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Complementarity in Training Practices: a Methodological Note

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Abstract

Based on the notion of complementarity among productive factors, the paper develops a conceptual framework aimed at analysing why a firm may find it profitable the financing of general training. The paper shows that a simple application of some contributions of theoretical analysis based on the lattice theory and the notion of supermodularity can provide a suitable framework of analysis to study and understand complementarity relationships which can be established among productive factors. Especially, the paper develops the analysis of complementarities between general and specific training providing, additionally, a theoretical reference for empirical analysis.

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0. Introduction

In his seminal contribution, Becker (1964) drew the crucial distinction between specific and general training and analysed its consequences. Assuming perfect competition in both the labour and the product market, perfect information and perfect mobility of productive factors, Becker showed that no employer is available to fund training of employees for the acquisition of skills/ knowledge that affect positively employees' productivity in the firm financing training, as well as in other comparable firms; namely no employer funds general training. On the contrary, employer's financing is available for specific training, namely the acquisition of knowledge/skill that affect positively employees' productivity solely in the firm providing the financial means supporting this training programme. In the case of specific training the burden of financing is sustained not only by the employer, but also by the employees benefiting from training support, who share with the employer direct training expenses and opportunity costs.

Departing from Becker's analysis, recent economic literature has shown that, if one abandons Becker's assumptions concerning perfect competition and information, the rationale for employers' funding of employees' general training can emerge. In their extensive and thorough survey Acemoglu, Pischke (1999) analyse this strategy of research. This paper adopts a marginally different strategy, as it investigates some features of production in firms. The basic idea is that the existence of complementary relationships among productive inputs can justify the employer's financing of general training. Especially, the paper emphasises the complementarity relationship, which can be established between specific and general training. If the assumption of complementarity among general and specific training is reasonable, we do not need many other technical hypothesis, since from that assumption relevant results directly follow. Quoting Milgrom and Roberts, "Once the reasonableness of the complementarity hypothesis is verified one hardly needs to write down a fully specified mathematical model. […] certain kinds of conclusions follow directly from the complementarity structure , without further technical assumptions" (Milgrom-Roberts, 1995, p. 200). In their model the profit function is supermodular in 12 variables, and this is sufficient to immediately derive that whatever change in one of the 12 variables (for instance a fall in the costs of flexible manufacturing equipment) will induce a systematic response in all the other 11 variables. What the theory does is just to establish "the complementarity

assumptions that are sufficient to imply the stated conclusions" (Milgrom, Roberts, 1995, p. 199).

Hence, what we still need is to empirically verify how realistic is the assumption of complementarity among general and specific training.

Our aim is just to present a methodological framework useful to empirically test complementarity among the two forms of training, keeping in mind that data about firms training practices, whether specific or general, are available and usually are discrete variables.

The analysis proposed in the paper builds up and brings together the contributions given by Topkis (1998), Milgrom, Shannon (1994), Milgrom, Roberts (1995) and Mohnen, Roller (2005).

The paper is organized as follows. The first section of the paper introduces the analysis of complementarity among productive factors recently developed. The second section of the paper analyses the consequences of complementarity in the process of skill development in firms. Particularly, this section stress the relevance of general training in the development of specific assets and provides a preliminary discussion about problems to deal with in empirical analysis when classifying and measuring training practices. In section three, a methodological framework is presented, wherein the complementarity among general and specific training is analysed through the supermodularity of firms average productivity function. The last section concludes the paper.

1. Complementarities in production

1.1 A definition

Milgrom, Roberts (1990, 1995) have developed a formal model that refines Edgeworth's approach to complementarity among productive factors. In their contributions Milgrom, Roberts never define a specific unit of analysis. They refer to either characteristic features of production (Milgrom, Roberts, 1995) or to "elements of the firm's strategy" (Milgrom, Roberts, 1990, p. 513) or in a broader sense to "groups of activities (Milgrom, Roberts, 1990, p. 514).

From a labour economics' perspective, complementarities among productive factors can be discussed with reference to four units of analysis:

a) employees' individual skills. In that case complementarity refers to both employees' knowledge and tasks carried out in productive activities;
b) division, shop floor, teams or, generically, autonomous sub-units of the productive unit;
c) organisational practices referring both to organisation of work in a broad sense (i.e.: teamwork, task and job rotation, training practices…) and to other defining features of production (i.e.: management of inventories, degree of vertical integration, outsourcing…);
d) capital equipment such as hardware (i.e.: lathe, computers…), software (i.e.; computer-aided design, word processing program…).
Complementarity among productive factors (inputs) can be observed when the level of a given productive factor affects positively marginal productivity of other productive factors. In technical terms that means that the second mixed derivative of the production function with respect to two productive inputs is always positive.

1.2 Complementarity and skills

Complementarity among inputs entails that the return of a single skill does not only depend on the skill itself, but also on other skills and inputs. For this reason it is useful to introduce the distinction between skills acquired and skills used. The former refer to the content of education, training and, in general, to the knowledge content transmitted to the employee. Skills acquired account for the stock of knowledge and previous working experience of an employee, definable regardless of the specific productive context in which she operates. Acquisition of skills occurs through both formal (formal education, training) and informal procedures of transmission. On the other hand, skills used refer to those skills actually used by employees in their working activities and define the set of tasks to perform. Skills used cannot be specified outside a well-defined productive context and their development can occur through some kind of formal and informal training.
Skills acquired and used result from a complex process of learning in which the specification of complementary relationships between both types of skills and the other inputs play a pivotal role. From an endowment of skills acquired, one can develop a set of skills used through the establishment of complementary relationships among this bundle of skills and the other productive inputs. These relationships convert skills acquired into skills used. However, these learning mechanisms also work in the opposite direction. In other words, after a series of skills acquired has developed into skills used, the process of conversion can continue in

reverse and proceed towards the acquisition of new skills and the consequent growth and sedimentation of the endowment of skills acquired.

This relationship between skills used and skills acquired implies that the effect on skills of specific and general training is different. As far as general training is concerned, it affects directly the endowment of skills acquired. As to skills used the story is different. As a matter of fact, the effect of general training on skills used depends on the complementary relationship with the other inputs. As mentioned in the previous paragraph, these relationships among inputs set, through the aforementioned process of conversion, the return of skills acquired. If general training favours the setting up of new complementary relationships with other productive inputs or improves the working of these relationships, this implies that the range of skills used has widened, resulting into a higher level of employees' productivity. Accordingly, if general training affects complementary relationships, positive effects can be observed on the productivity of other inputs, as well; i.e.: provision of general training may affect positively not only the productivity of the trained employed, but also the productivity of other employees.

As far as specific training is concerned, it affects the structure of the complementary relationships and, hence, the establishment and the working of links among inputs. For the same reasons as for general training, the observed effects may be positive on diverse inputs, as well as the employee trained.

2. Skills and asset specificity

The distinction between general and specific training has to be analysed in comparison with the notion of asset specificity. In Becker's analytical framework general training does not develop any specific asset and therefore the newly developed skills can be used in any workplace. Asset specificity stems from specific training only, giving rise to the opportunity for the employer to exploit an economic rent.

In the framework of analysis developed in this paper, things are different. Indeed, even though general training improves employees' productivity in any firm, training, favouring the establishment of new complementarity relationships, can also widen the range of skills used. The degree of asset specificity of the skills used increases, making the trainees' productivity firm specific. As a matter of fact, even though training can be general, its return, measured by

increases in employee's and other factors' productivity, depends on the complementarity relationships developed within the firm and, as such, is always firm specific. As a consequence of that, the development of specific assets through training does not depend on the nature of training, but on the cobweb of complementary relationships among inputs implemented in the organisation of production. Therefore, general training can develop specific assets.

Training is not provided, if it does not promote asset specificity. The discriminant is not the degree of specificity of training, but the framework of the complementarity relationships among productive inputs, stemming from the process of conversion of skills acquired into skills used. In a sense, general training, in the Becker's meaning of promotion of general assets, cannot be easily accommodated in this framework of analysis. It can be conceived as a very special case in which training is irrelevant with respect to production and does not affect productivity of inputs, at all. This can occur either because the content of training has no connections with inputs and production[1], or if, as a consequence of training, the firm does not promote the establishment and the strengthening of complementarity relationships among inputs [2]

This analysis of training, learning and skill development raises two crucial consequences. Firstly, general training affects productivity in the firm where the employee is currently employed (internal productivity) and productivity as perceived by employers in the external labour market (external productivity) in a different way . Divergence between internal and external productivity favours the setting up of internal labour markets, as they insulate the employers financing training from the underbidding of other employers. Secondly, the focus of the analysis shifts from the distinction between general and specific training to the analysis of complementary relationships among inputs. If general training can develop specific assets, this occurs through the interaction of this kind of training with other inputs. General training practices fit with other inputs and their interactions favour the process of skill development described in the previous paragraphs. Especially, as far as training practices are concerned, that means that general training has always to be analysed jointly with other training practices in order to understand its impact on the firm's productivity. It is useful to emphasize that the

[1] A bridge course for an electronic engineer.
[2] A course of word processing in a firm which has no computer.

effect of general training is not limited to individual productivity but spreads, due to the complementarity relationship among productive inputs.

Of course, that does not mean that employers are always available to finance general training. However, the distinction between skills acquired and used provides the rationale to understand the potential arising of a positive level of employer's rent, even when general training is provided and no special assumption about the level of wages is made as in Acemoglu, Pischke's (1999) analysis.

2.1 The measurement of training in empirical analysis.

This conception of training implies that the employee cannot be the proper unit of analysis, but that the firm can play this role. In the empirical analysis of training in firms problems are twofold. Firstly, the identification of the nature of training can be rather complicated. It seems that the two polar cases, "pure" general and specific training, cannot be easily observed and defined; training is a complex and articulated activity made up of different components which affect its nature in either direction. Of course, there are different degrees of generality/specificity, so that one can say that a dominant nature can be pointed out. In informal training practices such as apprenticeship or task rotation, the specific component prevails. On the other hand, in formal practices such as off-the-job training the general component can exceed the specific one. However, the dichotomies formal/informal and on-the-job/off-the-job do not overlap that between general and specific.

Secondly, a reliable and unbiased measurement of training might as well require data on single employees which can be hardly available or, when measured , can contain a high margin of error. In fact, a measurement of training practices should take account of both the percentage of employees involved in training practices and the time devoted to these practices.

Conclusively, the construction of a synthetic indicator of the nature and the amount of training practised in the firm would not be easily to conceive, either because the proper data could not be easily available or because it would require the introduction of arbitrary assumptions for the specification of the nature of training. Therefore, in empirical analysis it would make sense to consider the provision of training as a dummy variable, without intending to measure its intensity.

3. Testing complementarity between general ad specific training

In the analysis set out in the previous section, it emerges that doing more of general training can raise the return to doing more of specific training, that is, complementarity among general and specific training inside a firm seems to be a reasonable hypothesis.

Following Topkis (1998), Milgrom, Shannon (1994), Milgrom, Roberts (1990, 1995) one can assert that, since complementarity implies matters of order[3], the *lattice theory* is the branch of mathematics that better suites to the study of complementarity.

A *lattice* $(X, \geq)$ is a set X, with a partial order $\geq$, such that for any $x,y \in X$ the set X also contains a smallest element under the order that is larger than both x and y and a largest element under the order that is smaller than both x and y. Let $x \vee y$ denote the smallest element that is larger than both x and y, and let $x \wedge y$ denote the largest element that is smaller than both x and y. In the n-dimensional Euclidean space, R^n, $x \vee y$ and $x \wedge y$, are:

$x \vee y = (\max\{x_1,y_1\},\ldots, \max\{x_N,y_N\})$, and $x \wedge y = (\min\{x_1,y_1\},\ldots, \min\{x_N,y_N\})$.

When complementarity is expressed through the objective function, we say that a real-valued function F on a lattice X is supermodular in its arguments[4], if and only if:

$$F(x \vee y) + F(x \wedge y) \geq F(x) + F(y) \qquad \forall x,y \in X \tag{1}$$

Or, written in a different way:

$$F(y) - F(x \wedge y) \leq F(x \vee y) - F(x) \qquad \forall x,y \in X, \tag{2}$$

that is, the change in F from the minimum $(x \wedge y)$ to y (or x) is smaller than the change in F from x (or y) to the maximum $(x \vee y)$: having *more* of one variable *increases* the returns to having *more* of the other[5].

In our case, if general and specific training are complementary firm's objective function must be supermodular in these two variables.

In the specific, we consider firm's average productivity function (AP) as the objective function, that depends on firm's choices about general and specific training.

Each firm (indexed by j) is characterized by the average productivity function:

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[3] "Doing *more* of one thing *increases* the returns to doing *more* of another", Milgrom, Roberts (1995), p. 181

[4] That is its arguments are complements.

[5] From equations (1) and (2) it is evident that complementarity is symmetric: having *more* of *y increases* the returns to having *more* of x, as well as having *more* of *x increases* the returns to having *more* of y

(3) $$AP = \frac{Y}{N} = AP(t_{gk}, t_{s\gamma}, \theta_j) \qquad \forall j$$

that is, each firm can choose K types of general training and Γ types of specific training, $(t_g, t_s) = (t_{g1},...,t_{gK},t_{s1},...,t_{s\Gamma}) \in T$. θ_j represents firm-specific pre-determined factors.

The problem of firm j is to choose a set of policies for specific and general training, which maximizes her average productivity function, AP.

Complementarity between general and specific training may be analysed testing whether $AP(t_{gk}, t_{s\gamma}, \theta_j)$ is supermodular in t_{gk} and in $t_{s\gamma}$.

The maximization problem is the same for all the firms, but, since each firm is characterized by specific pre-determined factors (θ_j), - as product market or firm's size – the AP function may result supermodular in t_g and t_s for some firms, but not for others.

Following Mohnen, Roller (2005), and using the theory of *lattice*, our aim is to derive a set of inequalities (as those explicated in equations (1) and (2)), that can be used in empirical tests, to verify whether these inequalities are accepted by the data and, hence, whether complementarities among general and specific training is empirically confirmed, or in which specific circumstances (firm-specific pre-determined factors) complementarity holds[6].

Since in our case the objective function of each firm is the average productivity function, firm j's AP function on the lattice[7] T is supermodular in t_g and t_s (t_g and t_s are complements) if and only if, for any $t_g, t_s \in T$:

(4) $$AP(t_g \vee t_s,\ \theta_j) + AP(t_g \wedge t_s,\ \theta_j) \geq AP(t_g,\ \theta_j) + AP\ (t_s,\ \theta_j) \qquad \forall\ j$$

or:

(5) $$AP(t_g,\ \theta_j) - AP(t_g \wedge t_s,\ \theta_j) \leq AP(t_g \vee t_s,\ \theta_j) - AP\ (t_s,\ \theta_j) \qquad \forall\ j,$$

that is doing more of t_g increases the returns to doing more of t_s.

As an example we can think at two possible firm's decisions concerning general and specific training.

We can consider a firm which operates in the pharmacological sector. This firm can choose to organize (or not to organize) a refresher course in general chemistry and can choose to train (or not to train) her employees in the chemical reactions of human body to the adoption of

[6] What we want to investigate is whether t_{gk} and $t_{s\gamma}$ are complementary in any firm, or if they are complementary in some specific circumstances, e.g. in some specific product markets, or in large firms.

[7] Where the set T has dimension $K+\Gamma$

drugs. The first choice concerns general training, and the second choice concerns specific training[8].

It is obvious that in our example doing more of general training increases the returns to doing more of specific training, that is general and specific training are complements.

We can consider each of the two choices as binary decision variables. So, if a firm chooses to organize neither the course in general chemistry nor the training in the chemical reactions of human body to the adoption of drugs, we have t_g=0, t_s =0; in this case the element of the lattice T is $t_g \wedge t_s$ ={00}. If a firm chooses to organize both the course in general chemistry and the training in the chemical reactions of human body, we have t_g=1, t_s=1, and the element of the set T is $t_g \vee t_s$ ={11}. Including also the mixed cases, we have four elements in the lattice T={{00},{01},{10},{11}}.

From equations (4) and (5) we can assert that t_g and t_s are complementary and hence that the function AP is supermodular, if and only if:

$AP(00, \theta_j) + AP(11, \theta_j) \geq AP(10, \theta_j) + AP(01, \theta_j)$

or:

$AP(10, \theta_j) - AP(00, \theta_j) \leq AP(11, \theta_j) - AP(01, \theta_j)$,

that is, increasing one of the two forms of training (for instance t_g) increases the average productivity in a wider way if also the other form of training increases. Actually, the increases in AP due to an increase of t_g from {00} to {10} are less (or at least equal) to the increases in AP due to increases of both t_g and t_s from {01} to {11}.

If we considered four different kinds of choices concerning general and specific training, and if these choices could be considered as binary decision variables the elements of the lattice T would have been {0000}, {0001}, {0010}, ..., {1111}, in all 2^4 =16 elements. More generally, if the binary decision variables are N, 2^N would be the whole number of elements belonging to the lattice T.

With 16 elements belonging to the lattice T, the supermodularity of the function AP, would be granted by the set of inequalities:

$AP(0000, \theta_j) + AP(1100, \theta_j) \geq AP(1000, \theta_j) + AP(0100, \theta_j)$

$AP(0001, \theta_j) + AP(1101, \theta_j) \geq AP(1001, \theta_j) + AP(0101, \theta_j)$

$AP(0010, \theta_j) + AP(1110, \theta_j) \geq AP(1010, \theta_j) + AP(0110, \theta_j)$

[8] Where specific training may be intended in a widest meaning of sector specific, rather than firm specific. On

$AP(0011,\ \theta_j) + AP(1111,\ \theta_j) \geq AP(1011,\ \theta_j) + AP\ (0111,\ \theta_j)$

The above four inequalities insure complementarities between the first and the second choice variables. To these we have to add four inequalities necessary for complementarity between the first and the third choice variables:

$AP(0000,\ \theta_j) + AP(1010,\ \theta_j) \geq AP(1000,\ \theta_j) + AP\ (0010,\ \theta_j)$

$AP(0001,\ \theta_j) + AP(1011,\ \theta_j) \geq AP(1001,\ \theta_j) + AP\ (0011,\ \theta_j)$

$AP(0100,\ \theta_j) + AP(1110,\ \theta_j) \geq AP(1100,\ \theta_j) + AP\ (0110,\ \theta_j)$

$AP(0101,\ \theta_j) + AP(1111,\ \theta_j) \geq AP(1101,\ \theta_j) + AP\ (0111,\ \theta_j)$

In a similar way one can derive the set of inequalities which insure complementarity among the other choice variables. Considering binary variables, the number of inequalities for each variable is given by: $2^{(N-2)}$. Since the number of pairs of variables is given by: $\frac{N(N-1)}{2}$, then the amount of relevant inequalities is $2^{(N-3)}N(N-1)$.

In the above example of four decision variables, the whole number of inequalities is given by: 2·4·3=24.

Summing up, what we need is to check complementarity among pair wise choice variables of the lattice *T*. Complementarity among all the *N* decision variables exists if the *AP* function is shown to be supermodular in all the variables and this happens when all the $2^{(N-3)}N(N-1)$ inequalities are satisfied.

4. Concluding remarks

From a methodological perspective this paper has brought attention on two interrelated topics concerning the economics of training in firms. First of all, it introduces the notion of complementarity in the analysis of training practices managed in firms. In this framework of analysis training is not analysed per se, but it is studied as a component of a complex system such as a firm. Actually, this paper emphasises complementarity between general and specific training, but these are only two of the many elements interacting and matching in the firm. Training practices do not occur in vacuum but have to fit into a complex nexus of inputs, intended in the broad meaning implied in the analysis by Milgrom, Roberts. The application

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this subject, see the previous section and Acemoglu-Pischke, 1999.

of the notion of complementarity provides the rationale for employer's funding of general training.

Secondly, this work constitutes one more step towards the analysis of complementarity between general and specific training inside a firm, in the sense of providing a methodological support to the empirical analysis.

On the basis of the works by Topkis (1998), Milgrom, Shannon (1994), Milgrom, Roberts (1995), Mohnen, Roller (2005), complementarity between the two forms of training has been analysed through the supermodularity of the average productivity function. Adopting the principles of the *lattice theory*, the relevant inequalities which insure the supermodularity of the average productivity function and, hence, the complementarity among the variable choices concerning general and specific training, have been derived. In this context the variable representing firm-specific pre-determined factors (θ) deserves particular consideration. Actually, in comparative statics it could be discovered that complementarity between general and specific training holds only in presence of some firm-specific factors, as product markets or firm dimension.

The following necessary step will be to verify if the inequalities here derived are accepted by empirical data, and, hence, if complementarity between general and specific training is confirmed by empirical tests.

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