On the Role of Externalities in the Choice of Technology*

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Abstract

This paper examines technology adoption problems in a simple general equilibrium framework, characterized by the presence of a firm and a number of self-employed consumer. It is shown that the choice of technologies may be hindered, or even blocked when the firm is price maker on the labor market and price taker on the goods market. Two sources of externality are likely to determine inefficient technology choices and thus inefficiencies of market allocations. First, the firm's technology choice generates a positive externality on the production function of self employed workers. Second, this positive externality induces an increase in labor costs, hence implying a negative pecuniary externality on the firm.

Pareto efficient allocations that would be generated by a social planner internalizing all sources of externalities are discussed, and different mechanisms of policy intervention in order to overcome (or mitigate) market failure are studied, ranging from non linear (first best) subsidization to Pigouvian (second best) subsidies/taxes on labor input and technology adoption.

Keywords: Technology adoption, production externalities, efficiency wages, market failure, Pareto efficiency, Pigouvian subsidies.

JEL classification: J41, L20, O30

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1 Introduction

Technology is often referred to as one of the most important growth engines. It is therefore striking to observe that in many cases firms do not choose the best available technologies, sticking instead to inferior ones. It is similarly puzzling to note that, at a wider level, there are several countries that fail to use technology as a mechanism to promote economic progress. This, as pointed out by Lucas (1990), seems to be even more surprising in a world characterized by high capital mobility.

Many models studying technology adoption problems explain the adoption of older technologies in poorer countries emphasizing the role of transfer and adjustment costs.[1] There is, however, a stream of literature assuming that the knowledge of a technology spreads quite easily (or even instantaneously) and that the direct costs of its adoption are relatively small. Under these assumptions, some authors (e.g. Basu and Weil, 1998) claim that the adoption of a superior technology can be hindered by a low degree of development, while others (e.g. Zeira, 1998) establish a relationship between the adoption of different technologies in different countries and the prices of factors of production, as well as the increasing quantity of capital required by technological progress. Furthermore, recent empirical contributions focus on the role of available resources and infrastructures and the political regime as determinants of technology adoption.[2]

This paper — looking to the choice of technology at the firm level — aims to contribute to the debate by emphasizing the role of the consequences of technology adoption on the choice of the technology itself; thus proposing an endogenous mechanism different than those typically considered in the literature, mainly stressing the influence of exogenous factors. More precisely, we ask whether there exists a direct link between market power and technology adoption processes, claiming that the presence of perfectly competitive goods markets, but monopolistic factors markets can impede the adoption of superior technologies instead of favoring it.[3]

This view, that dates back to the Classics, has been recurrently investigated in the literature. Parente and Prescott (1999), for instance, show that the existence of a coalition of labor suppliers, selling their input under monopolistic conditions to all firms, can prevent the entry in the industry of other coalitions having access to a superior technology, over which however the incumbent coalition does not have monopoly rights,

[1] See, for example, Jovanovic and Lach (1989), Parente and Prescott (1994), Grossman and Helpman (1991, Chpts. 6 and 11), Anant, Dinapoulos and Segerstrom (1990), Barro and Sala-i-Martin (1997), as well as Colombo (2004, Chpt. 2), Comin and Hobijn (2004) and Geroski (2000) for surveys on technology adoption and diffusion.

[2] See, among others, Sachs and Warner (1997), Hall and Jones (1997), Sala-i-Martin (1997) and Colombo (2004, Chpt. 2) for a survey.

[3] There is a wide empirical support for the impact of market power on the technology adoption processes. For instance, it is well known that lack of competition is much more diffused in poorer countries, where old technologies tend to be most used, than it is in rich and developed countries.

blocking therefore the adoption of a better technology.

In this paper, we reach a similar conclusion without introducing a role for workers' coalitions or other forms of coordination in the labor market, by stressing instead the role of production externalities in a setting where the labor market is imperfectly competitive. We consider a general equilibrium economy with efficiency wages *à la* Shapiro and Stiglitz (1984) producing a single consumption good. There is a finite number of identical consumers; production is carried out by a price-taker firm (a "market sector" firm whose shares are equally held by all agents in the economy) and a number of self-employed entrepreneurs (workers). Excluding self-employment, the firm is a monopsonist in the labor market and, besides technology, labor is the only input of its production function. The labor productivity of the self-employed is influenced by the technology chosen by the firm. Consumers can therefore be employed by the firm, self–employed, or unemployed.

The central issue of the paper deals with the consequences of the technology adoption decision by the firm. On the one hand, when introducing a superior technology, the firm should observe an increase in its profits. On the other hand, the adoption of the technology generates positive externalities (spillovers) for the workers using it. By assuming that once introduced by a firm a technology and the knowledge it requires become easily available, nothing prevents workers to put it at work in other sectors of the economy as well.[4] In our model, self-employment will indeed be taken as a shortcut to model the set of outside options available to the firm's workers. The ability to exploit the externalities generated by the market sector firm increases workers' productivity in the self-employment sector and thus their reservation income and bargaining power. The firm is, therefore, forced to offer higher wages to its workers, if it wants them to accept its offer and exert the desired level of effort. The main contribution of the paper is to show that the associated increase in labor costs can be enough to induce the firm not to adopt the superior technology in all the cases in which it takes explicitly into account the link between externalities and technology choice. Note that, if the advantages and the costs associated to the choice of technology are common knowledge, one could design bargaining procedures to allocate and distribute the net gains from the adoption of a better technology between the firm and its workers. However, any such agreement must imply a wage high enough to meet the workers' participation and incentive compatibility constraints.

The channel through which market power on the labor market can block the choice of a superior technology has a strategic nature, depending on the relationship between the outside options of workers and the technology chosen by the firm, via the technology induced production externalities. Although the nature of these spillovers is not modeled explicitly in the paper, their existence can be motivated by observing that the adoption

[4] As observed by Acemoglu (2002), this is often the case in less developed countries where, because of the lack of intellectual property rights, new machine varieties invented in the North of the world can be copied without paying royalties.

of a superior technology is likely to increase the level of "transferable" human capital of workers, due to the worker's ability to operate it. A worker's expertise with the technology comes with a cost for the firm that can not be transferred on workers or on the good price (due to the price taking assumption). By learning how to operate the technology, workers acquire additional skills and increase their level of knowledge which, in turn, increases the value of their outside options; i.e. their revenue when self employed.[5] The idea of complementarity between technology and skills and that of the existence of a direct nexus between technology and wages are widely supported by the empirical evidence on technical change, which stresses how the knowledge of a technology is very costly to be produced but very easy to be reproduced and emphasizes that firms are typically unable to entirely appropriate the benefits deriving from the technological innovations they introduce.[6]

Note finally that, the consideration of a general equilibrium framework allows us to account endogenously for all feedback effects arising from the link between technology and outside options — and influencing the consumers' decisions on labor supply, the wages and the firm's choice of technology. At the same time, the economy is closed, in the sense that all monetary and real flows are accounted for, and the income determination process is both endogenous and complete, meaning that all income generated is used.

We focus on the choice of technology by the market sector firm under two different scenarios. First, we assume Cournot competition between the firm and the self-employed entrepreneurs, in the sense that the level of the externality induced by the technology choice of the firm is treated as an exogenous parameter both by the firm itself and by the self-employed. Under this assumption, which we take as our benchmark, we show that at a Cournot-Nash equilibrium the presence of the externality does not entail neither labor nor technology misallocation. Second, we replace Cournot competition with a competition *à la* Von Stackelberg among producers, in which the firm takes into account the impact of its technology choice on the self-employed entrepreneurs' profit maximization problem. In this case, two sources of externality are likely to produce inefficiencies of market allocations. On the one hand, there is a positive externality on the production function of self employed workers determined by the firm's choice of technology. On the other hand, this positive externality, induc-

[5] This observation is consistent with the observed wage increases for skilled workers, as documented by the U.S. college wage premium in the last sixty years (e.g. Acemoglu, 2002). For a survey of the impact of technology adoption on human capital see Booth and Snower (1996) and Colombo (2004, Chpts 2 and 3).

[6] See, for example, Nadiri (1993) proposing an extensive survey of the literature on the spillover effects induced by the adoption of superior technologies. As for the impact of spillovers on wages, Schultz (1975), Bartel and Lichtenberg (1987), Acemoglu (1998) and Machin and Van Reenen (1998), among others, investigate the relationships between technology, R&D activities and wages, while Goldin and Margo (1992), Katz and Murphy (1992) and Autor, Katz and Krueger (1998) analyze wage differentials between skilled and unskilled labor demand.

ing an increase in labor costs, generates a negative pecuniary externality on the firm. This double source of non-marketed relations may discourage technology adoption and produce labor misallocation across sectors.

The second part of the paper focuses on these issues by taking a normative perspective, exploring possible remedies to the inefficiencies stemming from the externalities associated to technology adoption. We first characterize the Pareto efficient allocations that would be generated by a social planner who internalizes all sources of externalities, and we then turn to the analysis of whether government intervention is able to overcome — or mitigate — market failure. In doing so, we consider a number of policy intervention from non linear (first best) sudsidization to second best — but eventually more realistic — policy instruments, showing their impact on allocations.

The paper is organized as follows. Section 2 describes the economy discussing the consumption and the production sector. Section 3 focuses on Cournot competition among producers and shows that at a Cournot - Nash equilibrium there is not technology misallocation. In Section 4, the Von Stackelberg case is introduced, so that technology misallocation becomes possible. A definition of the Von Stackelberg equilibrium is provided, as well as a discussion of its existence and uniqueness and of the possible technology adoption regimes. Section 5 takes a normative perspective and focuses on market failure, looking at Pareto efficient allocations obtained by internalizing all sources of externalities and turning then to policy analysis. Section 6 concludes by summarizing the results and suggesting avenues for further research. Finally, an appendix contains an exemplification of the paper main results for a Cobb-Douglas economy.

2 The economy

The economy is composed of $\bar{N}$ identical consumers and an industrial sector that produces a consumption good. Besides technology, labor is the only input of production and the price of consumption is normalized to unity without loss of generality. The choice of the numeraire good has no real effects within the general equilibrium framework here, although, in general, in economies with imperfect competition it affects the equilibrium allocation (see Böhm, 1994, and Myles, 1995, Chpt. 11).

The production sector is composed of a (market sector) firm, and a large number of self-employed entrepreneurs/workers (denoted with f). It is assumed that no other firms can enter the economy.[7] We first describe consumers' choices and then turn to producers' behavior.

[7] This might be due to institutional (e.g. the distribution of property rights and/or imperfections of financial markets) or political constraints rendering the entry of competitors impossible or unprofitable.

2.1 The consumption sector

Each consumer can supply one unit of labor (i.e. a fixed labor time given exogenously) to the firm or work as a self-employed. She derives income from labor and obtains an equal share of the profits generated by the firm. All consumers in the economy are characterized by a utility function of the type

$$V(x,h) = x - \varphi(h), \tag{1}$$

where x is consumption, $\varphi(h)$ is the (consumption-equivalent) disutility of labor and h denotes labor time, with

Assumption 1 *$\varphi \in C^2$, $\varphi(0) = \varphi'(0) = 0$, $\varphi'(h) > 0$, $\varphi''(h) > 0, h > 0$.*

Quasi-linearity in x implies that there are no income effects in the demand for the consumption good.

We assume that the firm has an imperfect monitoring technology and hence we allow for the possibility of shirking by workers employed by the firm. Without loss of generality, we take the disutility of labor for a shirker to be equal to 0, which is standard in the efficiency wage literature. Since labor time is exogenous and supplied inelastically when working for the firm, the disutility of labor can take only one of two values. If a worker does not exert effort it is $\varphi(h) = 0$; if she exerts the desired level of effort it is $\varphi(h) = e > 0$. For a self-employed worker, labor disutility is $\varphi(h_f)$, which depends on labor time. We assume that a self-employed does not have an incentive to shirk (or, which has the same consequences, that there is perfect monitoring in the self-employment sector). If an agent is unemployed (u) she does not exert any effort.

Hence, each consumer makes a choice among four options: work for the firm and shirk (s), work for the firm and not shirk (ns), to be self-employed (f) and, finally, to stay unemployed (u). The utility levels associated to the four options are derived from the corresponding expected utility maximization problems.

Consider first the case of workers employed by the firm. Two different utility maximization problems have to be studied for shirker and non-shirker workers. Since labor time is given exogenously, the disutility of effort can take one of two values: $\varphi(h) = e$ when the worker exerts the desired level of effort and 0 when she shirks. Therefore, a worker must choose her optimal consumption level x and her level of effort (where the latter is a binary choice).The non-shirkers are those who exert the required level of effort (e). Recalling that the consumption good price is set equal to 1, it is

$$\begin{cases} \max\limits_{x} V^{ns} = x - e \\ \text{s.t.} \quad x \leq w + \pi \end{cases}, \tag{2}$$

where w is the wage paid by the firm and π is the share of the firm profits going to each consumer. As for the latter, we assume that

Assumption 2 $\pi = \Pi/\bar{N}$, *where* Π *are total profits.*

We think of the firm as a corporation so that the $\bar{N}$ identical consumers, all making the same portfolio choice, hold a fraction $1/\bar{N}$ of shares and hence receive dividends π. This assumption on the ownership structure of the firm is obviously quite extreme and there are many possible alternative and more realistic profit distribution schemes that could be considered. One can assume for instance that profits are accruing to a subset of the population only. As far as a shareholder is not a worker of the firm (or better can not benefit directly or indirectly of the adoption of a superior technology by the firm), any profit distribution mechanism would not interfere with her decisions on labor allocation. However, as soon as an agent is at the same time a shareholder and a worker, she should take into account the impact of technology choices both on the share of profits (dividends) she is entitled to, and on the labor income she receives from the firm. A scheme of this type, by introducing additional feedback effects to be taken into account in a general equilibrium framework,would further complicate the analysis without however being central to our argument.

From Problem (2), it is immediate that $x = w + \pi$ and the corresponding expected utility level is specified by Equation (3).

$$V^{ns} = w + \pi - e. \tag{3}$$

Similarly, for the shirkers (exerting no effort) it is

$$\begin{cases} \max\limits_{x^0,x^1} V^s = (1-c)\,x^1 + cx^0 \\ \text{s.t.} \quad x^1 \leq w + \pi \\ \qquad\;\; x^0 \leq \pi \end{cases}, \tag{4}$$

where x^1 denotes consumption when a shirker is not caught shirking and x^0 when she is caught shirking and is fired, and $c \in (0,1)$ is the probability to be caught shirking when employed by the firm. We take the firm's monitoring technology as given exogenously, thus ruling out the possible links among the firm's technology choice and monitoring.[8] From Problem (4) it follows immediately that $x^1 = w + \pi$ and $x^0 = \pi$, and thus the corresponding expected utility level is

$$V^s = (1-c)(w+\pi) + c\pi. \tag{5}$$

Turning now to the self-employed consumers, they maximize expected utility both over labor time h_f and consumption x_f. By taking labor time as given (we will solve explicitly for it in Problem (8) in the next section), their expected utility level is derived in the same way as above, obtaining

$$V_f = w_f + \pi - \varphi(h_f), \tag{6}$$

[8] See Colombo (2004, Chpt. 4) for a discussion of the point.

where w_f is gross-income of a self-employed worker. Finally, for the unemployed agents it is

$$V_u = \pi. \tag{7}$$

Notice that π is the only source of income for the unemployed. In particular, there are no unemployment benefits.

Each consumer chooses the option that maximizes her welfare, among feasible options. Whenever any two options give the same utility level, we assume that preferences are such that:

Assumption 3 *If $V^{ns} = V^s$ then $ns \succ s$. If $V^s = V_f$ then $s \succ f$. If $V_f = V_u$ then $f \succ u$.*

2.2 The production sector

Self-employed entrepreneurs

Self-employed entrepreneurs are characterized by a production function incorporating a production externality via the technology adopted by the market sector firm of the form gh_f, where g captures the production externality. For any given g, hence, there are constant returns in labor. The self-employed agents are unable to influence the technology adopted by the firm and therefore take the production externality as a given parameter, i.e. $g = G$, when solving their decision problem.

Given that, as we will show below, the firm's problem includes the participation constraint of workers (which takes into account the outside option represented by self employment), by Assumption 3 only workers not employed by the firm are potentially interested in being self-employed, which acts therefore as an outside option.

Self-employed workers solve the following problem

$$h_f(G) := \underset{h_f}{\arg\max} \quad Gh_f - \varphi(h_f), \tag{8}$$

which gives $G = \varphi'(h_f)$ as a first order condition. By Assumption 1, the optimal h is unique and non negative. Denoting the (labor) income of a self-employed entrepreneur with $W_f(G)$, it is[9]

$$W_f(G) := Gh_f(G). \tag{9}$$

Hence the indirect utility of a self–employed is

$$V_f(G) := W_f(G) - \varphi(h_f(G)) + \pi. \tag{10}$$

[9]It is immediate to note that the marginal return on labor is equal to the (exogenous) marginal productivity G.

Notice, finally, that instead of self employment, we could have modelled sector f as a perfectly competitive industry that uses only labor as an input. This alternative specification is equivalent to the chosen one provided that: a) there are constant returns to scale, i.e. the production function of the individual firm j is of the type GN_f^j, where N_f^j is its labor input; b) there is perfect monitoring in sector f.

The market sector firm

Next, we consider the market sector firm — i.e. the externality producer — problem. The firm has a production function $\Phi(N,T)$ — where N denotes the labor input and T the technology adopted — satisfying the following assumptions:

Assumption 4 $\Phi \in C^2$, $\forall\,(N,T) \gg 0 : \dfrac{\partial\Phi}{\partial T} > 0,\ \dfrac{\partial\Phi}{\partial N} > 0,\ \dfrac{\partial^2\Phi}{\partial N^2} < 0,\ \lim\limits_{N\to 0}\dfrac{\partial\Phi}{\partial N} = +\infty.$

While we assume decreasing returns in labor input, we do not impose any restriction on technology returns. Moreover, we assume that technology adoption is a costless and continuous choice available within an exogenously given range.

Assumption 5 $T \in [0, T_{\max}]$. *There are no adoption costs and no price must be paid to install any available technology.*

The assumption that a superior technology can be chosen without suffering adoption (or adjustment) costs, although clearly simplistic, does not seem problematic in our framework, even though it requires some caution. On the one hand, the adoption costs often required in the literature to explain why superior technologies are not installed — besides being in many circumstances of too large a magnitude with respect to what reported by the available empirical evidence — reinforce the role of the externality (i.e. spillover) discussed here in slowing down the adoption of a higher technology grade.[10] On the other hand, the idea that once introduced by a firm a technology becomes freely available makes it easier for agents to put it at work elsewhere as well.[11] In our setting, the self-employment sector is just a compact way to model the set of outside options available to workers. The ability to exploit the externalities generated by the decision of the firm to adopt a better technology increases workers' productivity in the

[10] For a survey and discussion of the literature see Colombo (2004, Chpt. 2).

[11] As observed by Acemoglu (2002), this is often the case in less developed countries where, because of lack of intellectual property rights, new machine varieties invented in the North of the world can be copied without paying royalties.

Notice also that, in order to eliminate the possibility of heterogeneity between (skilled) employed and (unskilled) unemployed agents, we assume that once a superior technology has been introduced, its knowledge diffuses instantaneously. We could as well assume the presence of a union (or of institutional constraints) linking the firm's wage structure to the technology chosen and not to individual skills. If the firm does not have the opportunity to pay lower wages to the newly hired workers, the heterogeneity among skilled and unskilled workers disappears.

self-employment sector, and thus their reservation income and their bargaining power. Introducing an adoption cost (and/or a price) for the technology would therefore make it more difficult for workers to directly take advantage of the technology (for instance by adopting it as self-employed entrepreneurs). This would not imply, however, that a worker can not benefit elsewhere from the "skills" (i.e. technical knowledge) she acquired operating the technology (benefits that are here modeled in the form of a production externality), provided such skills are not entirely specific.

Note, finally, that $T_{\max}$ represents the best available technology given the "state of the art" of current scientific know-how, which is publicly available at no cost. Matters are different when the process of innovation is explicitly taken into account. In this case, the technology frontier ($T_{\max}$) becomes endogenous in the firm's investments in R&D (or managerial reorganization, and so on). Thus costs associated with moving the frontier can be substantial and likely to be among the most important factors behind firms' choices about technology developments. In this case, the spillover effects we emphasize can be of second order only. It is, however, worth to emphasize that there are many circumstances in which available technologies are not adopted by firms and adoption/adjustment costs are just not big enough to explain why. These are the cases in which the strategic interactions developed in this paper are likely to be important.

The firm solves the following profit maximization problem subject to the participation and non-shirking constraints of workers:

$$\begin{aligned} &\max_{N,T,w} \quad \Pi = \Phi(N,T) - wN \\ &\text{s.t.} \quad V^{ns} \geq V^{s}, \quad V^{ns} \geq V_f(G), \quad V^{ns} \geq V_u, \end{aligned}$$

where V^{ns}, V^{s}, $V_f(G)$ and V_u are defined respectively by Equations (3), (5), (10) and (7). The first constraint is the no-shirking (incentive compatibility) constraint and the other two are the participation (individual rationality) constraints.[12]

In solving the firm's decision problem, we consider two possible cases. In the first one that we denote as *Cournot-Nash*, the firm itself takes the externality it induces as a parameter given exogenously. In the second one, which we will refer to as *von Stackelberg case*, the firm knows the relationship between the technology it adopts and the production externality it induces and takes it into account while solving its profit maximization problem. In particular, we assume g to be increasing in T, so that the technology adopted by the firm increases the workers' productivity in the self-employment sector of the economy. More precisely:

Assumption 6 *$g \in C^2$, $g(0) = 0$, $g'(T) > 0$, $\forall T \geq 0$.*

[12] We are implicitly assuming that the firm's technology choices do not affect the decision of a worker to shirk or not to shirk. In a more sophisticated formulation, however, one could assume that a non-shirker worker can learn the technology faster and in a better way than a shirker, benefiting more of the spillover effects induced by the technology. This would make shirking more costly, contributing to relax the incentive compatibility constraint.

This assumption is meant to capture the positive impact of the spillover effects associated to the adoption of superior technologies.[13]

3 The Cournot-Nash case

This section introduces the benchmark case in which technology driven spillovers do not play any role, thus mimicking the behavior of the economy under perfect competition. The firm is assumed to act myopically, ignoring the consequence of its (technology) choice on the actions of self-employed entrepreneurs.[14] We show that when the firm treats the externality as an exogenous parameter G and not as a function of technology its profit maximization problem is not constrained by it. We assume, for the sake of simplicity, that the Cournot firm assigns the same value as the self-employed to the externality, and start focusing on the workers' individual rationality and incentive compatibility constraints. Assuming different evaluations of the externality might have an impact on the firm's ability to satisfy labor demand. This would be the case if its valuation of the externality is lower than the one by the self-employed. Insofar an higher externality transfers into a better outside option, the wage offer by the firm would not be enough to satisfy a self-employed's participation constraint. This would imply complete rationing of the firm on the labor market. In a framework of complete information, it seems natural to assume that the firm is knowledgeable about the outside options available to self-employed agents and thus it takes the relevant externalities into account when designing its wage offer.

Using (3), (5) and (10), the firm's constraints $V^{ns} \geq V^s$ and $V^{ns} \geq V_f(G)$ can be written respectively as

$$w \geq e/c, \tag{11}$$

$$w \geq W_f(G) + e - \varphi(h_f(G)). \tag{12}$$

There is no need to focus on the constraint $V^{ns} \geq V_u$ (i.e. $w \geq e$) since, being $0 < c < 1$, it is satisfied whenever the no-shirking constraint (11) is satisfied. It is obviously in the firm's interest to make w as small as possible, while satisfying (11) and (12). Hence, these constraints are to be taken as binding and written in compact form as

$$W(G) := \max \quad \left\{ \frac{e}{c}, W_f(G) + e - \varphi(h_f(G)) \right\}. \tag{13}$$

where W denotes the lowest wage compatible with the no-shirking and participation constraints.[15]

[13] See Colombo (2004, Chpt. 4) for an in-depth discussion of the link between the technology operated in the market sector and the productivity of workers in the self-employment sector

[14] This is a type of bounded rationality in that the firm is assumed to be unable to contemplate the strategic implications of its action.

[15] In the special case in which labor time is the same both when a consumer is self-employed or

Notice that the actual externality level will be determined in equilibrium. Given G, the firm's decision problem is[16]

$$\begin{cases} \max\limits_{N,T,w} \quad \Pi = \Phi(N,T) - wN \\ s.t. \;\; w \geq \max \quad \left\{\frac{e}{c}, W_f(G) + e - \varphi(h_f(G))\right\} \end{cases} . \tag{14}$$

We already know from the discussion of Equation (13) that the constraint in Problem (14) is binding. By Assumption 4, $\Phi(N,T)$ is an increasing function in T. Hence, from Problem (14), it follows immediately that the Cournot firm chooses to adopt the technology $T_{\max}$. This implies that labor demand follows from the first order condition of Problem (14) for an interior solution

$$\frac{\partial \Phi(N, T_{\max})}{\partial N} = W(G) := \max \quad \left\{\frac{e}{c}, W_f(G) + e - \varphi(h_f(G))\right\}, \tag{15}$$

which gives, by Assumption 4, $\hat{N}(T_{\max}, W(G))$ as a unique solution.

In order to avoid the possibility of rationing of labor demand by the firm, and without consequences on the generality of our results, we assume throughout that

Assumption 7 *$\bar{N}$ is sufficiently large so that $\hat{N} < \bar{N}$ for all admissible parameters values.*

By marking the equilibrium allocations with C, we can now define a *Cournot equilibrium* as follows.

Definition 1 *Given parameters e, c, $\bar{N}$ and $T_{\max}$, a triple $\{w^C, w_f^C, \pi^C\}$, a technology $T^C \geq 0$, employment levels $N^C \geq 0$, $N_f^C \geq 0$, and an externality level G^C constitute a Cournot equilibrium if the following conditions are fulfilled:*

(1) $w^C = W(G^C)$,

(2) $w_f^C = W_f(G^C) = G^C h_f(G^C)$,

(3) $\pi^C = \Pi^C/\bar{N}$, *where* $\Pi^C = \Phi(N^C, T^C) - w^C N^C$,

such that

(4) $T^C \leqq T_{\max}$,

(5) $N^C = \hat{N}\left(T^C, W(G^C)\right) < \bar{N}$, $\quad N_f^C = \bar{N} - N^C$,

(6) $G^C = g(T^C)$

hold.

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employed by the firm, i.e. $h = h_f(G)$, constraint (13) simplifies to

$$W = \max \quad \left\{\frac{e}{c}, Gh_f(G)\right\},$$

where $e = \varphi(h) = \varphi(h_f(G))$.

[16] We denote with w the generic wage level, and with W the wage level at the equilibrium.

Existence and uniqueness of the Cournot equilibrium follow immediately from the above discussion. Conditions (1) and (2) follow directly from Equations (13) and (9) respectively. Condition (3) derives from the profit distribution scheme introduced by Assumption 2. T^C and N^C solve Problem (14), from which it is apparent that $T^C = T_{\max}$; and the values N_f^C and N_u^C follow from the fact that all workers not employed by the firm prefer to work as self-employed instead of remaining unemployed since $V_f^C > V_u^C$. Finally, given Assumption 6, in equilibrium it must be $G^C = g(T^C)$, as stated by Condition (6).

Notice that, even if we adopt an efficiency wage setup, there is always full employment in equilibrium since workers not hired by the firm have the option to work as self employed. Notice as well that in equilibrium all workers employed by the firm are non-shirkers, since the wage paid by the firm satisfies the workers' incentive compatibility constraint. Moreover, given that the externality is treated as an exogenous parameter, there is no inefficiency in the technology adoption process as the firm does always adopt the best available technology. Since $T^C = T_{\max}$, in equilibrium the firm fails to internalize all the externalities it generates neglecting their impact, so that the first best outcome is achieved. In this sense, the analysis of technology adoption under the Cournot-Nash scenario achieves the same equilibrium and shares the same properties that would be attained in a perfectly competitive framework.

4 The von Stackelberg case

We now turn to the von Stackelberg case, in which the strategic interaction between the market sector firm and its workers — stemming from the technology driven externalities (i.e. the complementarity between technology and outside option) — is relevant and affects the equilibrium outcome of the economy. The firm considers now the strategic reaction of self-employed entrepreneurs in its response function, by taking into account the impact of its technology choice on the externality it induces.[17] We show that allocative inefficiencies may arise that were absent in the Cournot benchmark case.

By Assumption 6, the individual rationality and incentive compatibility constraints faced by the von Stackelberg firm require that

$$w \geq \max \quad \left\{ \frac{e}{c}, g\left(T\right) h_f\left(g\left(T\right)\right) + e - \varphi\left(h_f\left(g\left(T\right)\right)\right) \right\}. \tag{16}$$

Hence, the firm's decision problem can be written as[18]

$$\left\{ \begin{array}{l} \max\limits_{N,T,w} \quad \Pi = \Phi\left(N,T\right) - wN \\ s.t. \;\; w \geq \max \quad \left\{ \frac{e}{c}, g\left(T\right) h_f\left(g\left(T\right)\right) + e - \varphi\left(h_f\left(g\left(T\right)\right)\right) \right\} \end{array} \right. . \tag{17}$$

[17] Implicit in the von Stackelberg formulation is a *staggering* issue, as if one agent chooses ahead of the other: the forward looking firm internalizes the response of self-employed entrepreneurs in its optimal "reaction function".

[18] Problem (17) simplifies further in the special case in which labor time (and hence disutility of effort) is the same for both self-employed entrepreneurs and workers employed by the firm. In fact, in

By focusing on Constraint (16), given the assumptions made, namely $e > 0$, Assumption 6 and Assumption 1, it is immediate to see that

$$W(g(T)) := \begin{cases} e/c, & T \in [0, \tilde{T}), \\ g(T)h_f(g(T)) - \varphi(h_f(g(T))) + e, & T \in [\tilde{T}, T_{\max}], \end{cases} \tag{18}$$

where $\tilde{T}$ solves the following equation in T

$$\frac{e}{c} = g(T)h_f(g(T)) - \varphi(h_f(g(T))) + e. \tag{19}$$

If it exists, it is $\tilde{T} > 0$ and unique since the right hand side of Equation (19) is equal to $e < e/c$ at $T = 0$ and then is strictly increasing for $T > 0$.[19]

In order to rule out uninteresting cases, we assume that parameters c, e and $T_{\max}$ are such that

Assumption 8 $\tilde{T} < T_{\max}$.

It is immediate to observe that if Assumption 8 does not hold, the firm can always satisfy both the workers' individual rationality and incentive compatibility constraints by setting $W(g(T)) = e/c$, implying that technology spillovers would never influence the firm's wage setting.

Equation (18) is illustrated in Figure 1. Up to $\tilde{T}$, technological spillovers are irrelevant for wage setting, since the dominant effect is represented by the need to offer the no-shirking efficiency wage. Above $\tilde{T}$, on the contrary, technological spillovers become the main determinant of wage setting by the firm.

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this case, Constraint (16) becomes

$$w \geq \max\left\{\frac{\varphi(h_f(g(T)))}{c}, g(T)\, h_f(g(T))\right\}.$$

Both expressions into brackets are increasing in T and equal to 0 for $T = 0$. By differentiating them we get, respectively

$$\varphi'(h_f(g(T)))\frac{dh_f(.)}{dg}\frac{dg(T)}{dT} > 0, \qquad g(T)\frac{dh_f(.)}{dg}\frac{dg(T)}{dT} + \frac{dg(T)}{dT}h_f(g(T)) > 0.$$

Recalling that from the first order condition of Problem (8) it is $g = \varphi'(.)$ and given the assumptions made on $g(.)$ and $h(.)$, the expression in the second inequality above is always greater than the first one for all T greater than zero. Therefore, without loss of generality, one can write $W(T) = g(T)\, h_f(g(T))$. Thus, the leader's problem simplifies to

$$\max_{N,T} \Pi = \Phi(N,T) - W(T)\, N$$

By studying this problem, we get results that are qualitatively equivalent to those obtained for the general case.

[19] The first derivative of the right hand side is $g'(T)\, h_f(g(T)) + g(T)\frac{dh_f(.)}{dg(T)}g'(T) - \varphi'(h_f(g(T)))\frac{dh_f(.)}{dg(T)}g'(T)$, which reduces to $g'(T)\, h_f(g(T)) > 0$ since $g(T) = \varphi'(h_f(g(T)))$ from the first order condition of Problem (8). The fact that the firm exploits the latter property of $g(T)$ amounts implicitly to assume common knowledge of the economy's structure.

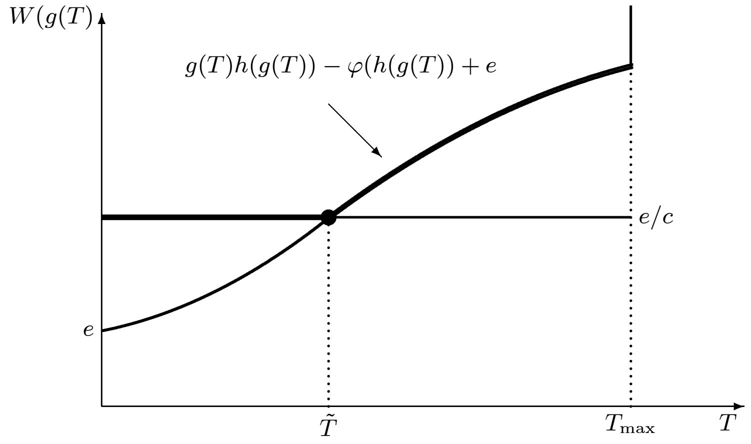

Figure 1: Wage setting by the leader

Before proceeding, it is worth stressing the impact of the workers' effort level and of the firm's monitoring (expressed in terms of the probability c) on the threshold technology grade $\tilde{T}$. By implicitly differentiating Equation (19) and after some algebra, we get

$$\frac{d\tilde{T}}{de} = \left(\frac{1}{c} - 1\right) \frac{1}{g'(T)\, h_f\,(g\,(T))} > 0$$

and

$$\frac{d\tilde{T}}{dc} = -\frac{e}{c^2} \frac{1}{g'(T)\, h_f\,(g\,(T))} < 0.$$

The technology level at which externalities start becoming relevant in the firm's wage setting is thus increasing in the effort exerted by workers and decreasing in the monitoring by the firm. The intuition behind these results is that the higher the level of effort required to workers, the higher is the wage necessary to satisfy their incentive compatibility constraint regardless of the technology operated by the firm. In this sense, an increase in effort mitigates the direct impact of spillovers. Conversely, a better monitoring has exactly the opposite effect. An increase in the probability that a shirker is caught shirking reduces the wage that the firm must pay in order to satisfy the workers' incentive compatibility constraint independently of the technology used.

We can now define a *von Stackelberg equilibrium* as follows.

Definition 2 *Given parameters e, c, $\bar{N}$ and $T_{\max}$, a triple $\{w^*, w_f^*, \pi^*\}$, a technology T^*, employment levels $N^* \geq 0$, $N_f^* \geq 0$, and an externality level g^* constitute a von Stackelberg equilibrium if*

(1) (N^*, T^*, w^*) *is a solution of Problem (17),*

(2) $w^* = W(g^*)$,

(3) $w_f^* = g^* h_f(g^*) = g^* h_f^*$,

(4) $\pi^* = \Pi^*/\bar{N}$, *where* $\Pi^* = \Phi(N^*, T^*) - w^* N^*$,

such that
(5) $\quad N^* = \hat{N}(T^*, w^*) < \bar{N}, \quad N_f^* = \bar{N} - N^*$ *and*
(6) $\quad g^* = g(T^*)$
hold.

In order to discuss the *existence* and *uniqueness* of the von Stackelberg equilibrium, we concentrate on Problem (17). From Equation (18), one can see that $W(g(T))$ is continuous in T in the relevant range $[0, T_{\max}]$ but presents a kink at $T = \tilde{T}$; hence, its derivative $dW(g(T))/dT$ is discontinuous at this point, jumping from $dW(g(T))/dT|_{T \to \tilde{T}^-} = 0$ to $dW(g(T))/dT|_{T \to \tilde{T}^+} = g'(\tilde{T})h_f(\tilde{T}) > 0$. Formally,

$$\frac{dW(g(T))}{dT} = \begin{cases} 0, & T \in [0, \tilde{T}) \text{ and } T \to \tilde{T}^- \\ g'(T)h_f(T), & T \in (\tilde{T}, T_{\max}] \text{ and } T \to \tilde{T}^+ \end{cases}. \tag{20}$$

Substituting for $W(g(T))$ into the firm's profit function, Problem (17) becomes

$$\max_{N,T} \quad \Pi = \Phi(N, T) - W(g(T))N. \tag{21}$$

Consider first the choice of labor input, given T. The first order condition for an interior solution is

$$\frac{\partial \Phi(N, T)}{\partial N} - W(g(T)) = 0, \tag{22}$$

which gives, by Assumption 4, $\hat{N}(T, W)$ as a unique solution.[20]

By differentiating (22) with respect to T we get[21]

$$\frac{\partial \hat{N}(T, W)}{\partial T} = \left(\frac{dW}{dT} - \frac{\partial^2 \Phi}{\partial N \partial T} \right) \Big/ \frac{\partial^2 \Phi}{\partial N^2}. \tag{23}$$

For $T \in [0, \tilde{T})$, (23) is positive whenever $\frac{\partial^2 \Phi}{\partial N \partial T} > 0$, since $\frac{dW}{dT} = 0$ and $\frac{\partial^2 \Phi}{\partial N^2} < 0$. In words, technology adoption brings about higher labor demand if a better technology augments the marginal productivity of labor. The relation between labor demand and technology adoption is less clear-cut when $T \in [\tilde{T}, T_{\max}]$. In this case, a better technology increases, via the spillover effect, the wage the firm must pay ($dW/dT > 0$), and this tends to reduce labor demand. Hence, if $\frac{\partial^2 \Phi}{\partial N \partial T} > 0$ the overall effect is ambiguous, whereas if $\frac{\partial^2 \Phi}{\partial N \partial T} < 0$ then (23) is negative.

Substituting labor demand $\hat{N}(T, W)$ into the profit function (21), the problem of optimal technology adoption can now be written as

$$\max_{T} \quad \hat{\Pi}(T, W) = \Phi(\hat{N}(T, W), T) - W\hat{N}(T, W). \tag{24}$$

[20] The possibility of rationing of labor demand by the firm is ruled away by Assumption 7.

[21] In order to save on notation, we write W instead of $W(g(T))$ and g instead of $g(T)$ whenever this is not misleading.

By differentiating, we have that

$$\frac{\partial \hat{\Pi}\,(T,W)}{\partial T} = \begin{cases} \dfrac{\partial \Phi(\hat{N},T)}{\partial T}, & T \in [0,\tilde{T}) \text{ and } T \to \tilde{T}^- \quad \text{(a)} \\ \dfrac{\partial \Phi(\hat{N},T)}{\partial T} - \hat{N}\dfrac{dW}{dT}, & T \in (\tilde{T},T_{\max}] \text{ and } T \to \tilde{T}^+ \quad \text{(b)} \end{cases}. \tag{25}$$

Since $\partial\Phi/\partial T > 0$, (25a) is strictly positive and hence the optimal level of technology adoption, T^*, is never lower than $\tilde{T}$. In other words, it always pays to expand technology as long as spillovers are irrelevant. Whether or not it is desirable to go further in the process of technology adoption it all depends on the sign of (25) for $T \to \tilde{T}^+$ and on its behavior for $T > \tilde{T}$. As for the sign of $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T\to\tilde{T}^+}$, from (25b) and (20), it is clearly ambiguous, as $\frac{\partial \Phi(\hat{N},T)}{\partial T} > 0$ and $\frac{dW}{dT} > 0$. Hence, it is positive if the marginal productivity of technology adoption is greater than marginal labor costs induced by the spillover effect, whereas it is negative when the latter effect dominates the former. As for the behavior of $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T>\tilde{T}}$, it is characterized by the following equation, obtained by differentiating (25b):

$$\frac{\partial^2 \hat{\Pi}}{\partial T^2} = \frac{\partial^2 \Phi}{\partial T^2} - \frac{\partial^2 \Phi}{\partial N^2}\left(\frac{\partial \hat{N}}{\partial T}\right)^2 - \hat{N}\frac{d^2 W}{dT^2}, \qquad T \in (\tilde{T}, T_{\max}], \tag{26}$$

where we have used (23) and the symmetry of cross partial derivatives of $\Phi\,(.)$ to obtain the second term.[22] The sign of Equation (26) depends on the sign of three terms. The first is negative (positive) whenever there are decreasing (increasing) returns in technology adoption. The second term is always positive, since we have assumed decreasing returns in labor inputs. Finally, the sign of the third term is ambiguous. A sufficient condition for it to be negative is that $g'' > 0$, meaning that technology adoption by the externality producer has an increasing marginal spillover effect on the self-employed productivity, since in this case $\frac{d^2W}{dT^2} = g''h_f + g'\frac{dh_f}{dT} > 0$. If, instead, it is $g'' < 0$, the sign of $\frac{d^2W}{dT^2}$, and hence that of the third term in (26), remains undetermined. Clearly, the overall sign of (26) is an empirical matter, as there are no theoretical explanations that can help to show which one of the three effects dominates over the others.[23]

In order to ensure a unique solution to the problem of technology adoption by the firm, it is sufficient to impose the following

[22] More precisely, from (25) it is

$$\begin{aligned}\frac{\partial^2 \hat{\Pi}}{\partial T^2} &= \frac{\partial^2 \Phi}{\partial T^2} + \frac{\partial^2 \Phi}{\partial T \partial N}\frac{\partial \hat{N}}{\partial T} - \hat{N}\frac{d^2 W}{dT^2} - \frac{\partial \hat{N}}{\partial T}\frac{dW}{dT} = \\ & \frac{\partial^2 \Phi}{\partial T^2} - \left(\frac{dW}{dT} - \frac{\partial^2 \Phi}{\partial T \partial N}\right)\frac{\partial \hat{N}}{\partial T} - \hat{N}\frac{d^2 W}{dT^2},\end{aligned}$$

and by making use of (23), we obtain Equation (26).

[23] The solution of our two-step maximization — Problem (21) with T fixed and then Problem (24)— is equivalent to the first order conditions of Problem (21), since Derivative (26) equals to (minus) the determinant of the Hessian matrix for Problem (21) and $\partial^2\Phi/\partial N^2 < 0$.

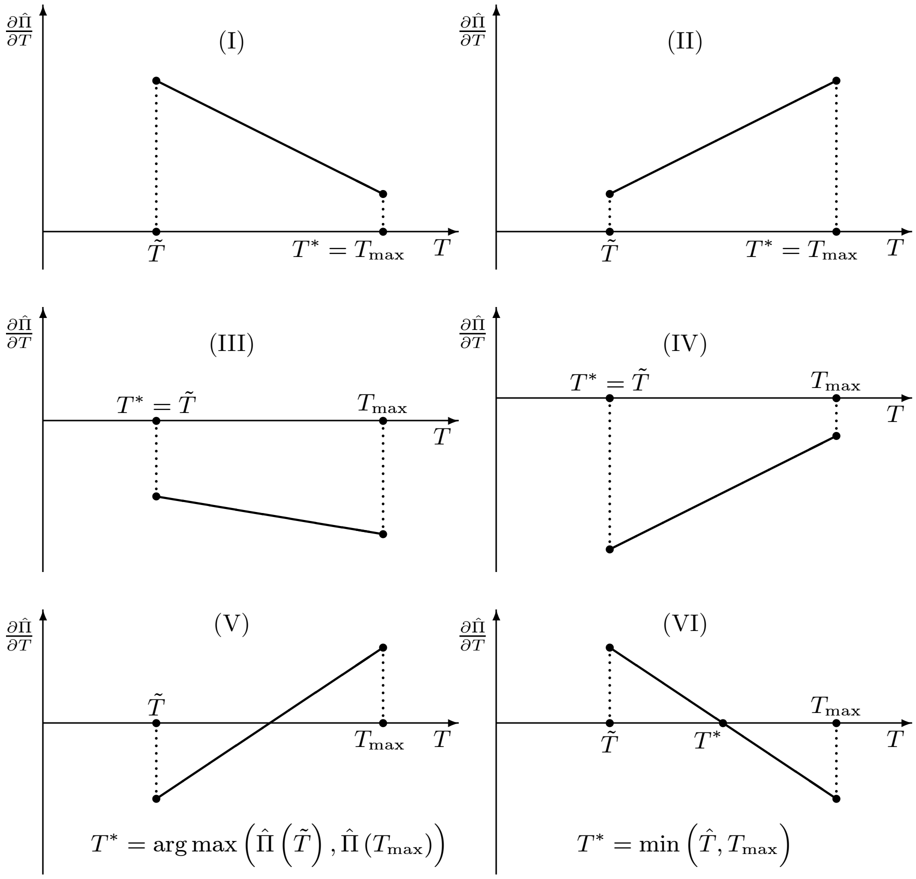

Figure 2: Optimal technology adoption

Assumption 9 $\partial\hat{\Pi}/\partial T$ *is monotone.*

In principle, one could argue that the impact of technology on profits is a function of the specific technology grade adopted. For example, spillovers can be completely irrelevant until a certain threshold technology. This, however, does not seem restrictive in the present framework. In fact, it is enough that the "regularity" Assumption 9 holds to restrict Equation (26) to have the same sign over the interval $(\tilde{T}, T_{\max}]$. This amounts to require that the marginal impact of technology on profits keeps going in the same direction as the technology grade improves, in the interval where production externalities are potentially relevant.

Under Assumption 9, the optimal T is then characterized by the signs of partial derivatives (25b) and (26). By Assumption 9, there are four possible cases that may arise, each with a unique optimal T, which are depicted in Figure 2. In the first one, it is either $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T\to\tilde{T}^+} > 0$, $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T=T_{\max}} > 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T>\tilde{T}} \leq 0$ (panel I) or

$\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} > 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T > \tilde{T}} \geq 0$ (panel II), hence $T^* = T_{\max}$. Spillovers are weak so that the firm always adopts the best available technology (*technological frontier regime*). In the second one, it is either $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} < 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T > \tilde{T}} \leq 0$ (panel III) or $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} < 0$, $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T = T_{\max}} < 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T > \tilde{T}} \geq 0$ (panel IV), hence $T^* = \tilde{T}$. Spillovers are so important to eliminate any incentive for the firm to adopt a superior technology and therefore technology adoption stops at $\tilde{T}$ (*blocked adoption regime*). In the third one $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} < 0$, $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T = T_{\max}} > 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T > \tilde{T}} > 0$ (panel V), so that $T^* = \arg\max_{T} \left(\hat{\Pi}\left(\tilde{T}\right), \hat{\Pi}\left(T_{\max}\right)\right)$. Spillovers are relevant, but they may be dominated by increased productivity in the firm's sector. Finally, in the last case (a possible outcome of which, corresponding to the case $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T = T_{\max}} < 0$, is shown in panel VI) it is $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} > 0$ and $\left.\frac{\partial^2 \hat{\Pi}}{\partial T^2}\right|_{T > \tilde{T}} < 0$, hence $T^* = \min(\hat{T}, T_{\max})$, where $\hat{T} > \tilde{T}$ solves the first order condition

$$\frac{\partial \hat{\Pi}}{\partial T} = \frac{\partial \Phi(\hat{N}, T)}{\partial T} - \hat{N}\frac{dW}{dT} = 0. \tag{27}$$

Spillovers are important, but not so as to prevent the adoption of a superior technology by the firm, although not necessarily the one at the frontier. Notice that the above conditions for the last case do not guarantee that the technology adopted by the firm is not the one at the technology frontier either, i.e. $T^* = \hat{T} \in (\tilde{T}, T_{\max})$, a case to which we refer to as *spillover regime*. It is, however, immediate to observe that a necessary and sufficient condition for this case to occur is to require that $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T = T_{\max}} < 0$ and $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}+} > 0$. This, combined with Assumption 9 on the monotonicity of $\partial\hat{\Pi}/\partial T$ — ensuring that there is one and only one T such that $\frac{\partial \hat{\Pi}}{\partial T} = 0$ —, guarantees that $\hat{T} \in (\tilde{T}, T_{\max})$ is the unique solution of the firm technology choice problem.[24]

The above discussion is summarized by the following proposition.

Proposition 1 *Under Assumptions 6 – 9, the technology chosen by the firm is unique. Any one of the following three regimes may arise:*

[24] Assumption 9 on the monotonicity of the profit function is not necessary. In Appendix A.2, we consider an example for a Cobb-Douglas economy, deriving a technology spillover regime without imposing any restriction on the sign of the second derivative of $\hat{\Pi}(T)$.

1. *Blocked adoption regime:* $T^* = \tilde{T}$;

2. *Technological frontier regime:* $T^* = T_{\max}$;

3. *Spillover regime:* $T^* = \hat{T} \in (\tilde{T}, T_{\max})$.

The three graphs in Figure 3 — depicting in the $(W(g(T)), T)$-space the firm's profit contours and the (incentive compatibility and individual rationality) constraint on wages it faces — illustrate the choice of technology by the firm under the three regimes identified in Proposition 1: spillover regime (graph a), technological frontier (graph b) and blocked adoption (graph c).

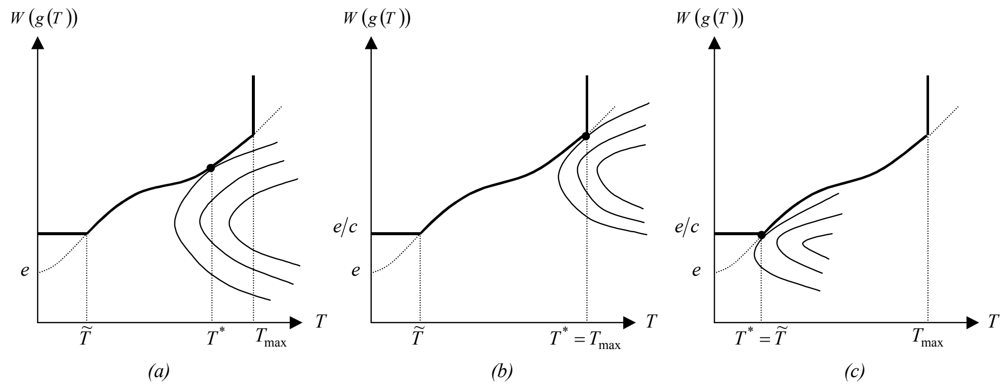

Figure 3: The firm's technology choice

Proposition 1 identifies the unique T^* solving Problem (17) and characterizes the different types of (unique) equilibria possibly arising in our economy. Given T^*, w^* is uniquely defined by Equation (18), and $N^* = \hat{N}(T^*, w^*)$. Furthermore, the unique equilibrium values $g^* = g(T^*)$, w_f^* and π^* follow from Assumption 6, Equation (9) and Assumption 1, and Assumption 2 respectively. Notice that also in the von Stackelberg case (as in the Cournot one investigated in the previous section), albeit the presence of an efficiency wage in the market sector guaranteeing that the firm does not employ shirkers, in equilibrium there is always full employment, since workers not hired by the firm have an incentive to make an earning with self employment, where they never shirk.[25] This follows directly from Assumption 3 and by inspection of Equation (10). Hence, at the von Stackelberg equilibrium, $N^* = \hat{N}(T^*, w^*)$, $N_f^* = \bar{N} - N^* \geq 0$, and $N_u^* = 0$. Notice, furthermore, that whenever superior technologies are labor saving the number of workers hired by the firm will be higher in the von Stackelberg than in the Cournot equilibrium. In the latter regime, in fact, it will always be $T^C = T_{\max}$ at the equilibrium.[26]

[25] Recall that we refer to the firm's labor market with the expression "market sector", as opposed to "self-employment sector".

[26] This amounts to saying that, although in both cases there is no unemployment in equilibrium,

Finally, a subtle point is worth noting. Throughout the paper we assume that the firm's monitoring is not affected by the choice of technology. However, one could argue that the adoption of a higher grade technology may have an impact on the firm's ability to detect shirkers, influencing monitoring costs. This, in turn, would affect the incentive compatibility constraint and hence the wage the firm must pay to workers. In this sense, the impact of technology adoption on monitoring can either reinforce or weaken its effect on the workers' outside options (captured by their productivity as self-employed entrepreneurs). In the case that better technologies improve monitoring, for our argument to affect firms' decisions (by increasing wages), it is necessary that the impact of technology choice on incentive compatibility (i.e. on the probability to be caught shirking) is of second order with respect to that on individual rationality (i.e. on workers' outside options).

5 Market failure

When the market sector firm behaves like a von Stackelberg firm, there are two sources of externality that are likely to produce inefficiencies of market allocations. On the one hand, technology adoption by the firm exerts a positive externality on the production function of self employed workers. On the other hand, this positive externality generates a negative pecuniary externality on the firm, determined by increasing labor costs for all technologies above a certain threshold level (due to spillover effects). This double source of non-marketed relations may dampen technology adoption by the firm and may also affect the labor distribution across sectors. It is, therefore, natural to ask whether there is a role for the government in trying to overcome the inefficiencies induced by the presence of externalities and to support Pareto efficient allocations. This section focuses on the consequences of such non-marketed relations and conducts a normative analysis of the welfare implications of the presence of externalities in the technology adoption process. We first characterize the Pareto efficient allocations that would be generated by a social planner internalizing all sources of externalities, and we compare them with the market allocations derived in the previous sections. We then examine whether government intervention is able to overcome market failure, studying various types of policy intervention schemes to overcome the possible inefficiencies in market allocations: from non-linear (first best) subsidization mechanisms to second best — but eventually more realistic — policy instruments based on Pigouvian subsidies/taxes on labor input and/or technology adoption.

there is a different distribution of workers between the market sector and the self-employment sector of the economy under the von Stackelberg and the Cournot-Nash regimes, with lower employment in the market sector under the latter.

5.1 Pareto efficient allocations

Pareto efficient allocations are characterized as follows. As a first step, we recognize that Pareto efficiency must be compatible with the resource constraint of the economy, in the obvious sense that aggregate consumption must not exceed aggregate output, that is:

$$\omega N + \omega_f N_f + \omega_u N_u \leq \Phi(N, T) + g(T) h_f N_f \tag{28}$$

where ω, ω_f and ω_u denote the total consumption of the agent working for the firm, the self-employed and the unemployed, respectively; N, N_f, N_u denote the number of workers employed by the firm, of self-employed and of unemployed respectively, with

$$N \geq 0, N_f \geq 0, \ N_u \geq 0, \text{ and } N + N_f + N_u = \bar{N}. \tag{29}$$

From (28), subtracting from both sides $eN + \varphi(h_f) N_f$ we obtain

$$\begin{aligned} (\omega - e) N + [\omega_f - \varphi(h_f)] N_f + \omega_u N_u \quad \leq \quad & \Phi(N, T) - eN + \\ & + (g(T) h_f - \varphi(h_f)) N_f. \end{aligned} \tag{30}$$

The left hand side of (30) is aggregate social welfare (according to a utilitarian social welfare function), whereas the right hand side is aggregate production net of aggregate social cost, represented by labor effort disutility which is expressed, by assumption, in equivalent consumption units.

From a normative point of view, Pareto efficient allocations must be characterized by the absence of unemployment (i.e. $N_u = 0$), since the labor productivity of all agents in the economy is strictly greater than zero. Moreover, Pareto efficiency requires that there are no resources that remain unused in equilibrium. Hence the third constraint in Condition (29) must read $N + N_f = \bar{N}$. Turning to the optimal allocation problem, irrespective of distributional choices (i.e. the choice of total consumption levels), Pareto efficiency requires to maximize the right hand side of Inequality (30) with respect to T, N, h_f and N_f. In this respect, a first result is immediately apparent: since $\partial\Phi/\partial T > 0$ and $g' > 0$, the right hand side of (30) is strictly increasing in T for all (N, h_f, N_f) triples provided that at least N or N_f is non-zero (with also $h_f > 0$ in the latter case), and hence at the optimum $T = T_{\max}$. Also, for any given (T, N_f), h_f must be chosen so that $g(T) h_f - \varphi(h_f)$ is maximized, which gives $h_f(g)$ as a solution as in market equilibrium. Hence, at the social optimum, $T^{**} = T_{\max} \geq T^*$ and $h_f^{**} = h_f^*$. To distinguish Pareto efficient allocations from market allocations, the former are marked with a double asterisk.

Next we turn to the optimal allocation of labor. By maximizing the right hand side of (30) with respect to N at $T^{**} = T_{\max}$ and recalling that $N_f = \bar{N} - N$, it is

$$\frac{\partial \Phi(N, T_{\max})}{\partial N} - e - g(T_{\max}) h_f^{**} + \varphi(h_f^{**}) = 0. \tag{31}$$

Let $\breve{N}$ be the value of N that solves (31). Given Assumption 4, $\breve{N}$ is positive and unique; also, under Assumption 7, $\breve{N} < \bar{N}$, meaning that there will be self-employed entrepreneurs at the Pareto efficient equilibrium.

Notice that Condition (31) requires that the marginal return on labor is the same in the market sector and in the self-employment sector of the economy. In fact, by defining the aggregate net output in the right hand side of (30) as Y, we get

$$\left.\frac{\partial Y}{\partial N}\right|_{T=T_{\max}} \equiv \frac{\partial \Phi(N, T_{\max})}{\partial N} - e = g(T_{\max})h_f^{**} - \varphi(h_f^{**}) \equiv \left.\frac{\partial Y}{\partial N_f}\right|_{T=T_{\max}}. \tag{32}$$

We can summarize the previous discussion by characterizing Pareto efficient allocations through the following proposition:

Proposition 2 *Given parameters e and $T_{\max}$, Pareto efficient allocations are as follows: $T^{**} = T_{\max}$, $g^{**} = g(T^{**})$, $h_f^{**} = h_f^*$, $N^{**} = \breve{N}$, $N_f^{**} = \bar{N} - \breve{N}$, $N_u^{**} = 0$.*

In order to compare market allocations with Pareto allocations, we prove the following proposition.

Proposition 3 *When $T^* < T_{\max}$ there is technology misallocation but not labor misallocation. When $T^* = T^C = T_{\max}$ there are neither technology nor labor misallocation.*

Proof. When $T^* < T_{\max}$ we only need to prove that there is no labor misallocation. We know, by Proposition 1, that in a market equilibrium it must be $T^* \geq \tilde{T}$. Therefore, by Equation (18) it is $W(g(T^*)) = g(T^*)h_f^* - \varphi(h_f^*) + e$. By substituting this expression for $W(g(T^*))$ into Equation (22), it is immediate to check that it reads exactly as Equation (32), which proves the claim.

Also when $T^* = T_{\max}$ we only need to prove that there is no labor misallocation. This follows directly by the comparison of Equations (22) and (32) using Condition (18). ■

Given Proposition 3, policy intervention is called for only when $T^* < T_{\max}$. In other words, it turns out to be useful only in the von Stackelberg case, while in the Cournot case market and Pareto allocations coincide.[27]

[27] Note that, having determined the allocations of inputs that maximize total output, the social planner could move on focusing on distributional issues and on incentive compatibility. In our framework, these objectives are achieved by choosing the ω's under the constraint of Pareto efficiency. Formally, this requires the planner to choose ω and ω_f under the constraints $\omega - e \geq 0$, $\omega_f - \varphi(h_f^{**}) \geq 0$ and

$$(\omega - e)\, N^{**} + [\omega_f - \varphi(h_f^{**})] N_f^{**} \leq Y^{**},$$

where $Y^{**} = \Phi(N^{**}, T_{\max}) - eN^{**} + g(T_{\max})h_f^{**}N_f^{**} - \varphi(h_f^{**})N_f^{**}$.

Due to the focus on technology adoption problems, however, our modelling of the economy abstracts from many important issues that should, instead, be considered when dealing with income distribution (thus rendering the study of problems like the one outlined by the above inequality a special case and a quite limited one in terms of economic insights). Hence, we do not further pursue these topics, turning instead to policy analysis.

5.2 Policy analysis

We now examine how government intervention is able to overcome (or at least mitigate) market failure offsetting the production externality that is not internalized by the firm, by considering various types of subsidization policies.

Non-linear (first best) subsidization

It is a matter of algebra to show that the government can achieve a Pareto efficient outcome by introducing non-linear subsidies. Suppose the government grants the firm a subsidy S for each employee, which is conditional on the level of technology adoption and on the level of employment, of the form

$$S(T,N) := \begin{cases} e/c - e + [g(T_{\max})h_f\left(g\left(T\right)\right) + \\ -\varphi(h_f\left(g\left(T\right)\right))]\frac{\bar{N}-N}{N}, & T \in [0,\tilde{T}) \\ g(T)h_f\left(g\left(T\right)\right) - \varphi(h_f\left(g\left(T\right)\right)) + \\ \left[g(T_{\max})h_f\left(g\left(T\right)\right) - \varphi(h_f\left(g\left(T\right)\right))\right]\frac{\bar{N}-N}{N}, & T \in [\tilde{T},T_{\max}] \end{cases}. \quad (33)$$

The policy maker is assumed to move first by setting tax policy, and then producers make their choices as described in Section 2.2. Government's budget is assumed to balance; in particular any subsidy (tax) paid (levied) to producers is financed with a lump sum tax (subsidy) on consumers. The use of a lump sum tax is without loss of generality, as other non-distortive tax instruments are available within this framework. For instance, a proportional tax on the firm's gross profits or on consumers' dividends does not affect the choices made by the firm, by the self-employed workers and by consumers, and hence is equivalent to a lump sum tax on consumers.

Under the subsidy defined in Equation (33), the firm's profit function becomes

$$\Pi = \Phi\left(N,T\right) - \left(W\left(g\left(T\right)\right) - S\left(T,N\right)\right)N$$

where $W\left(g\left(T\right)\right)$ is defined as in (18). Since we have assumed away income effects (by assuming quasi-linearity in consumption of the utility function), the lump sum tax on consumers does not affect the participation and incentive compatibility constraints. Hence, by Equations (18) and (33), and after some algebra, we have

$$W\left(g\left(T\right)\right) - S\left(T,N\right) = e + \left[\varphi(h_f\left(g\left(T\right)\right)) - g(T_{\max})h_f\left(g\left(T\right)\right)\right]\frac{\bar{N}-N}{N}$$

for all T, and thus the problem of the firm reduces to

$$\max_{T,N} \quad \Pi = \Phi\left(N,T\right) - eN + \left[g(T_{\max})h_f\left(g\left(T\right)\right) - \varphi(h_f\left(g\left(T\right)\right))\right]\left(\bar{N}-N\right). \quad (34)$$

The solution of Problem (34) gives $T = T_{\max}$ and a first order condition for the choice of N that, once evaluated at $T = T_{\max}$, is identical to (32) characterizing Pareto efficient allocations. This follows immediately from the observation that, by differentiating Π with respect to T (recalling that $\partial\varphi\left(.\right)/\partial h_f\left(.\right) = g\left(T\right)$), it is

$$\frac{\partial\Pi}{\partial T} = \frac{\partial\Phi\left(N,T\right)}{\partial T} + \frac{dh_f\left(g\left(T\right)\right)}{dg}\frac{dg\left(T\right)}{dT}\left[g(T_{\max}) - g\left(T\right)\right]\left(\bar{N}-N\right) > 0,$$

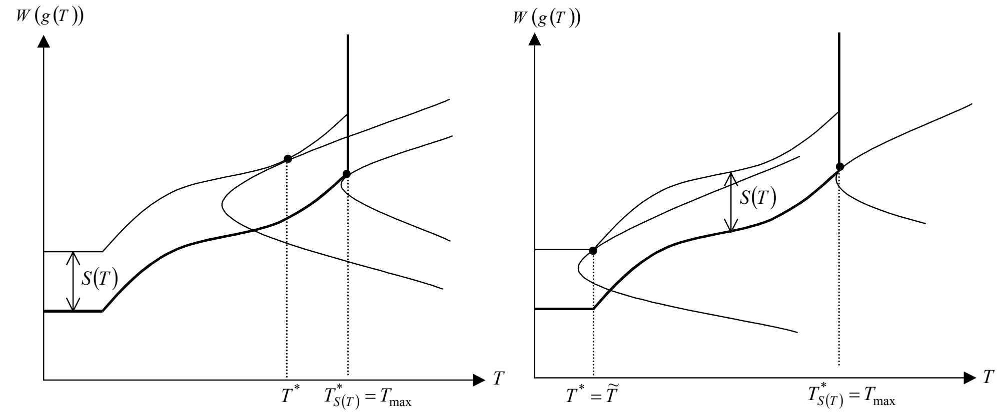

Figure 4: Non-linear (first best) subsidization

and, moreover, that the first order conditions of Problem (34) with respect to N is given by

$$\frac{\partial \Pi}{\partial N} = \frac{\partial \Phi (N, T)}{\partial N} - e - g(T_{\max}) h_f (g(T)) + \varphi(h_f (g(T))) = 0.$$

Hence, with the non-linear subsidy (33), the decentralized market equilibrium achieves a Pareto efficient allocation. Policy intervention corrects for market failure and achieves a first best allocation. Indeed, the externality producer is induced to maximize aggregate net output as in the social planner problem, since the objective function in (34) is identical to the right hand side of (30).

The above discussion is summarized in the following proposition:

Proposition 4 *Policy subsidization through the non-linear subsidies $S(T, N)$ — defined in Equation (33) — corrects for market failure allowing the economy to achieve the Pareto efficient equilibrium defined in Proposition 2.*

Figure 4 shows how the introduction of a non-linear subsidy per employee leads to Pareto efficiency. Both graphs in the figure show that the introduction of the subsidy S (Equation 33) affects the wage constraint faced by the firm (Equation 18) shifting it downward to the point at which the optimal technology choice by the firm becomes $T_{\max}$.

To implement the non-linear subsidy, the policy maker needs, however, to have a great deal of information; indeed it needs to know the entire structure of the economy, as is standard in optimal policy analysis. The point is that it observes and can enforce truthful revealing at no cost (i.e costless monitoring) of both N and T, which are the choice variables on which the transfer to the firm is contingent. These information requirements are in many cases so demanding that the actual implementability of such first best policy instruments is greatly reduced if not impaired, which suggests to look at instruments imposing a smaller informational burden on the policy maker.

Second best policy instruments

We now consider two less sophisticated, but more realistic, policy instruments affecting the marginal returns to work and technology. The first one is a fixed unit subsidy, at rate s, on workers employed by the externality producer; the second one is a fixed unit subsidy, at rate σ, on each unit of technological adoption. Both s and σ are simple to implement, since it is reasonable to assume that both the employment level and the type of technology adopted are observed. Also, these kind of instruments are widely employed in real tax systems: s can be assimilated to a (negative) payroll tax, whereas σ resembles the kind of incentive schemes that governments grant to induce firms to dismiss old equipments for new ones.[28] Moreover, the introduction of second best policy instruments is needed whenever there is imperfect observability (or possibility of cheating on) of T.

As a first step in addressing the effects of second best policy measures, we start focusing on the problem faced by the firm, that becomes

$$\max_{N,T} \quad \Pi = \Phi\left(N,T\right) + \sigma T - \left(W\left(g\left(T\right)\right) - s\right)N, \tag{35}$$

where $W\left(g\left(T\right)\right)$ is defined as in (18).

Consider first the choice of labor input, given T. The first order condition for an interior solution is

$$\frac{\partial \Pi}{\partial N} = \frac{\partial \Phi\left(N,T\right)}{\partial N} - W\left(g\left(T\right)\right) + s = 0, \tag{36}$$

which gives $\hat{N}(T, W\left(g\left(T\right)\right), s)$ as a solution. It is straightforward to see that

$$\partial \hat{N}/\partial s = -\left(\partial^2 \Phi/\partial N^2\right)^{-1} > 0,$$

and that labor demand is independent of σ.

Substituting $\hat{N}$, $\hat{N} \equiv \hat{N}(T, W\left(g\left(T\right)\right), s)$, into the profit function, the problem of technological adoption (35) can now be written as

$$\max_{T} \quad \hat{\Pi} = \Phi(\hat{N},T) + \sigma T - \left(W\left(g\left(T\right)\right) - s\right)\hat{N}.$$

Thus, given $\hat{N}$, the first order condition for an interior solution is

$$\frac{\partial \hat{\Pi}}{\partial T} = \frac{\partial \Phi(\hat{N},T)}{\partial T} + \sigma - \hat{N}\frac{dW\left(g\left(T\right)\right)}{dT} = 0. \tag{37}$$

Let the solution be $T(s,\sigma)$. By totally differentiating (37) with respect to s — and recalling that $\hat{N}$ is a function of s — we get

$$\frac{\partial^2 \hat{\Pi}}{\partial T^2}\frac{dT}{ds} + \frac{\partial^2 \Phi(\hat{N},T)}{\partial T \partial N}\frac{\partial \hat{N}}{\partial s} - \frac{\partial \hat{N}}{\partial s}\frac{dW\left(g\left(T\right)\right)}{dT} = 0. \tag{38}$$

[28] A third tax instrument that can be used to indirectly affect the firm choices is a tax or subsidy on self-employed workers' labor input. It is immediate to show that this is equivalent to the subsidy s on the firm labor inputs.

From Equation (23) it is

$$\frac{dW\left(g\left(T\right)\right)}{dT}=\frac{\partial\hat{N}}{\partial T}\frac{\partial^{2}\Phi(\hat{N},T)}{\partial N^{2}}+\frac{\partial^{2}\Phi(\hat{N},T)}{\partial T\partial N},$$

and substituting into (38), we get

$$\frac{dT}{ds}=\frac{\partial^{2}\Phi}{\partial N^{2}}\frac{\partial\hat{N}}{\partial T}\frac{\partial\hat{N}}{\partial s}\bigg/\frac{\partial^{2}\hat{\Pi}}{\partial T^{2}} \tag{39}$$

Finally, by differentiating (37) with respect to σ, we obtain

$$\frac{dT}{d\sigma}=-\left(\frac{\partial^{2}\hat{\Pi}}{\partial T^{2}}\right)^{-1} \tag{40}$$

where, again by making use of (23), it is[29]

$$\frac{\partial^{2}\hat{\Pi}}{\partial T^{2}}=\frac{\partial^{2}\Phi}{\partial T^{2}}-\frac{\partial^{2}\Phi}{\partial N^{2}}\left(\frac{\partial\hat{N}}{\partial T}\right)^{2}-\hat{N}\frac{\partial^{2}W\left(g\left(T\right)\right)}{\partial T^{2}}. \tag{41}$$

By inspection of Equations (39) and (40), it is immediate to notice that the signs of $\frac{dT}{ds}$ and $\frac{dT}{d\sigma}$ are undecided, depending on the sign of $\frac{\partial^{2}\hat{\Pi}}{\partial T^{2}}$ that remains an empirical matter.

As for policy analysis, let us now consider each tax instrument in turn.

Pigouvian subsidy on labor input

Let $\sigma=0$. We wish to analyze whether social welfare can be increased by using the subsidy on labor input, s, while balancing the budget with the lump sum tax, Θ, that has no influence on work incentives because of the linearity assumption. Assuming a utilitarian social welfare function, the policy maker solves the following problem

$$\max_{s,\Theta}\quad \mathcal{V}=V^{ns*}N^{*}+V_{f}^{*}N_{f}^{*} \tag{42}$$

$$\text{s.t.}\quad sN^{*}=\Theta\bar{N}.$$

Since, in a market equilibrium, $N_{f}^{*}=\bar{N}-N^{*}$ and $V^{ns*}=V_{f}^{*}$, the social welfare function can be written as

$$\begin{aligned}\mathcal{V} &= \left(w_{f}^{*}-\varphi(h_{f}^{*}\left(g\left(T^{*}\right)\right))+\frac{\Pi^{*}}{\bar{N}}-\Theta\right)\bar{N}= \\ &\quad [g(T^{*})h_{f}^{*}\left(g\left(T^{*}\right)\right)-\varphi(h_{f}^{*}\left(g\left(T^{*}\right)\right))]\bar{N}+\Pi^{*}-\Theta\bar{N}. \end{aligned} \tag{43}$$

[29] The following equation is the same as Equation (26), but (41) is defined for all T whereas (26) only for $T>\tilde{T}$.

Substituting in (43) the budget constraint, we can finally write the optimal tax Problem (42) as

$$\max_s \mathcal{V} = [g(T^*(s))h_f^*(g(T^*(s))) - \varphi(h_f^*(g(T^*(s))))]\bar{N} + \Pi^*(s) - sN^*(s). \quad (44)$$

Differentiating $\mathcal{V}$ with respect to s, the first order condition of Problem (44) is

$$\begin{aligned}
&\frac{d\mathcal{V}}{ds} = [h_f^*(g(T^*(s)))\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s} + \\
&+ g(T^*(s))\frac{\partial h_f^*(g(T^*(s)))}{\partial g}\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s} + \\
&- \frac{\partial \varphi(h_f^*(g(T^*(s))))}{\partial h_f}\frac{\partial h_f^*(g(T^*(s)))}{\partial g}\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s}]\bar{N} + \\
&+ \frac{\partial \Pi^*(s)}{\partial s} - N^*(s) - s\frac{\partial N^*(s)}{\partial s} = 0
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
&\left\{ h_f^*(g(T^*(s)))\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s} + \right. \\
&\left. \left[g(T^*(s)) - \frac{\partial \varphi(h_f^*(g(T^*(s))))}{\partial h_f} \right] \frac{\partial h_f^*(g(T^*(s)))}{\partial g}\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s} \right\} \bar{N} + \\
&+ \frac{\partial \Pi^*(s)}{\partial s} - N^*(s) - s\frac{\partial N^*(s)}{\partial s} = 0. \qquad (45)
\end{aligned}$$

By the envelope theorem it is $\frac{\partial \Pi^*(s)}{\partial s} = N^*(s)$, and by the first order condition of the self-employed workers' utility maximization problem (i.e. Problem (8)) it is $g(T^*(s)) = \frac{\partial \varphi(h_f^*(g(T^*(s))))}{\partial h_f}$. Thus Equation (45) reduces to

$$h_f^*(g(T^*(s)))\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s}\bar{N} - s\frac{\partial N^*(s)}{\partial s} = 0.$$

Therefore, if an interior solution exists, s is implicitly defined by

$$s = \frac{\bar{N}h_f^*(g(T^*(s)))\frac{\partial g(T^*(s))}{\partial T}\frac{\partial T^*(s)}{\partial s}}{\frac{\partial N^*(s)}{\partial s}}. \qquad (46)$$

The optimal s can be both negative or positive, meaning that social welfare can be increased by using either a fixed unity subsidy or tax on labor input depending on whether it is a subsidy ($s > 0$) or a tax ($s < 0$) that induces higher technology adoption than in the *laissez faire* equilibrium. This matter can not be solved analytically. In fact, s is greater or smaller than zero depending on the sign of $\partial T^*/\partial s$ at the numerator of Equation (46). The sign of $\partial T^*/\partial s$, defined by Equation (39), depends in turn on the sign of $\frac{\partial \hat{N}}{\partial T}$ and $\frac{\partial^2 \hat{\Pi}}{\partial T^2}$. While, as noticed by discussing Equation (23), it is easy to characterize the sign of $\frac{\partial \hat{N}}{\partial T}$, it is not possible to provide general conditions for the sign of $\frac{\partial^2 \hat{\Pi}}{\partial T^2}$ which remains an empirical matter, as emphasized when studying Equation (26).

The above discussion is summarized, slightly abusing notation, in the following proposition.

Proposition 5 *If there exists an interior optimal s^*, then s^* satisfies the necessary condition*

$$s^* = \frac{g' h_f^* \bar{N}(\partial T^* / \partial s)}{\partial N^* / \partial s}.$$

Therefore, s^* is negative (a tax) if $\partial T^*/\partial s$ and $\partial N^*/\partial s$ have opposite sign; otherwise it is positive (a subsidy).

Pigouvian subsidy on technology adoption

Let $s = 0$. The tax instrument used by the policy maker is now the fixed unit subsidy on technology σ. Using (43), and after substituting for the budget constraint $\sigma T = \Theta \bar{N}$, the optimal tax problem is

$$\max_{\sigma} \mathcal{V} = [g(T^*(\sigma)) h_f^*(g(T^*(\sigma))) - \varphi(h_f^*(g(T^*(\sigma))))]\bar{N} + \Pi^*(\sigma) - \sigma T^*(\sigma). \quad (47)$$

The first order condition of Problem (47) can be written as

$$\begin{aligned}
\frac{d\mathcal{V}}{d\sigma} &= [\frac{\partial g(T^*(\sigma))}{\partial T} \frac{\partial T^*(\sigma)}{\partial \sigma} h_f^*(g(T^*(\sigma))) + \\
&+ g(T^*(\sigma)) \frac{\partial h_f^*(g(T^*(\sigma)))}{\partial g} \frac{\partial g(T^*(\sigma))}{\partial T} \frac{\partial T^*(\sigma)}{\partial \sigma} + \\
&- \frac{\partial \varphi(h_f^*(g(T^*(\sigma))))}{\partial T} \frac{\partial h_f^*(g(T^*(\sigma)))}{\partial g(T^*)} \frac{\partial g(T^*(\sigma))}{\partial T} \frac{\partial T^*(\sigma)}{\partial \sigma}]\bar{N} + \\
&+ \frac{\partial \Pi^*(\sigma)}{\partial \sigma} - T^*(\sigma) - \sigma \frac{\partial T^*(\sigma)}{\partial \sigma} = 0.
\end{aligned}$$

Recalling that $\frac{\partial g(T^*(\sigma))}{\partial T} = \frac{\partial \varphi(h_f^*(g(T^*(\sigma))))}{\partial T}$, we get

$$\frac{\partial g(T^*(\sigma))}{\partial T} \frac{\partial T^*(\sigma)}{\partial \sigma} h_f^*(g(T^*(\sigma))) \bar{N} + \frac{\partial \Pi^*(\sigma)}{\partial \sigma} - T^*(\sigma) - \sigma \frac{\partial T^*(\sigma)}{\partial \sigma} = 0. \quad (48)$$

Since by using the envelope theorem it is $\frac{\partial \Pi(\sigma)}{\partial \sigma} = T(\sigma)$, we obtain the following implicit equation for σ

$$\sigma = \frac{\partial g(T^*(\sigma))}{\partial T} h_f^*(g(T^*(\sigma))) \bar{N} > 0, \quad (49)$$

which shows that a Pigouvian subsidy unambiguously gives the proper incentive to foster technology adoption. In this sense, it is better than a Pigouvian subsidy on labor input since it gives rise unambiguously to a welfare improvement. Moreover,

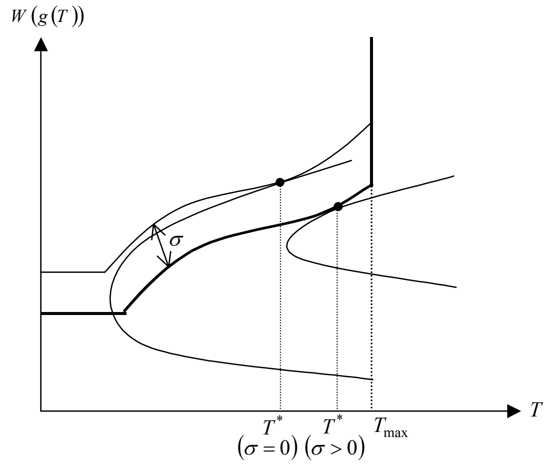

Figure 5: A second best subsidy on technology

being levied on the variable that the policy maker needs to affect (i.e. T), it is more direct than a fixed unity subsidy (or tax) on labor input that acts only indirectly through N^*. Figure 5 illustrates how a subsidy σ on technology adoption affects the technology chosen by the firm, and Proposition 6 summarizes the above arguments.

Proposition 6 *A Pigouvian subsidy on technology, σ^*, defined implicitly by Condition (49), always fosters technology adoption. Differently from a Pigouvian subsidy on labor input, it gives rise unambiguously to a welfare improvement.*

6 Concluding remarks

We have shown, in a simple general equilibrium efficiency wage framework with a monopsonistic labor market, that a firm may continue to use old and inefficient technologies even when better ones are available. This is due to the presence of production externalities — and of negative pecuniary externalities associated to them — that are responsible for the emergence of a lock-in problem in the choice of technology.

The externalities we focus on stem from technology driven spillovers entailing an upskilling of workers, which causes an increase of their productivity. This, in turn, determines a widening of their outside options set, so that — as long as workers have some bargaining power in their relationship with the firm— the choice of a better technology by the firm determines an increase in the wage it must pay to retain workers. The negative pecuniary externality represented by the increase in the level of wages can discourage (or block) the adoption of a better technology.

From a normative perspective, the comparison of market allocations with the Pareto-efficient ones achieved by a social planner internalizing all sources of non-marketed relations has shown the possibility of technology misallocation, which introduces a clear scope for government intervention in order to overcome or mitigate market failure. Without the pretension of being exhaustive, as government intervention may span several dimensions, we have shown that a policy maker is able to re-establish Pareto-efficiency by means of first-best (non linear) subsidization. Furthermore, when non-linear subsidies prove too cumbersome to be implemented, welfare improvements can always be achieved by means of second-best instruments as Pigouvian subsidies on technology adoption, while the effects of interventions on firms' labor demand are ambiguous in that either a (Pigouvian) tax or a subsidy can be welfare improving, depending on the relative impact of the subsidy on labor and technology.

Although the links between technology and wages (as well as the role of market power and strategic interaction) have been abundantly investigated, our results bear important differences with the conclusions of popular streams of the literature. For instance, workers' skills play a completely different role here than the one they play in human vintage capital models — often referred to as providing for a major engine of technology adoption.[30] In our model it is the existence of transferrable human capital to hinder the choice of a superior technology via its impact on wages, while in the vintage human capital model the transferability of knowledge favors the adoption of superior technologies, whereas specific human capital would be responsible for firms' lock-in in inefficient technologies.

In spite of the fact that our results are derived in a static setup, they seem to qualitatively fit several characteristics of the observed technical change patterns.[31] It is well known that the diffusion process of technologies depicts a S-shape. Our theoretical argument suggests that the adoption of the frontier technology is small at first due to relevance of the externalities it generates, which render the marginal benefit of choosing it smaller than the marginal increase in wages it induces. As the technical knowledge (the skills) required by the technology becomes more abundant, the impact of technology on outside options diminishes and so does that on wages, increasing firms' willingness to adopt. Finally, the market becomes saturated, eventually slowing down adoption rates.

Our framework appears also to be consistent both with the observation of an increasing uniformity of adoption rates among richer countries and with the trickle down hypothesis, implying that technologies are first adopted by the richer and more developed countries and diffuse in the poorer and less developed countries only at a later stage (thus explaining the observed divergence in cross-country rates of adoption between industrialized and less developed countries). The choice of a superior technology is in fact more likely to spill over in richer economies due to a greater homogeneity of the ex-ante technical knowledge that may render new knowledge more easily transferable. At the same time, however, the induced externalities are likely to have a lower impact

[30] See, for example, Jovanovic and Nyarko (1996), Chari and Hopenhayn (1991) and Brezis, Krugman and Tsiddon (1993).

[31] See Colombo (2004, Chpt. 2) for a survey.

on workers' outside options and hence on wages exactly for the same reason. Note that this is, once more, in sharp contrast with the predictions of the vintage human capital theory, according to which the more advanced economies should be those having the most to loose from the choice of superior technologies, due to the loss of the specific human capital embodied in older vintages.

Our setting could be extended in several respects. The process and costs of skills acquisition in relation to the firm's technology choices could be accounted for explicitly. Furthermore, our assumptions — and most notably those on the probability to be fired or re-hired, on consumers' risk neutrality, and on the modelling of the market sector — as well as the absence of capital market imperfections and of uncertainty about the size and relevance of the technology induced spillovers allow us to highlight in the simplest possible way the potential impact of production externalities and of the associated negative pecuniary externalities. Quite intuitively, in a richer framework, the introduction of risk aversion, the presence of imperfect capital markets and/or uncertainty on the relevance of externalities, and a low degree of technological complementarity between different sectors of the economy would all be factors reducing the impact of technology choices on workers' outside options, thus reducing the costs of innovating for the firm.

Finally, and at a greater level of generality, a dynamic version of the model might prove useful to further investigate — both from a positive and a normative perspective — the relevance of technology induced externalities in the study of technology diffusion and innovation, as well as for growth.

A Appendix: An application to a Cobb-Douglas Economy

In this appendix, we exemplify the main results of the paper for a Cobb-Douglas general equilibrium economy, using it to perform comparative statics exercises.

The firm's production function is described by the Cobb-Douglas production function

$$\Phi(N,T) = AT^{\alpha}N^{\beta}, \tag{A.1}$$

where $0 \leq T \leq T_{\max}$, $0 \leq N \leq \bar{N}$ and A is a scale parameter, that can be interpreted, for example, as an exogenous component of technical progress, which in the following we will normalize to 1, without loss of generality.

We further describe the (technology-induced) production externality by letting

$$g(T) := T^{\gamma}, \tag{A.2}$$

and we model the self-employed entrepreneurs (workers) labor disutility by assuming

$$\varphi(h_f) := h_f^2/2. \tag{A.3}$$

We finally make the following assumptions on parameters:

Assumption A.1 $\gamma > 1 > \alpha > 0$.

Assumption A.2 $1 > \beta > 0$.

Assumption A.3 $\beta\gamma > \alpha$.

It is immediate to see that these assumptions meet those made for the general case discussed in the previous sections, although we obviously account for a subset only of the cases that can emerge in the general equilibrium framework studied in the paper

All notation remains as in the main text and, whenever without ambiguities, we slightly abuse it in order to ease the exposition.

A.1 The Cournot-Nash case

In the Cournot-Nash case, the firm behaves as the self-employed entrepreneurs, in that it takes the externality as a given parameter (i.e. it does not take into account the impact of its decisions on the externality level). We denote, without loss of generality, this externality level with G. Following the discussion in Section 3, the firm's decision problem can be written as

$$\begin{cases} \max\limits_{N,T,w} & \Pi(N,T) = T^{\alpha}N^{\beta} - wN \\ s.t. & w \geq \max \left\{ \frac{e}{c}, Gh_f(G) + e - \frac{h_f(G)^2}{2} \right\} \end{cases}. \tag{A.4}$$

Since $\Pi(N,T)$ is increasing in T, the profit maximizing technology adopted by the Cournot firm is $T^C = T_{\max}$. As for the optimal wage, we already know that the constraint in the above maximization problem is always binding and therefore it is

$$w^C = W(G) = \max \quad \left\{ \frac{e}{c}, Gh_f(G) + e - \frac{h_f(G)^2}{2} \right\}. \tag{A.5}$$

Finally, given $T^C = T_{\max}$, the employment level is determined by the first order condition

$$\beta T_{\max}^{\alpha} N^{C\beta-1} = W(G),$$

and thus

$$N^C = \left(\frac{W(G)}{\beta T_{\max}^{\alpha}} \right)^{\frac{1}{\beta-1}}. \tag{A.6}$$

It is immediate to see that externalities do not play any role in the choice of technology by the firm. On the other hand, they do affect parametrically the equilibrium level of wage and hence the firm's employment.

A.2 The von Stackelberg case

We now apply to the Cobb-Douglas economy the analysis of the von Stackelberg case studied in Section 4. In this scenario, the firm does take into account the impact of the externalities it generates through its technology choice. Knowing the labor choice of the self-employed, $h_f = g(T)$,[32] and substituting for $g(T) = T^{\gamma}$, the decision problem of self-employed entrepreneurs/workers (Equation (8) in Section 2.2) yields $h_f = T^{\gamma}$, and hence the corresponding utility level of a self-employed is

$$\hat{V}_f = \frac{T^{2\gamma}}{2} + \pi. \tag{A.7}$$

Given the specific functional forms we consider, the workers' participation and incentive compatibility constraint (Equation (13)) becomes

$$W(T) = \max \quad \left\{ \frac{e}{c}, \frac{T^{2\gamma}}{2} + e \right\} \tag{A.8}$$

and the decision problem faced by the firm is

$$\max_{T,N,w} \quad \Pi = T^{\alpha} N^{\beta} - wN \tag{A.9}$$

$$\text{s.t.} \qquad w = \max \quad \left\{ \frac{e}{c}, \frac{T^{2\gamma}}{2} + e \right\}.$$

[32] $h_f := \arg\max g(T)\, h_f - \frac{h_f^2}{2}$.

The technology threshold $\tilde{T}$ — at which externalities start becoming relevant — follows immediately by solving

$$\frac{T^{2\gamma}}{2} + e = \frac{e}{c},$$

i.e.

$$\tilde{T} = \left(\frac{2e\,(1-c)}{c}\right)^{\frac{1}{2\gamma}}. \tag{A.10}$$

Since Π is monotonically increasing in T for $T \in [0, \tilde{T})$ and for any N, it is $T^* \geqq \tilde{T}$. Hence, the optimal level of technology is never lower than $\tilde{T}$. In order to understand if and when it pays to expand technology over $\tilde{T}$ when $T \in (\tilde{T}, T_{\max}]$, we need to study the sign of $\frac{\partial \Pi}{\partial T}$ for $T \to \tilde{T}^+$ and its behavior for $T > \tilde{T}$. From the first order condition with respect to N, given $T \in (\tilde{T}, T_{\max}]$, of Problem (A.9) it is immediate to get

$$\hat{N}\,(T) = \left(\frac{\frac{T^{2\gamma}}{2} + e}{\beta T^{\alpha}}\right)^{\frac{1}{\beta-1}}. \tag{A.11}$$

By differentiating Problem (A.9) with respect to T, and using $\hat{N}\,(T)$, one obtains

$$\frac{\partial \hat{\Pi}\left(\hat{N}\,(T)\,, T\right)}{\partial T} = \alpha T^{\alpha-1}\hat{N}^{\beta} - \gamma T^{2\gamma-1}\hat{N} \quad T \in (\tilde{T}, T_{\max}] \text{ and } T \to \tilde{T}^+. \tag{A.12}$$

By applying Proposition 1, we know that two regimes are possible when it pays to expand technology over $\tilde{T}$: either the firm adopts the best available technology (i.e. the technological frontier case in which $T^* = T_{\max}$) or it improves its technology, but not up to the frontier (i.e. the spillover case, with $T^* = \hat{T} \in (\tilde{T}, T_{\max})$). In order to determine $\hat{T}$ in the latter regime, by substituting $\hat{N}\,(T)$ into Equation (A.12) and after some algebra, it is

$$\left(\frac{\frac{T^{2\gamma}}{2} + e}{\beta T^{\alpha}}\right)^{\frac{1}{\beta-1}} \left[\frac{\alpha\left(T^{2\gamma}/2 + e\right)}{\beta T} - \gamma T^{2\gamma-1}\right] = 0,$$

from which, being $T \neq 0$, it is[33]

$$\hat{T} = \left(\frac{2\alpha e}{2\beta\gamma - \alpha}\right)^{\frac{1}{2\gamma}}. \tag{A.13}$$

[33] Notice that $\hat{T} > 0$ requires $2\beta\gamma - \alpha > 0$, which is satisfied whenever Assumption A.3 holds. Moreover, in order to have $\hat{T} > \tilde{T}$, the following condition must be satisfied:

$$2\beta\gamma - \alpha < \frac{c}{1-c}.$$

One can immediately check that the latter is also a necessary and sufficient condition for $\left.\frac{\partial \hat{\Pi}}{\partial T}\right|_{T \to \tilde{T}^+} > 0$. One can also easily show that $\hat{T}$ is a global maximum of the firm's problem in technology.

In general, for a technology spillover regime to emerge, by applying the logic behind Proposition 1 we need to require that $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T \to \tilde{T}^+} > 0$ and $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T=T_{\max}} < 0$ are simultaneously satisfied. More precisely, since $\hat{\Pi}(T) \in \mathcal{C}^2$ in $(\tilde{T}, T_{\max}]$ and there exists a unique $\hat{T}$ — given by Equation (A.13) — such that $\frac{\partial \hat{\Pi}(\hat{T})}{\partial T} = 0$, requiring that the two conditions $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T \to \tilde{T}^+} > 0$ and $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T=T_{\max}} < 0$ hold guarantees that $\hat{T} \in (\tilde{T}, T_{\max})$ is a maximum of the firm's decision Problem (A.9).

We check the two conditions on the first derivative of $\hat{\Pi}(T)$ in turn. As for $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T \to \tilde{T}^+}$, since T is approaching $\tilde{T}$ from above, after substituting (A.11) into (A.12) — where we made use of the envelope theorem — and evaluating it at $T \to \tilde{T}$, where $\tilde{T}$ is given in Equation (A.10), it is (after some algebra)

$$\frac{\partial \hat{\Pi}(T)}{\partial T}\mid_{T \to \tilde{T}^+} = \left(\hat{N}\left(\tilde{T}\right)\right)^{\beta} \tilde{T}^{\alpha-1}\left[\alpha - 2\beta\gamma(1-c)\right]. \tag{A.14}$$

The term in square brackets is positive if and only if $\alpha > 2\beta\gamma(1-c)$. Since under Assumptions A.1 and A.3 it is $\alpha < \beta\gamma$, we can immediately conclude that a necessary, but not sufficient, condition for the above inequality to hold is $c > \frac{1}{2}$. For a spillover regime to emerge in this Cobb-Douglas economy, it is therefore necessary for the firm to have a good monitoring technology. In particular, *coeteris paribus*, the higher the probability to detect shirkers, the more likely the emergence of a spillover regime.[34]

As for $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T=T_{\max}}$, again by substituting (A.11-b) into (A.12-b) and after some algebra, we get

$$\frac{\partial \hat{\Pi}}{\partial T}\mid_{T=T_{\max}} = \left(\hat{N}(T_{\max})\right)^{\beta} T_{\max}^{\alpha-1}\left[\alpha - \beta\gamma\frac{T_{\max}^{2\gamma}}{T_{\max}^{2\gamma}/2+e}\right]. \tag{A.15}$$

It is immediate to see that $\frac{\partial \hat{\Pi}}{\partial T}\mid_{T=T_{\max}} < 0$ if and only if $\left[\alpha - \beta\gamma\frac{1}{1/2+e/T_{\max}^{2\gamma}}\right] < 0$. Since $\gamma > 1$, for $e > 0$ and finite, a necessary and sufficient condition for this inequality to hold requires $T_{\max} \to +\infty$. However, this is obviously a more restrictive condition than needed. One can notice, for instance, that since $\beta\gamma > \alpha$ by Assumption A.3, a sufficient condition for it to be negative is that

$$e < \frac{1}{2}T_{\max}^{2\gamma}. \tag{A.16}$$

As already noticed in the general framework discussed in the previous sections, Condition (A.16) highlights that it is the interplay between the parameter values for $T_{\max}$, γ and e to be responsible for the possible emergence of a spillover regime.

In the framework developed in the paper, having assumed $T_{\max}$ and γ as exogenously given parameters seems rather innocuous. There is a large literature investigating the innovation processes, that has developed several mechanisms explaining the arrival

[34] Since $\alpha > 0$, the above condition is obviously satisfied in the special case in which the firm has a perfect monitoring ability, i.e. $c = 1$, that is therefore a necessary and sufficient condition, even though a restrictive one.

rates of new technologies. Hence, the factors affecting the technology frontier $T_{\max}$ are well debated and understood. Furthermore, it is often assumed that the arrival rate of new technologies is exogenous.[35] As for γ, it captures the entity of the production externalities induced by the firm's technology choice and it is therefore natural to treat it as a parameter.

We need, nevertheless, to be more cautious in treating the level of effort exerted by workers. Throughout the paper we have considered it as an exogenous parameter to keep our framework simple. In general, however, it is reasonable to assume that e is a variable under the firm's control (at least up to a certain extent, and if it is possible to write appropriate incentive-compatible contracts as in our efficiency wage setup). Thus, it is reasonable to claim that it is related in specific ways to the technology adopted by the firm, i.e. it is a function $e(T)$ of the technology. Under the assumption that the firm is aware of the specific form of $e(T)$, this implies that it should take it into account in its decision problem, by considering explicitly the impact that the adoption of a certain technology has on the effort workers are required to exert. This, in turn, would affect the type of equilibrium that emerges, without however implying that some of the three possible regimes become unfeasible.

A.3 Comparative statics

We first look at the factors affecting the threshold at which production externalities start distorting the firm's decisions, focussing especially on the role of the disutility of effort. As it is for the general case discussed in the paper, and obviously for the same reasons, it is immediate to conclude that for the Cobb-Douglas economy we are studying a rise in the level of effort exerted by workers implies an increase of the technology grade at which externalities become relevant. In the same way, a sharpening of the firm's monitoring (as captured by an increase in the probability, c, of catching a shirker) determines a decrease in the threshold technology level $\tilde{T}$. Analytically, both findings follow immediately, by differentiating Equation (A.10) with respect to e and c respectively, i.e.

$$\frac{\partial \tilde{T}}{\partial e} = \frac{1-c}{c\gamma}\left(\frac{2e\,(1-c)}{c}\right)^{\frac{1-2\gamma}{2\gamma}} > 0, \qquad \frac{\partial \tilde{T}}{\partial c} = -\frac{e}{\gamma c^2}\left(\frac{2e\,(1-c)}{c}\right)^{\frac{1-2\gamma}{2\gamma}} < 0. \quad \text{(A.17)}$$

In order to assess the impact of a change in the level of effort on the firm's labor demand, when the blocked technology adoption regime applies (i.e. $T^* = \tilde{T}$), we substitute for $\tilde{T}$ into Equation (A.11). By differentiating with respect to e, after some algebra, we get

$$\frac{\partial \hat{N}\left(\tilde{T}\right)}{\partial e} = \frac{e\,(1-c)\,(\alpha-2\gamma)}{\beta\gamma c^2\,(1-\beta)}\left(\frac{e}{\beta c}\right)^{\frac{2-\beta}{\beta-1}}\left(\frac{2e\,(1-c)}{c}\right)^{\frac{\alpha-2\gamma(1-\beta)}{2\gamma(1-\beta)}}. \qquad \text{(A.18)}$$

[35] See Colombo (2004, chpt. 2) for a survey and references to this literature.

Since $0 < (c, \beta) < 1$ and $\alpha < 2\gamma$ by Assumption A.3, it is immediate to notice that the first term in Equation (A.18) is negative, while the other two are positive. Thus $\frac{\partial \hat{N}(\tilde{T})}{\partial e} < 0$, meaning that an increase in the level of effort exerted by workers has a negative impact on equilibrium employment. This is a result following directly from the structure of the efficiency wage framework. As is standard in the efficiency wage literature, an higher disutility of effort requires the firm to pay an higher wage (at the equilibrium) in order to meet workers' incentive compatibility and individual rationality constraints. A higher wage, in turn, implies a lower labor demand by the firm.

We turn now to the impact of effort on the firm's technology choice when the spillover regime applies (i.e. $T^* = \hat{T} \in (\tilde{T}, T_{\max})$). By differentiating Equation (A.13), we get

$$\frac{\partial \hat{T}}{\partial e} = \frac{\alpha}{\gamma (2\beta\gamma - \alpha)} \left(\frac{2\alpha e}{2\beta\gamma - \alpha} \right)^{\frac{1}{2\gamma} - 1}, \tag{A.19}$$

that is greater than 0 under Assumptions A.1 and A.3. An increase in the workers effort determines an upward movement on the wage paid by the firm (i.e. $w^* = \frac{{T^*}^{2\gamma}}{2} + e$). This, in turn, is responsible for reducing the impact of the adoption of a superior technology on wages, determining an improvement of the technology grade chosen in equilibrium. By inspection of (A.13) it is apparent that, for the Cobb-Douglas economy we are examining, the firm's monitoring has no impact on the technology it chooses in the spillover regime. As for the impact of effort on the equilibrium level of employment, by substituting for (A.13) and differentiating the relevant part of (A.11), after some algebra, it is

$$\begin{aligned} \frac{\partial \hat{N}\left(\hat{T}\right)}{\partial e} &= \frac{1}{\beta - 1} \left(\frac{\frac{e}{2\beta\gamma - \alpha} + e}{\beta \left(\frac{2e}{2\beta\gamma - \alpha} \right)^{\frac{\alpha}{2\gamma}}} \right)^{\frac{2-\beta}{\beta-1}} \cdot \\ &\quad \left[e\beta^{-1} \left(\frac{2\beta\gamma - \alpha + 1}{2\beta\gamma - \alpha} \right) \left(\frac{2e}{2\beta\gamma - \alpha} \right)^{-\frac{2\gamma+\alpha}{2\gamma}} \left(\frac{2\gamma - \alpha}{\gamma (2\beta\gamma - \alpha)} \right) \right]. \end{aligned} \tag{A.20}$$

It is easy to see that the term in square brackets in Equation (A.20) is greater than 0 by Assumptions A.2 and A.3, and since the first term is negative (by Assumption A.2), an increase in the disutility of effort has a negative impact on equilibrium employment.[36]

A.4 Welfare Analysis

We finally briefly focus on Pareto efficiency and on policy intervention, for the latter investigating non-linear first best subsidies only.[37]

[36] As for the impact and cross-effects on technology and employment of other relevant parameters, see the analysis in Colombo (2004, chpts. 4 and 5).

[37] The study of second best policy measures proves to be algebraically demanding under the Cobb-Douglas specification studied here. The problems at hand can be solved only for specific parameters

In order to determine Pareto efficient allocations, we need to maximize aggregate production net of aggregate social costs, as defined by the right hand side of Inequality (30), that for the Cobb Douglas economy under scrutiny specializes into:

$$\max_{T,N,h_f,N_f} \quad Y(T,N,h_f,N_f) := T^{\alpha}N^{\beta} - eN + \left(T^{\gamma}h_f - h_f^2/2\right)N_f \tag{A.21}$$

By mimicking the same arguments developed in the previous sections for the general case, it is immediate to notice that Y is strictly increasing in T, for all (N,h_f,N_f) triples, provided N or (h_f,N_f) are different from zero. Thus $T^{**} = T^{*} = T_{\max}$. (Recall that we use a double asterisk to denote Pareto efficient allocations). Moreover, for any (T,N_f), h_f is chosen so as to maximize $T^{\gamma}h_f - h_f^2/2$, and thus $h_f^{**} = h_f^{*} = T_{\max}^{\gamma}$. As for the optimal labor allocation, it must be that the marginal return on labor is the same in the market sector and in the self-employment sector of the economy (see Equation (32)), and hence:

$$\beta T_{\max}^{\alpha}N^{\beta-1} - e = T_{\max}^{2\gamma} - \frac{T_{\max}^{2\gamma}}{2}, \tag{A.22}$$

i.e.,

$$\beta T_{\max}^{\alpha}N^{\beta-1} = e + \frac{T_{\max}^{2\gamma}}{2}, \tag{A.23}$$

from which it follows

$$N^{**} = \breve{N} = \left(\frac{e + T_{\max}^{2\gamma}/2}{\beta T_{\max}^{\alpha}}\right)^{\frac{1}{\beta-1}}. \tag{A.24}$$

Policy intervention is called for whenever the market equilibrium is such that $T^{*} < T_{\max}$. This implies that there is a role for an active fiscal policy only under the von Stackelberg scenario. Indeed, it is immediate to notice that in the Cournot-Nash framework, market allocations and Pareto allocations coincide.[38]

As already stated above, we only consider the set of instruments proposed in Section 5.2 in order to overcome market failure: that is, *non-linear first best subsidies*.

By introducing a per-employee subsidy $S(T,N)$, the firm's profit function is

$$\Pi = T^{\alpha}N^{\beta} - (W(T) - S(T,N))N, \tag{A.25}$$

configurations. A full characterization of parameter regions, however, does not add much in terms of economic insights, and thus we omit it.

[38] The technology adopted by the firm is $T_{\max}$ in both cases and, comparing Equations (A.24) and (A.6) where

$$W(G) = e + \frac{T_{\max}^{2\gamma}}{2},$$

by making use of Equation (A.23), it is immediate to note that also the employment level is the same.

where $W(T)$ is defined in Equation (A.8) and the subsidy $S(T,N)$ is equal to

$$S(T,N) = \begin{cases} \frac{e}{c} - e + \left(T_{\max}^{\gamma} T^{\gamma} - \frac{T^{2\gamma}}{2}\right) \frac{\bar{N}-N}{N} & T \in \left[0, \tilde{T}\right) \\ \left(T_{\max}^{\gamma} T^{\gamma} - \frac{T^{2\gamma}}{2}\right) \frac{\bar{N}-N}{N} + \frac{T^{2\gamma}}{2} & T \in \left[\tilde{T}, T_{\max}\right] \end{cases}. \tag{A.26}$$

By substituting for $W(T)$ and $S(T,N)$, the firm's problem becomes

$$\max_{T,N} \quad T^{\alpha} N^{\beta} + \left(T_{\max}^{\gamma} - \frac{T^{\gamma}}{2}\right) T^{\gamma} \frac{\bar{N}-N}{N} - eN. \tag{A.27}$$

Since the above program is increasing in T, it is immediate to observe that, following the introduction of the non-linear subsidy, it is $T^{*} = T_{\max}$. Moreover, it is also straightforward to notice that at $T = T_{\max}$, Problem (A.27) gives a first order condition for the choice of employment identical to the condition characterizing the optimal (Pareto efficient) allocation of labor, i.e.

$$\beta T_{\max}^{\alpha} N^{\beta-1} = e + \frac{T_{\max}^{2\gamma}}{2}. \tag{A.28}$$

Hence, non-linear subsidization conditional on the level of technology and employment allows to eliminate the distortion in technology adoption without introducing any distortion in the allocation of labor.

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