed or not). This modifications, by allowing for heterogeneity in subjects' preferences, break the symmetry of the equilibrium of Proposition 1. However, as before, the equilibrium remains the same under both frames: PGP and PPGD. Section **??** look at our theoretical framework from the

[3] Another interpretation is that a minimum number of agents must contribute to the provision of the public good, when all individuals contribute equally (Olson [10]).

point of view of Prospect theory. Under this alternative paradigm, players' preferences crucially depends on *a reference point* they use to evaluate costs and benefits of contribution. As a consequence, different frames yield different equilibria, which we characterize in a sequence of propositions as functions of all reference points and contribution thresholds induced by our framework. In Section 3 we calibrate the model to provide theoretical predictions for our experiment, whose basic design is describe in Section 4. Section 5 describes our experimental results, followed by our conclusions and an Appendix in which we report the experimental instructions..

2 The basic model

There is a group of N individuals. An individual is indexed by $i \in \{1, .., N\}$. Individual i has wealth w_i. Each individual has one unit of input that she can either consume privately or contribute. The public good is provided if and only if at least k units of input are contributed, where $1 \leq k \leq N$. The input of individual i has a private value of c_i. Each individual knows her own value c_i. Regarding the value of the input of other individuals, each individual knows only that they come from some Cumulative Distribution Function $F(\cdot)$ which is continuous on some interval $[0, \bar{c}]$, with $\bar{c} > 0$ and $F(0) = 0$, and $F(\bar{c}) = 1$. By analogy with our experimental conditions, in what follows we shall assume that this distribution is uniform. We normalize the value that the public good has for every individual as $+g$ (-g).

Figure 1 describes player i's monetary payoffs in case of VCTG when the number of individuals other than i that are contributing is n and the contribution threshold is k. In Figure 1, the letter C (NC) denote the action of (non) contributing.

States of the world	$n > k - 1$	$n = k - 1$	$n < k - 1$
Probabilities	p_1	p_2	$1 - p_1 - p_2$
C	g	g	0
NC	$g + c_i$	c_i	c_i

Fig. 1. Voluntary Contribution Threshold Game

State probabilities are determined in equilibrium. A symmetric Bayesian Nash Equilibrium has the form of a cutpoint rule: Individual i contributes if and only if her cost c_i is less than a cutpoint value c^*, common to all individuals. Given a c^* and the assumed uniform distribution of the c_i, the probabilities p_j can be computed. To solve for the value of c^*, we note that it must be that value that makes an individual to be indifferent between C and NC. Then, it must satisfy:

$$p_1 + p_2 = p_1(1 + c^*) + p_2 c^* + (1 - p_1 - p_2)c^*. \tag{1}$$

From this we get:

$$c^* = p_2 = \Pr(n = k - 1).$$

Then c^* is defined implicitly by the following

Proposition 1 *In the unique BNE of the game of Figure 1, all player contribute whenever $c_i < c^*$, and do not contribute whenever $c_i > c^*$, where*

$$c^* = \binom{N-1}{k-1}\left\{\left(\frac{c^*}{\bar{c}}\right)^{k-1}\left(1 - \frac{c^*}{\bar{c}}\right)^{N-k}\right\}. \tag{2}$$

By analogy with our experimental conditions, consider a situation where $N = 3, k = 2$, and c_i is uniformly distributed on $[0, \bar{c}]$, with $\bar{c} = 1.1$. In this case, By (2), we then have $c^* = .495$ and the predicted frequency of contribution is $F(c^*) = .45$.

Notice that Figure 1 not only represents the case of PGP. The representation of the case of PPGD is exactly identical. In other words, the (BN) equilibria of the contribution game induced by Figure 1 are absolutely identical, independently of the frame.

2.1 Altruism

In this section, we follow Andreoni and Miller [1] by considering a situation in which players

(i) derive some extra utility from contributing (they feel better just because they contribute) and/or

(ii) derive some extra utility from knowing that the public good is there, so other people enjoy it.

If these two effects are individual specific constants which sum up to the monetary payoffs in the game of Figure 1, then Figure 2 represents the payoff matrix this new strategic situation.

States of the world	$n > k-1$	$n = k-1$	$n < k-1$
Probabilities	p_1	p_2	$1 - p_1 - p_2$
C	$w_i + 1 + \mu_i + \beta_i$	$w_i + 1 + \mu_i + \beta_i$	$w_i + \mu_i$
NC	$w_i + 1 + c_i + \beta_i$	$w_i + c_i$	$w_i + c_i$

Fig. 2. Basic game with altruistic preferences

The parameter μ_i of Figure 2 represents the utility that player i derives from the fact that she is contributing. Note that this extra utility is independent of whether the public good is provided or not. By the same token, the parameter β_i represents the additional utility that i derives when the public good is provided, presumably because all other individuals are enjoying it. Again, this extra utility is independent of whether player i has contributed or not.

Under these new assumptions, player i will contribute if

$$p_2(1 + \beta_i) + \mu_i \geq c_i.$$

This, in turn, defines a cutoff value

$$c_i^* = p_2(1 + \beta_i) + \mu_i. \tag{3}$$

Notice that c_i^* can now vary across players, since they may hold heterogenous values for μ_i and β_i. However, like in Proposition 1, the same equilibrium value for c_i^* is common to both frames, PGP and PPGD.

2.2 Frames

If players evaluate risky prospects in terms of gains and losses with respect to a reference point, the game of provision of the public good can be framed into four natural forms. The successful provision of the public good can be put as the realization of a gain (PGP), or instead, as the elimination of a loss (PPGD). Because of loss aversion, the value of the provision of the public good will be seen as larger if it is put in the latter framing. Similarly, the cost of contributing can be put as a loss, or instead, as a lack of realizing a gain. Because of loss aversion, the former will tend to be more discouraging of contributing. These two dimensions can be combined into four forms that can be seen as the game seen from four natural reference points: $x_0 = 0$, $x_0 = c_i$, $x_0 = g$, and $x_0 = g + c_i$, where g is the value of the public good and c_i is the individual cost of contributing. As it will be seen below, the effect of the change in the reference point will not be due only to loss aversion but also to the nonlinearity of the probability weighting function as well as to the curvature of the value function in gains and or in losses. Let G_{x_0} define the induced prospect game when the reference point is equal to x_0.

Reference point $x_0 = 0$ (G_0). From this reference point the provision of the public good is seen as a gain and contributing to the public good involves a lack of realizing a gain, that is, not contributing involves a gain. This is exactly the case of the matrix form of Figure 1.

	$n > k-1$	$n = k-1$	$n < k-1$
C	g	g	0
NC	$g + c_i$	c_i	c_i

Fig. 3. Game G_0

In this case, player i has to choose between two prospects involving gains,

$$C = (g, p + q) \text{ or } CN = (g + c_i, p; c_i, q + r) \tag{4}$$

Where $p = \Pr(n > k - 1)$, $q = \Pr(n = k - 1)$, and $r = \Pr(n < k - 1)$. Loss aversion will then not be playing any role in this choice.

Reference point $x_0 = c_i$ (G_c). In G_c the provision of the public good is seen as a gain, as in the previous case. However, now contributing to the public good involves a loss. The payoff matrix of Figure 1 is modified by substracting $x_0 = c_i$ to every cell.

	$n > k-1$	$n = k-1$	$n < k-1$
C	$g - c_i$	$g - c_i$	$-c_i$
NC	g	0	0

Fig. 4. Game G_c

Player i has now to choose between two prospects,

$$C = (g - c_i, p; -c_i, q + r) \text{ or } CN = (g, p) \quad (5)$$

Notice that the payoff $g - c_i$, corresponding to contributing when sufficient number of others also contribute, may be a gain or a loss, since it may be $c_i > g$. In this latter case, NC is a strictly dominant strategy.

Reference point $x_0 = g$ (G_g). If $x_0 = g$, the provision of the public good is not seen as a gain, but instead, as avoiding a loss. On the other hand, the cost of contributing is not seen as a loss, but instead, it is seen as a lack of realizing a gain. As a consequence, payoffs in this situation are those of Figure 1 in which the value of the public good $x_0 = g$ is subtracted to every cell.

	$n > k - 1$	$n = k - 1$	$n < k - 1$
C	0	0	$-g$
NC	c_i	$-(g - c_i)$	$-(g - c_i)$

Fig. 5. Game G_g

In this case, player i has to choose between two prospects,

$$C = (-g, r) \text{ or } CN = (c_i, p; -(g - c_i), q + r) \quad (6)$$

C involves a risky loss, and CN that may result in a gain or a loss, in the non trivial case where $g > c_i$.

Reference point $x_0 = g + c_i$ (G_{g+c}). In this latter case, the provision of the public good is again not seen as a gain, but instead, as avoiding a loss. On the other hand, the cost of contributing is now seen as involving a loss. As a consequence, payoffs in this situation are those of Figure 1 in which $x_0 = g + c_i$ is subtracted to every cell.

	$n > k - 1$	$n = k - 1$	$n < k - 1$
C	$-c_i$	$-c_i$	$-g - c_i$
NC	0	$-g$	$-g$

Fig. 6. Game G_{g+c_i}

Player i has now to choose between two prospects involving only losses,

$$C = (-c_i, p + q; -g - c_i, r) \text{ or } CN = (-g - c_i, q + r) \tag{7}$$

Since these prospects involve only losses, loss aversion plays no role in this choice.

2.3 One contribution is enough (Γ_1)

In this subsection we consider the polar case where the public good is produced whenever there is at least a contribution. As we shall see this is a simple case where it is clear both the relevance of the reference point and the contrast with the predicted behavior under Expected Utility Theory.

We focus on symmetric pure strategy equilibria. Let s be a cutoff strategy profile where each individual j contributes if her cost $c_j < c$ and she does not contribute if her cost $c_j > c$. Let $q(c)$ be the probability that no individual other than i contributes, that is,

$$q(c) = (1 - c)^{N-1} \tag{8}$$

Note that q is decreasing in c, from 1 at $c = 0$ to 0 at $c = 1$.

We analyze equilibria from the four natural reference points: $x_0 = 1 + c$, $x_0 = 1$, $x_0 = c$, and $x_0 = 0$.

(a) Reference Point $x_0 = 1 + c$. Equilibrium Conditions,

$$v(-c) = w(q)v(-1)$$

$$\frac{v(-c)}{v(-1)} = w(q) \tag{9}$$

Since the left-hand side is increasing from 0 to 1, and the right-hand side is decreasing from 1 to 0, there is a unique symmetric equilibrium $c^*_{1+c} \in (0, 1)$.

(b) Reference Point $x_0 = c$. Equilibrium Conditions,

$$v(1-c) = w(1-q)v(1)$$

$$\frac{v(1-c)}{v(1)} = w(1-q) \tag{10}$$

Since the left-hand side is decreasing from 1 to 0, and the right-hand side is increasing from 0 to 1, there is a unique symmetric equilibrium $c_c^* \in (0,1)$.

(c) Reference Point $x_0 = 0$. Equilibrium Conditions,

$$v(1) = v(c) + w(1-q)\left[v(1+c) - v(c)\right] \tag{11}$$

Note that the right-hand side has a positive derivative so that it is increasing and at 0 is 0, so less than the left-hand side, which is less than the right-hand side at 1 that is $v(2)$. Then, there is a unique symmetric equilibrium $c_0^* \in (0,1)$.

(d) Reference Point $x_0 = 1$. Equilibrium Conditions,

$$0 = w(1-q)v(c) + w(q)v(c-1) \tag{12}$$

Since the right-hand side is increasing from $v(-1) < 0$ to $v(1) > 0$, it follows that there is a unique symmetric equilibrium $c_1^* \in (0,1)$.

We have shown the following Proposition.

Proposition 1. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, for each of the four reference points $x_0 = 1+c$, $x_0 = c$, $x_0 = 0$, and $x_0 = 1$, there exists a unique symmetric equilibrium. These equilibria are interior to $(0,1)$ and are the unique solution to the equations (9), (10), (11), and (12), respectively.*

2.3.1 Ranking probability of contribution by reference point

We next rank the probability of contribution resulting from the four natural reference points. We begin with the comparison of c^*_{1+c} and c^*_c.

Suppose $a = c^*_{1+c} \geq c^*_c = b$, then $q(a) \leq q(b)$, $w(q(a)) \leq w(q(b))$, and by subcertainty of the probability weighting function, (9) and (10) imply,

$$\frac{v(-a)}{v(-1)}\frac{v(1-b)}{v(1)} = w(q(a)) + w(1-q(b)) \leq w(q(b)) + w(1-q(b)) < 1 \quad (13)$$

On the other hand, from the concavity in gains of the value function v it follows,

$$v(1-b) \geq bv(0) + (1-b)v(1)$$

so

$$\frac{v(1-b)}{v(1)} \geq 1-b, \quad (14)$$

with strict inequality if the concavity is strict. Similarly, from the convexity in losses of v it follows,

$$v(-a) \leq (1-a)v(0) + av(-1),$$

and since $v(-1) < v(0) = 0$,

$$\frac{v(-a)}{v(-1)} \geq a \quad (15)$$

with strict inequality if convexity is strict.

Thus, by (13), (14), and (15),

$$a + 1 - b < 1.$$

$$a < b.$$

A contradiction of our assumption that $a \geq b$.

We have proved that in equilibrium, there is more contribution when the reference point is $x_0 = c$ than when it is the high reference point $x_0 = 1 + c$.

Proposition 2. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, there is more contribution with a low reference point $x_0 = c$ than with a high reference point $x_0 = 1 + c$.*

$$c_c^* > c_{1+c}^*. \tag{16}$$

For this result it suffices that any of the following holds: subcertainty of the probability weighting function w, strict concavity in gains of the value function v, or strict convexity in losses of v.

Note that since from these two reference points, any of the prospects at choice is nonnegative or nonpositive, loss aversion plays no role.

When the reference point is $x_0 = 0$, the unique symmetric equilibrium is the solution to equation (11), which we can also write,

$$\frac{v(1) - v(c)}{v(1+c) - v(c)} = w(1 - q(c)). \tag{17}$$

By concavity of v in the positive domain,

$$\frac{v(1) - v(c)}{1 - c} \geq \frac{v(1+c) - v(c)}{1 + c - c},$$

so,

$$\frac{v(1) - v(c)}{v(1+c) - v(c)} \geq 1 - c. \tag{18}$$

Thus, by the same argument that showed $c_c^* > c_{1+c}^*$, we have $c_0^* > c_{1+c}^*$.

Proposition 3. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, there is more contribution with a low reference point $x_0 = 0$ than with a high reference point $x_0 = 1 + c$.*

$$c_0^* > c_{1+c}^*. \tag{19}$$

For this result it suffices that any of the following holds: subcertainty of the probability weighting function w, strict concavity in gains of the value function v, or strict convexity in losses of v.

Loss aversion again plays no role in this comparison.

If the value of the public good is relatively small, as it is likely to be in an experimental setting, the value function can be taken as linear in gains and linear in losses. From the equilibrium conditions (10) and (11) corresponding to the two low reference points $x_0 = c$ and $x_0 = 0$, it is immediate that if the value function is linear in gains on the interval $[0, 1]$ then at both reference points the equilibrium is the same.

Proposition 4. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, if the value function v is linear in the domain of gains, $[0, 1]$, then*

$$c^*_c = c^*_. \tag{20}$$

Proposition 4 can be generalized. When v is linear in gains the *payoffs* and so the equilibria are not affected when the reference point moves down from c. Similarly when v is linear in losses, nothing changes if the reference point moves up from $1 + c$.

Next we compare the equilibrium at an intermediate reference point $x_0 = 1$ and at a high reference point $x_0 = 1 + c$. We show that $c^*_1 > c^*_{1+c}$ by showing that the right-hand side of (12), the equilibrium condition for reference point $x_0 = 1$, evaluated at c^*_{1+c} is negative.

Let $a = c^*_{1+c}$, so by (9),

$$\frac{v(-a)}{v(-1)} = w(q(a)). \tag{21}$$

Let $R(\cdot)$ be the right-hand side of (12). From (21) and subcertainty of the probability weighting function w,

$$R(a) = w(1-a)v(a) + w(a)v(a-1) \tag{22}$$

$$R(a) < \left(1 - \frac{v(-a)}{v(-1)}\right) v(a) + \frac{v(-a)}{v(-1)} v(a-1) \tag{23}$$

For $x > 0$ let $\lambda(x) := -v(-x)/v(x)$, be the variable loss aversion. It is generally assumed that $\lambda \geq 1$. Empirical evidence suggests it is close to a value of 2. No loss aversion corresponds to $\lambda = 1$.

From (23),

$$R(a) < -\left(\frac{v(-1) - v(-a)}{v(-1)}\right)\frac{v(-a)}{\lambda(a)} + \frac{v(-a)}{v(-1)}v(a-1) \tag{24}$$

So,

$$R(a)\frac{v(-1)}{v(-a)} < \left(\frac{v(-a) - v(-1)}{\lambda(a)}\right) + v(a-1) \leq v(-a) - v(-1) + v(a-1). \tag{25}$$

With the last inequality being strict if there is loss aversion, i.e., if $\lambda(a) > 1$. Now, since v is convex in losses and $0 < a < 1$, by Proposition 1, we have,

$$v(a-1) \leq av(0) + (1-a)v(-1) = (1-a)v(-1), \tag{26}$$

and

$$v(-a) \leq (1-a)v(0) + av(-1) = av(-1). \tag{27}$$

So,

$$v(a-1) + v(-a) \leq v(-1), \tag{28}$$

and $R(a) < 0$. We conclude that $c_1^* > c_{1+c}^*$.

Proposition 5. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, there is more contribution with an intermediate reference point $x_0 = 1$ than with a high reference point $x_0 = 1 + c$.*

$$c_1^* > c_{1+c}^*. \tag{29}$$

For this result it suffices that any of the following holds: loss aversion, subcertainty of the probability weighting function w, strict concavity in gains of the value function v, or strict convexity in losses of v.

Finally we compare c_1^* with c_0^* and c_c^*.

Evaluating the right-hand side of (12) at $b = c_c^*$.

$$R(b) = w(1-q(b))v(b) + w(q(b))v(b-1) = \frac{v(1-b)v(b)}{v(1)} + w(q(b))v(b-1) \tag{30}$$

and since $\lambda(1-b) = -v(b-1)/v(1-b)$,

$$R(b) = \frac{v(1-b)v(b)}{v(1)} - \lambda(1-b)w(q(b))v(1-b) \tag{31}$$

Since (31) is linear and decreasing in $\lambda(1-b)$ and $R(b)$ is increasing in b, it follows that for $\lambda(1-b)$ greater than the threshold

$$\overline{\lambda}_c = \frac{v(c_c^*)}{v(1)w(q(c_c^*))} \tag{32}$$

$R(b) > 0$ so that $c_1^* > b = c_c^*$, while the inequality is reversed if $\lambda(1-b)$ is less than $\overline{\lambda}_c$.

Similarly, evaluating the right-hand side of (12) at $c = c_0^*$.

$$R(c) = w(1-q(c))v(c) + w(q(c))v(c-1) = \frac{v(1)-v(c)}{v(1+c)-v(c)}v(c) + w(q(c))v(c-1), \tag{33}$$

and since $\lambda(1-c) = -v(c-1)/v(1-c)$,

$$R(c) = \frac{v(1)-v(c)}{v(1+c)-v(c)}v(c) - \lambda(1-c)w(q(c))v(1-c). \tag{34}$$

Since (34) is linear and decreasing in $\lambda(1-c)$ and $R(c)$ is increasing in c, it follows that for $\lambda(1-c)$ greater than the threshold

$$\overline{\lambda}_0 = \left(\frac{v(1)-v(c)}{v(1+c)-v(c)}\right)\left(\frac{v(c)}{v(1-c)}\right)\left(\frac{1}{w(q(c))}\right). \tag{35}$$

$R(c) > 0$ so that $c_1^* > c = c_0^*$, while the inequality is reversed if $\lambda(1-c)$ is less than $\overline{\lambda}_0$.

We have proved the following proposition.

Proposition 6. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, there is more contribution with a reference point $x_0 = 1$ than with a lower reference point $x_0 = c$ if and only if loss aversion is high enough,*

$$c_1^* > c_c^* \text{ iff } \lambda(1 - c_c^*) > \overline{\lambda}_c. \tag{36}$$

where the threshold $\overline{\lambda}_c$ is defined by (32).

Similarly, there is more contribution with a reference point $x_0 = 1$ than with a lower reference point $x_0 = 0$ if and only if loss aversion is high enough,

$$c_1^* > c_0^* \text{ iff } \lambda(1 - c_0^*) > \overline{\lambda}. \tag{37}$$

where the threshold $\overline{\lambda}_0$ is defined by (35).

In the case that the value function v is linear in gains, both thresholds collapse,

$$\overline{\lambda} := \overline{\lambda}_0 = \overline{\lambda}_c = \frac{c}{w(q)}, \tag{38}$$

where $c = c_0^* = c_c^*$ and $q = q(c)$.

If in addition, the probability weighting function is linear, then $\overline{\lambda} = 1$, so that if there is loss aversion, i.e., $\lambda > 1$, then c_1^* is greater than the equilibrium probability of contribution with the other reference points.

2.3.2 Comparison with expected utility

Let c_{eu} be the symmetric equilibrium probability of individual contribution for linear VNM utility in the game where a single contribution is enough for the public good to be provided. Clearly, c_{eu} is the unique solution to the equilibrium condition ,

$$c_{eu} = q(c_{eu}) = (1 - c_{eu})^{N-1}. \tag{39}$$

Since q is decreasing both in c and N, it is clear from (39) that c_{eu} is decreasing in N. The larger the group, the smaller the equilibrium probability of contribution for each individual. Some values of c_{eu} for small groups:

$$c_{eu}(2) = 1/2$$

$$c_{eu}(3) = .382$$

$$c_{eu}(4) = .318$$

$$c_{eu}(5) = .276$$

We start comparing the equilibrium probability of contribution for a high reference point $x_0 = 1 + c$ with the expected utility probability of contribution c_{eu}. At c_{eu} the left-hand side and the right-hand side of the equilibrium condition for c^*_{1+c}, expression (9), becomes,

$$R(c_{eu}) = w(c_{eu})$$

$$L(c_{eu}) = \frac{v(-c_{eu})}{v(-1)} \geq c_{eu}$$

The last inequality follows from convexity of v on the negative domain, and the inequality becomes an equality if v is linear in losses.

Let c_f be the interior fix point of the probability weighting function, i.e., $w(c_f) = c_f$, $0 < c_f < 1$. Then, if $c_{eu} > c_f$, $w(c_{eu}) < c_{eu}$, and since L is increasing and R is decreasing it must be $c^*_{1+c} < c_{eu}$.

Proposition 7. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, the probability of contribution with a high reference point $x_0 = 1+c$ is less than according to Expected Utility Theory if the latter is less than the fix point of the probability weighting function.*

$$c^*_{1+c} < c_{eu} \text{ if } c_{eu} > c_f. \tag{40}$$

In addition, if the value function v is linear in losses then

$$c^*_{1+c} > c_{eu} \text{ if } c_{eu} < c_f. \tag{41}$$

Prelec (1998) reports estimates of the fix point c_f that range from .30 to .39. Then it follows from Proposition 7 that if there are only $N = 2$

individuals, $c^*_{1+c} < c_{eu}$, and if there are more than 4 individuals in the group then $c^*_{1+c} > c_{eu}$ if v is linear in losses.

To compare c^*_c with c_{eu}, evaluate the left- and right-hand sides of the equilibrium condition of the former at the latter.

$$L(c_{eu}) = \frac{v(1-c_{eu})}{v(1)} \geq 1 - c_{eu}, \tag{42}$$

by concavity of v in gains, and with equality if v is linear in gains.

$$R(c_{eu}) = w(1 - c_{eu}). \tag{43}$$

If, as all estimates suggest, $c_f < 1/2$, $c_f < 1 - c_{eu}$ for all $N > 1$, so that $1 - c_{eu} > w(1 - c_{eu})$. Thus, $L(c_{eu}) > R(c_{eu})$ and we have the following proposition.

Proposition 8. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, the probability of contribution with reference point $x_0 = c$ is greater than according to Expected Utility Theory , if the fix point of the weighting function $c_f < 1/2$.*

$$c^*_c > c_{eu}, \tag{44}$$

The result also holds if the probability weighting function w is linear but the value function v is strictly concave in gains.

Comparing c^*_0 and c_{eu} we similarly get the following proposition.

Proposition 9. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, the probability of contribution with a low reference point $x_0 = 0$ is greater than according to Expected Utility Theory , if the fix point of the weighting function $c_f < 1/2$.*

$$c^*_0 > c_{eu}, \tag{45}$$

The result also holds if the probability weighting function w is linear but the value function v is strictly concave in gains.

Proof. This is immediate for v linear in gains, by Proposition 4 and 8. It remains to prove the result for v nonlinear in gains.

Evaluate the right-hand side of the equilibrium condition of c_0^*, expression (11), at c_{eu},

$$R(c_{eu}) = v(c_{eu}) + w(1 - c_{eu}) \left[v(1 + c_{eu}) - v(c_{eu}) \right]. \tag{46}$$

If $c_f < 1/2$, then $1 - c_{eu} > c_f$, so$w(1 - c_{eu}) < 1 - c_{eu}$,

$$R(c_{eu}) < v(c_{eu}) + (1 - c_{eu}) \left[v(1 + c_{eu}) - v(c_{eu}) \right], \tag{47}$$

and, since by concavity of v in gains,

$$v(1) \geq v(c) + (1 - c) \left[v(1 + c) - v(c) \right], \tag{48}$$

it follows that $v(1) > R(c_{eu})$, and since R is increasing, it must be $c_0^* > c_{eu}$. □

Finally, we compare c_1^* with c_{eu}.

Proposition 10. *Suppose the public good is provided as long as at least one individual contributes. Under Prospect Theory, at the symmetric equilibrium, the probability of contribution with reference point $x_0 = 1$ is greater than according to Expected Utility Theory ,*

$$c_1^* > c_{eu} \text{ iff } \lambda(1 - c_{eu}) > \overline{\lambda}_{eu}, \tag{49}$$

where the threshold,

$$\overline{\lambda}_{eu} = \left(\frac{w(1 - c_{eu})}{v(1 - c_{eu})} \right) \left(\frac{v(c_{eu})}{w(c_{eu})} \right). \tag{50}$$

In particular, for $N = 2$, $\overline{\lambda}_{eu} = 1$. For $N > 2$, $\overline{\lambda}_{eu} < 1$, if v is linear in gains and $c_{eu} \leq c_f$.

Proof. At c_{eu}, the right-hand side of the equilibrium condition for c_1^*, expression (12) becomes,

$$R(c_{eu}) = w(1 - c_{eu}) v(c_{eu}) + w(c_{eu}) v(c_{eu} - 1). \tag{51}$$

Then (49) follows since R is increasing.

For $N = 2$, since $c_{eu} = 1/2 = 1 - c_{eu}$, clearly $\overline{\lambda_{eu}} = 1$.

For $N > 2$, if v is linear in gains,

$$\overline{\lambda}_{eu} = \left(\frac{w(1-c_{eu})}{v(1-c_{eu})}\right)\left(\frac{v(c_{eu})}{w(c_{eu})})\right) = \left(\frac{w(1-c_{eu})}{1-c_{eu}}\right)\left(\frac{c_{eu}}{w(c_{eu})})\right), \tag{52}$$

is less than $c_{eu}/w(c_{eu})$ since $1 - c_{eu} > 1/2 > \underline{c}_f$, by regressiveness and subcertainty of w at c_f. Thus, if $c_{eu} \leq c_f$ then $\overline{\lambda}_{eu} < 1$ since $c_{eu} \geq w(c_{eu})$. □

2.4 All contributions required (Γ_N)

In this section we consider the other polar case, where the provision of the public good requires that all of the players contribute.

So far we have considered that the value of the public good is $g = 1$, the same as the supremum of the support of the distribution of private costs. As it will be apparent below, it is convenient that we allow the value of the public good $g \leq 1$. When $g < 1$ it is clearly dominated the strategy of contributing regardless of cost. This way we get a sharper contrast between EUT and PT that can be tested experimentally. We focus our analysis in the limit of equilibrium as g tends to 1 from below.

As in the previous section, let $q(c)$ be the probability that a player is decisive (i.e., pivotal) given that all the other players contribute if and only if their cost is less than c. Then, in this case, $q(c) = c^{N-1}$. Clearly, q is an increasing function in c from 0 at $c = 0$ to 1 at $c = 1$, and decreasing in N.

We start with the equilibrium analysis in the reign of Expected Utility Theory with linear VNM utility function. In equilibrium,

$$gc^{N-1} = c \tag{53}$$

Proposition 11. *Suppose the public good is provided as long as all players contribute. Under Expected Utility Theory with linear utility function, the symmetric equilibria are the solutions $c \in [0, g]$ of equation (53), that is,*

(i) For $g < 1$ the unique equilibrium is $c_{eu} = 0$

(ii) For $N = 2$, and $g = 1$ there is a continuum of equilibria $[0, 1]$.

(iii) For $N > 2$ and $g = 1$, there are only two equilibria 0 and 1.

We now turn to analyzing equilibrium under Prospect Theory for the four reference points we have considered in the previous section.