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Consumer Demand Composition, Human Capital and R&D Activity in Simple Model of Endogenous Growth

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Abstract

Within a context of endogenous growth driven by the accumulation of human capital, we investigate the effect on the steady state distribution of skills of the allocation of total consumption between a traditional good and a composite commodity, whose variety dimension can be increased through R&D. Moreover, we introduce a transitional dynamic, through an external effect of education on individual's preferences: we assume that the accumulation of human capital shifts tastes toward a stronger liking for the "luxury" commodity. The resulting short run effects of the changhing shares of income allocated between the two commodities are analized.

Keywords: endogenous growth, human capital, R&D, composition of demand.

JEL Classification: J24, O31, O41

1 Introduction

The endogenous growth literature that emerged in the late eighties has evolved along two fundamental lines. First, the idea-based approach (Romer [1987, 1990], Grossman-Helpman [1991], Aghion-Howitt [1992]) obtains endogenous growth by introducing an external effect in the R&D activity. This is due to the fact that newly patented innovations embody knowledge in a non rival manner, that contributes positively to the capacity to innovate. Secondly, the human capital approach (Lucas[1988]) prevents the occurence of diminishing return through the accumulation of human capital, that raises the productivity of both labor and physical capital. Mostly, these two approaches have developed separately[1]. However, the earliest class of R&D models, above mentioned, relies their endogenous growth mechanism upon a scale effect at odds with the empirical evidence (Jones[1995a, 1995b]), Barro and Sala-i-Martin [1995]). Prompeted by this evidence, various contributions have adjusted the theoretical setting of the original R&D-based growth models to avoid the scale effect prediction determined by the *level* of innovations (Jones [1995b], Kortum [1997], Segerstrom [1998], Jones [1999], Aghion-Howitt [1998; Ch.12], Peretto [1998], Dinoupolous-Thompson [1998]).

More recently, several papers have proposed an alternative model setup which combines human capital accumulation as the device for generating sustained growth in the long run, avoiding scale effects due to the increase in the stock of knowledge. These contributions have provided a fertile ground to investigate various aspects of the interaction between human capital and innovation. Blackburn *et al.*[2000] consider a constant returns to scale technology for the education sector, while diminishing returns characterize the R&D sector. As a result, the steady state growth rate is completely independent from R&D activity, so the model avoids the scale effect prediction[2]. Bucci [2003a] neglects any spillover effect deriving from the process of invention of new intermediate goods. Therefore, growth is driven by human capital accumulation. This simple setup is employed to investigate how the market power influences growth and the sectoral distribution of skills. Ribeiro [2000] develops a very similar theoretical framework to that of Bucci [2003a, 2003b], except for the fact that the choice between working and education is separated from the choice of allocating a fixed amount of labor among the various sectors.

In the present context we develop a model, which integrates human capital accumulation and innovation activity, that departs from the previous cited contributions, since it considers the process of innovation aimed at expanding the variety of consumer goods rather than the number of intermediate inputs. Under this respect, our model combines the framework of Grossman-Helpan [1991, Ch. 3] with the Lucas [1988] setup.

Our first purpose is to examine to what extent the demand oriented R&D

[1] Few remarkable exceptions include Stockey [1988], Grossman-Helpman[1991, ch. 5.2], Young [1993], Eicher [1996] and Redding [1996].

[2] Arnold [1998] develops a model, that combines R&D actvity with human capital accumulation, obtaining similar results.

generates different results, as compared to the producer intermediates approach of Bucci [2003a, 2003b]. Under this respect, it will be shown that, despite many similarities, several remarkable differences arise. Secondly, we will use our theoretical frame to analyze the impact of the consumption expenditure shares on the steady state distribution of skills. Finally, we'll introduce a transitional dynamics through a peculiar representation of preferences, that displays non homotheticity over the transition.

The model describes a three sectors economy. A traditional industry produces an ordinary good within a perfectly competitive environment. An advanced sector offers a refined consumption bundle consisting of many varieties, each produced by a single firm within a market of monopolistic competition. Any new variety entering the market is provided by a competitive innovation sector through investments in R&D. The only required factor of production in the three sectors is effective labor, given by the amount of working time augmented by the existing stock of human capital. Technology in the three industries is characterized by constant returns to scale. On the preferences side, a single representative individual maximizes his lifetime utility over an infinite time horizon. The arguments of the instantaneous utility function are the quantities of a traditional good and of the "refined" commodity, available in many varieties. At any period of time, the decision process faced by the individual involves two allocation problems. First, he must decide the optimal composition of his consumption bundle. Second, he must allocate a single unit of time endowment between production activities (two consumption goods industries, and the R&D sector) and education. We assume that the education sector employes only effective labor to increase the existing stock of human capital.

The individual is motivated to invest in education, because he recognizes that the increased productivity will be rewarded by higher labor income levels. However, besides this obvious purpose related to market activity, education undoubtedly affects individuals under many other aspects. Not only the "technology" to extract "utility" from the consumption activity requires some knowledge input, that can be acquired through education (for example, reading a book, or understanding the instruction manual of a new generation portable phone), but the whole system of preferences is the result of the complex dynamics of the individual within the social and cultural environment, including education as an essential component [3]. We assume that the external effect of the investment in education aimed at increasing the productivity of labor is not rationally percieved by the individual. Thus, the taste modification due to the accumulating human capital is a completely unconscious process. In our model we capture this effect in a very simple way. We consider a Cobb-Douglas specification for the utility function, where the parameters representing the expenditure shares on the two goods depends on the stock of human capital. As human capital accumulates, the individual moves away from the "primitive and rude" tastes of the early stages of development and acquires more refined and cultivated

[3] Experiments involving reared apart twins shows that the influence of the social environment, of which the education process represents a prominent aspect, may even overwhelm the genetic attitudes of the individual.

preferences. As a result he shows a stronger liking for the "luxury" composite commodity, continually enriched by new varieties, and tends to disregard the traditional "ordinary" good. Given the device implied by our specification of the utility function, we model this effect through varying expenditure shares. The ongoing investments in education induce the individual to allocate higher shares of his income resources to the purchase of the composite coommodity. Or, conversely, the progressive distaste for the traditional good results in lower shares of income allocated to this commodity.

This process, however, does not end up with a linear utility function, i.e. with a zero demand for the ordinary good, as we assume that asymptotically the expenditure shares will stabilize to constant levels. Therefore, in our model preferences are non homothetic in the short run, but retain homotheticity in the steady state.

The idea that the consumption pattern reflects in some degree the educational curriculum of individuals is not new. Michael[1972, 1973] and Morris [1976] explicitely consider education within the utility function, and test the theoretical prediction, obtained in a partial equilibrium and static context, of a positive education elasticiy for luxury good and of a negative education elasticity for necessities, finding supporting evidence.

Our main results are the following. Given constant the parameters of the Cobb-Douglas utility function, the model displays a unique steady state balanced growth path with no transition. As we neglect any spillover effects in the research sector, the long run growth rate depends only on the parameters characterizing technology in the education sector and preferences. A static comparative analysis is performed to invesigate the effect of the market power and of the composition of consumption on the allocation of human capital across sectors. We find that an increase in the level of competition in the advanced sector (i.e. a lower mark-up) unambiguously determines a reallocation of productive resources towards the consumption goods sectors. In other words, a decrease in the market power enjoyed by monopolists crowds out resources devoted to innovation, and increases the activity level in the consumption sectors. Therefore, contrary to Bucci (2003b), it is not true that "*in the presence of human capital accumulation and when all sectors emply skilled workers it is non longer obvious that in the decentralized steady state equilibrium the R&D share is always increasing in the mark-up level* (Bucci (2003b; p. 431))", as in our setup a consumer goods innovation activity suffices to restore an unambiguos result, linking positively market power and incentive to innovate (see for eample Jones-Williams (2000)). On the demand side, we find that decreasing the share of consumption devoted to the traditional good, i.e. increasing the share on the composite commodity, has a beneficial effect on innovation, and decreases the amount of effective labor employed in the consumption industries.

Assuming varying expenditure shares due to the effect of human capital accumulation introduces a transitional dynamics in the model. In particular, the short run evolution of demand doesn't affect the growth rates of human capital and total expenditure, but it does affect the composition of total expenditure, and, as a consequence, the growth rate of innovation. We show that the transition towards the long run balanced growth path is accompanied by a growth rate of new varieties above its steady state level, while the growth rate of the traditional good remains below its steady state value.

2 The Consumer Allocation Problem

A single representative consumer faces the following intertemporal maximization problem:

$$\max U(0) = \int_0^\infty \exp(-\rho t)\left[\alpha(H)\ln(D(t)) + (1-\alpha(H))\ln(C(t))\right] dt \;;$$

$$D(t) = \left[\int_0^n x(i,t)^\epsilon di\right]^{1/\epsilon} = \left[\int_0^n x(i,t)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \;;\; \sigma = \frac{1}{1-\epsilon}$$

subject to the following dynamic equations and initial conditions:

$$\begin{gathered} \dot{a}(t) = r(t)a(t) + w(t)u(t)H(t) - P_C(t)C(t) - P_D(t)D(t) \\ \dot{H}(t) = B_H[1-u(t)]H(t) \\ H(0),\, a(0) \text{ given} \end{gathered} \tag{1}$$

σ is the (constant) elasticity of substitution. C represents a homogeneous good (traditional sector). D represents an aggregate commodity available in many varieties (innovative industry). $\alpha(H)$ represents the expenditure share allocated to the commodity D. It depends on the stock of human capital (H) accumulated by the individual. The crucial assumption is that preferences are fundamentally shaped (in an unconscious way) by the process of cultivation, which characterizes the formation of human capital. In particular, we assume the following properties to hold:

$$\begin{aligned} \frac{\partial\alpha(H)}{\partial H} &> 0 \\ \frac{\partial^2\alpha(H)}{\partial H^2} &< 0 \\ \lim_{H\to\infty} \alpha(H) &= a \text{ constant} \end{aligned}$$

We assume that the conscious (under the control of rationality) motivation to education is the expectation of higher wage levels. However, the education effort aimed at increasing the stock of productive human capital has an external effect on preferences, which is not percieved by the individual, so that it is not taken into account within his maximization strategy. In particular higher level

of education shifts individual tastes towards more sophisticated goods. In the present context we do not model explicitely the preferences response to human capital accumulation We expect this externality to work in actual behaviours through two possible channels. First, to the extent that education determines more cultivated individuals, probably they will be more discriminating in tastes: consuming the D bundle might be considered a more refined activity as compared to the "ordinary" C good. Second, handling new technologically sophisticated goods requires some knowledge, such as reading and understanding the "user manual". If new goods require some skills to use them, then more educated individuals might find it easier to extract utility from innovative goods than low literacy people. As a result of the lower user cost, a greater share of income might be allocated to purchase the more advanced or innovative goods.

$u(t)$ represents the fraction of human capital allocated to the labor market. This activity yields a wage rate $w(t)$. The remaining fraction $(1-u(t))$ is devoted to the accumulation of human capital through investment in education. B_H is the productivity of one unit of human capital in the education sector. P_C and P_D are the prices of C and D respectively. $a(t)$ is a riskless asset, yielding an interest rate $r(t)$, which represents the financial instrument, enabling individual's intertemporal substitution. The individual faces two allocation problems:

- Due to (weak) separability of preferences he first allocates his income between D and C. Then he decides the optimal composition of the D bundle.
- He allocates a fraction u of a single unit of time endowment to productive activities (production of D, C, and R&D) and the remaining $(1-u)$ to non productive activities (human capital formation through education). Given this optimal time allocation, the individual decides to optimally distribute the fraction of time u among the alternative productive activities available.

We set up the current value hamiltonian (we omit the time variable t from now on):

$$J = e^{-\rho t}(\alpha\ln(D) + (1-\alpha)\ln(C)) + \lambda[ra + wuH - P_C C - P_D D] + \mu[B_H(1-u)H]$$

with λ and μ representing the shadow prices of one unit of income and human capital respecitvely.

The first order and the transversality conditions :

$$\frac{\partial J}{\partial C} = 0 \Rightarrow e^{-\rho t}(1-\alpha)C^{-1} = \lambda P_C \tag{F1}$$
$$\frac{\partial J}{\partial D} = 0 \Rightarrow e^{-\rho t}\alpha D^{-1} = \lambda P_D \tag{F2}$$
$$\frac{\partial J}{\partial u} = 0 \Rightarrow \lambda w - \mu B_H = 0 \tag{F3}$$
$$-\frac{\partial J}{\partial a} = \dot{\lambda} \Rightarrow -\lambda r = \dot{\lambda} \tag{F4}$$
$$-\frac{\partial J}{\partial H} = \dot{\mu} \Rightarrow \lambda w u + \mu B_H(1-u) = -\dot{\mu} \tag{F5}$$
$$\lim_{t\to\infty} \lambda a = 0 \;;\; \lim_{t\to\infty} \mu H = 0$$

Equations [F1, F2] establish the necessary conditions for an otpimal allocation of income between the two consumption goods, implying that the marginal rate of substitution betewen C and D must be equal to the relative prices (static Euler equation).

Equation [F3] represents the static condition for the optimal allocation of human capital between education and market activity. In equilibrium the marginal benefit of one unit of human capital employed in the labor market (λw) must be equal to the marginal (opportunity) cost in terms of foregone human capital (μB_H).

Equations [F4] and [F5] represent the necessary dynamic conditions for the optimal time paths of consumption (Keynes-Ramsey rule) and human capital respectively.

2.1 Static Properties of the Demand Side

From now on we consider variables in real terms. To this purpose, we set the price of the traditional commodity to one $P_C = 1$.

As preferences are separable in D and C (Cobb-Douglas), the following static demand relationships must hold:

$$C = (1-\alpha)E \tag{2}$$
$$D = \alpha\frac{E}{P_D} \;;\; P_D = \left[\int_0^n p(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \tag{3}$$

With $E = C + P_D D = ra + wuH - \dot{a}$ representing the flow of (real) income. As far as the single variety is concerned, the following demand function must hold:

$$x(i) = \frac{\alpha E p(i)^{-\sigma}}{P};\; P = \int_0^n p(j)^{1-\sigma} dj \tag{4}$$

with P representing the price index of the aggregate commodity D.

2.2 Dynamic Implications

We explore the dynamic implications derived from the first order conditions [F1-F5].

We differentiate with respect to time the first two equations [F1] and [F2]. Using $-\lambda r = \dot{\lambda}$ we get ($\gamma_z \equiv \dot{z}/z$):

$$\gamma_C = r - \rho \tag{5}$$
$$\gamma_D + \gamma_{P_D} = r - \rho \tag{6}$$

Equations [5] and [6] require that the real expenditure on C and D grows at the same rate equal to the difference between the market interest rate and the subjective discount rate. Furthermore, differencing $E = C + P_D D$ with respect to time, and using [5] and [6] we get that the time path of total real expenditure is driven by the difference between r and ρ :

$$\gamma_E = r - \rho$$

As varieties of the composite commodity enter symmetrically the utility function, in equilibrium must be verified that $x_i = x$ and $p_i = p$ for any $i \in [0, n]$. Therefore equation [3] rewrites:

$$D(t) = \left[\int_0^n x(i,t)^{\epsilon} di\right]^{1/\epsilon} = n^{1/\epsilon}x \tag{7}$$
$$P_D = \left[\int_0^n p(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} = n^{\frac{\epsilon-1}{\epsilon}}p \tag{8}$$

Time differencing [7] and [8] obtains:

$$\gamma_D = \frac{1}{\epsilon}\gamma_n + \gamma_x$$
$$\gamma_{P_D} = \frac{\epsilon-1}{\epsilon}\gamma_n + \gamma_p$$

Finally, differencing [F3] and applying [F4] and [F5] we get:

$$\gamma_w = r - B_H \tag{9}$$

3 The Supply Side

We consider a three sector economy. Each sector is characterized by a constant returns technology, which employs human capital as the sole factor of production.

There exists a traditional industry, producing a homogenous consumption good in a perfectly competitive environment and a monopolistic sector offering, at any time t, n differentiated varieties. New varieties are introduced into the market through investments in the R&D sector.

3.1 The Traditional Sector

The undifferentiated good C is produced by a single representative firm according to the following technology:

$$C = B_C H_C;\ H_C = u_C H$$

with H_C representing the stock of human capital employed and B_C the exogenous factor's productivity. H_C is defined by the fraction u_C of the total stock of human capital available (H) allocated to the production of the C good. Profit maximization implies the following supply schedule:

$$w_C = B_C$$

with w_C defining the real wage rate paid by the firm. Given equation [2], the market clearing condition determines the following equilibrium quantity of human capital (H_C) employed in the traditional sector:

$$H_C = (1-\alpha)E\frac{1}{w_C} \tag{10}$$

3.2 The Advanced Sector

Firms in the advanced sector operate within a market of monopolistic competition. Each firm manufactures a single brand, retetainig a perpetual monopoly power over the variety it produces. Producer i maximizes profits, subject to a constant returns technology which employs human capital as the only factor of production. Thus, given the exogenous human capital productivity, the relevant marginal cost reflects only the unit wage rate (w_x) payed to the fraction of human capital employed in the i-th sector. The technology of the i-firm:

$$x(i) = B_x h(i) = B_x u(i) H$$

B_x measures the productivity of the human capital, $h(i)$ is the fraction ($u(i)$) of H employed in the production of the i-th variety.

The optimal price setting rule implies a constant mark-up over marginal cost:

$$p(i) = \frac{1}{\epsilon}\frac{w_x}{B_x} \tag{11}$$

If the wage rate $w_x(i)$ is uniform across sub-sectors[4], then $p(i) = p$ for $i \in [1, n]$. Given the optimal price rule [11] and the demand function [4], we derive the equilibrium quantity of each of the n varieties available in the market:

$$\begin{aligned} x(i) &= \frac{\alpha E p^{-\sigma}}{P};\ P = \int_0^n p^{1-\sigma} dj = n p^{1-\sigma} \\ x(i) &= x = \frac{\alpha E p^{-\sigma}}{n p^{1-\sigma}} = \alpha(H)\frac{E}{np} \\ x &= \alpha(H)\frac{B_x E \epsilon}{n w_x} \end{aligned}$$

and the demand of human capital:

$$x = \alpha(H)\frac{\epsilon E B_x}{n w_x};\ x(i) = B_x h(i) = B_x u(i) H$$
$$h(i) = \alpha(H) E \frac{\epsilon}{n w_x} \tag{12}$$

Integrating [12] over varieties we derive the total stock of human capital employed in the D sector:

$$H_D = \int_0^n h(i) di = \alpha(H) E \frac{\epsilon}{w_x} \tag{13}$$

Given [11] and [12], we calculate the per brand operating profits:

$$\begin{aligned} \pi(i) &= \pi = \left(p - \frac{w}{B_x}\right) x \\ \pi &= \left(\frac{1}{\epsilon}\frac{w}{B_x} - \frac{w}{B_x}\right)\alpha\frac{\epsilon E B_x}{n w} \\ \pi &= E\frac{(1-\epsilon)\alpha}{n} \end{aligned}$$

[4] As in equilibrium the individual must be indifferent about the allocation of his stock of human capital among alternative uses, a uniform wage rate in the economy is a necessary condition for an equilibrium in the labor market.

or, in terms of elasticity of subsitution:

$$\pi(i) = \pi = E\frac{\alpha}{\sigma n}$$

profits depends positively on the aggregate expenditure share allocated to the commodity D, negatively on the size of the market and negatively on the degree of subsitution between varieties.

3.3 The Innovation Sector

The innovation sector is competitive. New blueprints are produced according to the following constant returns technology:

$$\dot{n} = B_n H_n = B_n u_n H \tag{14}$$

H_n represents the amount of human capital employed in the innovation sector, B_n measures the productivity of the the skilled labor in the R&D activity. Equation [14] says that H_n units of human capital determine a flow of new blueprints per interval of time dt equal to $dn = B_n H_n dt$, bearing a cost of $w_n H_n dt$. As the R&D sector is competitive a zero profit condition must hold (free entry condition). This implies that the cost of creating a new blueprint must equalize the discounted value of profits associated to the new variety:

$$\frac{w_n}{B_n} = V_n \ \ ; \ \ V_n(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}\pi(j,\tau)d\tau \ ; \ \tau > t$$

V_n is, equivalentely, the patent price. so that $V_n = \frac{w_n}{B_n}$ represent the price - marginal (average) cost equality.

4 Static General Equilibrium

As far as the labor market is concerned, since human capital is pefectly homogeneous it must be paid the same wage rate in equilibrium:

$$w_C = w_x = w_n = w$$

Moreover, the total amount of human capital required by the three sectors must be equal to the fraction of human capital devoted to production activities:

$$H_C + H_D + H_n = (u_C + u_D + u_n)H$$

Firms operating in the R&D sector finance new investment projects through equity issuance. Since the R&D effort successfully results in a new variety with certainty, stocks and bonds are perfect substitutes. As a result, the process of new varieties entering the market determines the total value of asset held by the representative individual to coincide with whole stock issued by the firms:

$$a = nV_n \tag{15}$$

Equation [15] implies that the two assets must pay the same rate of return. The total return accruing to an investment V_n $(\dot{V}_n + \pi)$ must be equal to the return obtained by investing the same amount V_n in the riskless asset. Thus the price arbitrage condition writes as follows:

$$V_n r = \dot{V}_n + \pi$$

5 Steady State Analysis

A balanced growth path is defined by a common (constant) growth rate driving the long run evolution of H and n. Given our particular specification of preferences, a steady state equilibrium implies that the expenditure shares allocated to the consumption goods C and D are constant. As a consequence, preferences display homotheticity in the long run.

From Eq. 1, a constant growth rate of H implies that the fractions of human capital employed in market and non market activites, u^* and $(1 - u^*)$, are constant. Since $u^* = u_C^* + u_D^* + u_n^*$ is constant, also the fractions of H (u_C^*, u_D^*, u_n^*) allocated to alternative market uses must be constant.

Given the uniform wage condition, with $w = B_C$ implied by the price normalization, equilibrium in the goods markets (Eq. 10 and 13) gives:

$$\gamma_H = \gamma_E \tag{16}$$

Thus, from Eq. 16 and Eq. 9 we obtain the long run growth rate of human capital:

$$\gamma_H = B_H - \rho \tag{17}$$

Given Eq. 17 we derive the optimal steady state fractions of human capital allocated to market activity u^* and to human capital accumulation $(1 - u^*)$:

$$\frac{\dot{H}}{H} = B_H[1 - u] = B_H - \rho$$
$$u^* = \frac{\rho}{B_H}$$
$$(1 - u^*) = \frac{B_H - \rho}{B_H} \tag{18}$$

We now turn to the R&D sector. If in the long run $\dot{n} > 0$, then $\frac{w}{B_n} = V_n$[5]. Differencing we obtain:

$$\dot{V}_n = \gamma_w V_n$$

Using the arbitrage condition $V_n r = \dot{V}_n + \pi$, with $\pi = E\frac{\alpha(1-\epsilon)}{n}$, and Eq. 9 we rewrite the previous condition:

$$\begin{aligned} V_n r &= \gamma_w V_n + \pi \\ V_n(r - \gamma_w) &= \pi \\ V_n(r - r + B_H) &= \pi \\ V_n B_H &= \pi \end{aligned} \tag{19}$$

since single firm profits in the advanced sector must be constant in the steady state, Eq. 19 implies that:

$$\begin{aligned} \dot{V}_n &= 0 \\ r &= B_H \end{aligned}$$

and

$$\gamma_n = \gamma_E = B_H - \rho$$

In the steady state the rate of growth of real expenditure equals the rate of growth of n (and H). Therefore the ratios E/n and E/H are constant in the long run.

From Eq. 19 and the price marginal cost equality condition $w = V_n B_n$ we derive the E/n ratio:

$$\frac{E}{n} = \frac{wB_H}{(1-\epsilon)\alpha B_n} \tag{20}$$

To compute the H/n ratio, lets recall the human-labor market equilibrium condition:

$$uH = u_C H + u_D H + u_n H$$

[5] $w/B_n > V_n$ means that the marginal cost of a new blueprint exceeds the flow of profits that can be generated by the new variety. This would imply $\dot{n} \leq 0$.
$w/B_n < V_n$ would shift the whole stock of human capital to the R&D sector, determining a zero growth rate in the commodity sectors.
Therefore, in a steady state equilibrium must be verified $w/B_n = V_n$ with $\dot{n} > 0$.

Given technology in the R&D sector, to sustain a positive $\dot{n}$ the following input of human capital is needed:

$$\begin{aligned} H_n &= u_n H = \frac{\dot{n}}{B_n} \\ u_n H &= \frac{\dot{n}}{B_n}\frac{n}{n} = \gamma_n \frac{n}{B_n} \end{aligned}$$

We already know that in steady state $\gamma_n = B_H - \rho$, so Eq. 21 can be rewritten:

$$u_n H = n\frac{B_H - \rho}{B_n} \tag{21}$$

From the human-labor market equilibrium condition, taking into account Eq. 21 and Eq. 18:

$$\begin{aligned} uH &= u_C H + u_D H + u_n H \\ \frac{\rho}{B_H} H &= (1-\alpha)\frac{E}{w} + \alpha\epsilon\frac{E}{w} + n\frac{B_H - \rho}{B_n} \end{aligned} \tag{22}$$

Substituting from Eq.[20] E/w into Eq. 22 we obtain the steady state H/n ratio:

$$\begin{aligned} \frac{H}{n} &= \frac{B_H}{\rho}\left[\frac{(1-\alpha+\alpha\epsilon)B_H}{(1-\epsilon)\alpha B_n} + \frac{B_H - \rho}{B_n}\right] \\ \frac{H}{n} &= \frac{B_H}{\rho}\frac{B_H - \rho\alpha(1-\epsilon)}{(1-\epsilon)\alpha B_n} \\ H &= \Phi n;\ \Phi = \frac{B_H}{\rho}\frac{B_H - \rho\alpha(1-\epsilon)}{(1-\epsilon)\alpha B_n} \end{aligned}$$

Given Eq. 21 we obtain the steady state level of u_n:

$$u_n = \frac{\alpha(1-\epsilon)(B_H - \rho)}{B_H - \rho\alpha(1-\epsilon)}\frac{\rho}{B_H}$$

Now it is possible to calculate the steady state values of u_C and u_D . As $u_C H = (1-\alpha)\frac{E}{w}$ and $u_D H = \alpha\epsilon\frac{E}{w}$ it follows that:

$$u_C = \frac{(1-\alpha)}{\alpha\epsilon} u_D$$

Substituting into the labor-human capital equilibrium condition, we get:

$$u_D = \frac{\alpha\epsilon}{1-\alpha+\alpha\epsilon}\left[\frac{\rho}{B_H} - \frac{1}{\Phi}\frac{B_H-\rho}{B_n}\right] = \frac{\alpha\epsilon\rho}{B_H-\rho(1-\epsilon)\alpha}$$
$$u_C = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\left[\frac{\rho}{B_H} - \frac{1}{\Phi}\frac{B_H-\rho}{B_n}\right] = \frac{(1-\alpha)\rho}{B_H-\rho(1-\epsilon)\alpha}$$

Finally, we turn to the steady state growth rate of C, x and D. It is straightforward to derive:

$$\gamma_C = (B_H-\rho)$$
$$\gamma_x = 0$$
$$\gamma_D = \frac{1}{\epsilon}(B_H-\rho)$$

6 The effect of Competition and Demand Composition on the Sectoral Distribution of Skills

In this section we want to investigate the impact of competition and of the composition of demand on the steady state allocation of effective labor across sectors. The price rule in the monopolistic sector implies a constant mark-up over the marginal cost, given by $(1-\epsilon)/\epsilon$, where ϵ represents the average degree of subsitution betwee any pair of variety in D. Therefore, $1/\epsilon$ can be viewed as a measure of the market power enjoyed by firms operating within the luxury goods sector. Specifically, a higher value of ϵ implies a lower demand elasticity faced by the single firm, a lower mark-up, and, in turn, an increased competitiveness within the market. Conversely, a lower ϵ increases the distance between varieties, which enhances the market power of firms and reduces the degreee of competition among firms. On the demand side, to see how the composition of real expenditure affects the distribution of production resources, we simply consider the effect of α on the various steady state fractions of working time. A higher value of α implies that a greater share of real income purchases the refined composite commodity, and a lower fraction is devoted to the traditional ordinary good.

It is easy to check the signs of the following derivatives:

$$\frac{\partial u_C}{\partial\epsilon} < 0 \; ; \; \frac{\partial u_C}{\partial\alpha} < 0$$
$$\frac{\partial u_D}{\partial\epsilon} > 0 \; ; \; \frac{\partial u_D}{\partial\alpha} > 0$$
$$\frac{\partial u_D}{\partial\epsilon} > 0 \; ; \; \frac{\partial u_D}{\partial\alpha} < 0 \; ; \; u_E = u_C + u_D$$
$$\frac{\partial}{\partial\epsilon}\frac{H}{n} > 0 \; ; \; \frac{\partial}{\partial\alpha}\frac{H}{n} < 0$$
$$\frac{\partial u_n}{\partial\epsilon} < 0 \; ; \; \frac{\partial u_n}{\partial\alpha} > 0$$

An increase in the elasticity of substitution between varieties, reduces the market power of monopolists. The increased degree of competition results in lower mark-up and lower selling prices for the available varieties. As C and D are normal goods, the change in relative prices induces a revision in the individual optimal consumption plan, through a lower demand for the traditional good and an increased demand for D. The reduced profitability in the luxury goods market, however, lowers the incentive to innovation. A lower fraction of the existing productive resources will be devoted to innovate. As a result, the steady state ratio between human capital and knowledge decreases. This implies that the increased demand for the composite good occurs through a higher demand for any single variety available. The opposite effect on u_C and u_D do not cancel out. Since ϵ does not affect in our setup the steady state growth of n, the responsiveness of u_D to a change in ϵ offsets the negative impact on u_C, so that the overall effect of an increased competitivenss in the monopolistic sector is a higher fraction of human capital devoted to production in the consumption sector.

7 Transitional Dynamics

We consider the evolution of real variables along the transition to their steady state values.

With α constant the model economy does not display any transition[6]; i.e. starting from the initial conditions, the growth process immediately jumps to its long run balanced growth path. Second, we investigate the short run properties of the model, as the influence of human capital accumulation on consumption is explicitely taken into account.

We assume that, the accumulation of human capital is accompanied by (unconsious) changes in α. As a consequence, the individual moves a higher share of income to more sophisticated goods.

We differentiate with respect to time the first order conditions [F1] and [F2]. Taking into account the dynamic of $\alpha(H(t))$, by using [F4], and considering that $r = B_H$ and $\dot{H} = (B_H-\rho)H$ we obtain:

[6] See the Appendix for a formal proof.

$$\gamma_C = (B_H - \rho) - \frac{\alpha'}{1-\alpha}\dot{H} = (B_H - \rho)\left(1 - \frac{\alpha'}{1-\alpha}H\right) \tag{T1}$$

$$\gamma_D + \gamma_{P_D} = (B_H - \rho) + \frac{\alpha'}{\alpha}\dot{H} = (B_H - \rho)\left(1 + \frac{\alpha'}{\alpha}H\right) \tag{T2}$$

We differentiate $E = C + P_D D$:

$$\gamma_E = (1-\alpha)\gamma_C + \alpha(\gamma_D + \gamma_{P_D})$$

subsituting to γ_C and $\gamma_D + \gamma_{P_D}$ their respective expressions [T1] and [T2] we obtain:

$$\gamma_E = B_H - \rho \tag{T3}$$

Changes in α do not affect the time path of aggregate consumption, but just the composition of total real expenditure. Since α approaches a constant for H going to infinity, α' asymptotically tends to zero. Therefore, the transition of C and D is characterized by a growth rate of the traditional good below $(B_H - \rho)$, which gradually increases to its long run growth path, and a growth rate of the aggregate commodity D above $(B_H - \rho)$, which gradually moves downward its steady state level.

Since w and r are constant, from the arbitrage condition we get that the ratio $\frac{E\alpha}{n} = \frac{B_C B_H}{B_n(1-\epsilon)}$ is constant as well. Differentiating $\frac{E\alpha}{n}$ with respect to time we obtain the transition growth rate of n:

$$\gamma_n = \gamma_D + \gamma_{P_D} = (B_H - \rho)\left(1 + \frac{\alpha'}{\alpha}H\right) \tag{T4}$$

γ_n exceeds the steady state growth rate by the factor $\frac{\alpha'}{\alpha}\dot{H}$. Therefore, along the transition the change in the composition of total expenditure stimulates the R&D activity.

Finally, given Eq.[T4] and the equilibrium condition on the labor-human capital market, we derive the fraction of labor allocated to the various sectors. We know that:

$$\frac{\dot{n}}{B_n} = u_n H$$

from Eq. [T4]

$$u_n H = \frac{(B_H - \rho)}{B_n}\left(1 + \frac{\alpha'}{\alpha}H\right)n$$

Considering the equilibrium condition in the labor market $uH = (H_C + H_D + H_n)$ we get:

$$\frac{\rho}{B_H}H = E\frac{(1-\alpha+\alpha\epsilon)}{B_C} + n\frac{B_H - \rho}{B_n}\left(1 + \frac{\alpha'}{\alpha}H\right)$$

which allow us to determine the H/n ratio:

$$\frac{H}{n} = \frac{B_H}{\rho}\left[\frac{(1-\alpha+\alpha\epsilon)B_H}{(1-\epsilon)\alpha B_n} + \frac{B_H - \rho}{B_n}\left(1 + \frac{\alpha'}{\alpha}H\right)\right]$$

Given H/n we obtain the fraction of human capital allocated to the innovation sector:

$$u_n = \frac{n}{H}\frac{(B_H - \rho)}{B_n}\left(1 + \frac{\alpha'}{\alpha}H\right) = \frac{\rho(1-\epsilon)\alpha(B_H - \rho)\left(1 + \frac{\alpha'}{\alpha}H\right)}{B_H[(1-\alpha+\alpha\epsilon)B_H + (1-\epsilon)\alpha(B_H - \rho)\left(1 + \frac{\alpha'}{\alpha}H\right)]} \tag{T5}$$

From [T5] it is easy to check that the fraction of effective labor allocated to the innovation sector exceeds the steady state level.

We know that: $u_C = \frac{1-\alpha(t)}{\alpha(t)\epsilon}u_D$. Therefore, from the labor market clearing condition

$$\frac{\rho}{B_H} = u_D\left[\frac{1-\alpha+\alpha\epsilon}{1-\alpha}\right] + u_n$$

the transition levels of u_D and u_C are easily derived:

$$u_D = \left[\frac{\rho}{B_H} - u_n\right]\frac{1-\alpha}{1-\alpha+\alpha\epsilon} \tag{T6}$$

$$u_C = \left[\frac{\rho}{B_H} - u_n\right]\frac{\alpha\epsilon}{1-\alpha+\alpha\epsilon} \tag{T7}$$

The intensifying preference for the advanced consumption commodity, as the individual tastes are shapened by education, prompts the individual to allocate a higher fraction of his productive human capital to the R&D sector. This crowds out the resources devoted to the production of the traditional ordinary good, and also, somewhat surprisingly, the resources devoted to the production of the existing varieties (equations [T7] and [T6])[7]. In other words, due to the external effect of education on preferences, the individual is willing to allocate

[7] It is easy to check that the quantity of each variety purchased is constant along the transition. Differentiate with respect to time the expression for x. With [T3] and [T4] the $\dot{x} = 0$ result is obtained.

higher fractions of his income to purchase the refined commodity, only to the extent that it offers a higher number of varieties. This seems consistent with the character of present consumerism. As the society evolves towards more mature stages of development, the labor productivity, and consequently the purchasing capacity, increases. In the present context this aspect of the observed evolution of modern economic systems is described by the ongoing process of human capital accumulation. At the same time, more advanced societies involve patterns of consumption, in which the hedonistic motivation tends to dominate the primary needs of the material subsistence. In our model this changing style of consumption reflects the process of human capital accumulation. More educated individuals not only are more productive workers, but they are also more demanding and discriminating consumers. The increased willigness to spend relatively more for the luxury commodity, however, does not lead to the purchasing of more quantities of the existing varieties, but to the purchase of *new* types. In this respect our result conforms to the wisdom that the lure for the new constitutes a distinctive feature of modern consumerism.

8 Concluding Remarks

Endogenous growth literature retains the classical setup, in which consumption activity plays a passive role. It simply represents the reward, in terms of utility, resulting from the accumulation of productive resources. As a result, it is the real side of the economy, such as work, accumulation and production, that causes, in a logical sense, consumption. This view can be shared in many respects, but it should be underlined, that it mainly reflects the single homogeneous good setting and the metaphor of the representative agent. In other words, with no hetoregeneity characterizing consumption, the *level* of the consumer expenditures passively reflects the real decisions on the supply side.

Indeed, the pattern of real world consumption results from the complex social and economic dynamics, involving individuals' heterogeneity, different technologies for the production of different goods and so on. This process continually directs the choices of firms, particularly as far as the strategies of product differentiation and innovation are concerned. In the present paper we have retained the representative agent set up, but we have introduced heterogeneity in the consumption activity, assuming that different commodities embody a different degree of R&D sophistication. In particular, we have assumed that individuals derive utility from the consumption of two goods. A traditional or ordinary sector, such as "food", that does not require any innovation effort, and a composite "refined" commodity, whose variety dimension can be increased through R&D investment.

The model integrates human capital accumulation and innovation activity within an endogenous growth framework. We depart from the previous literature, in that we consider the R&D effort aimed at increasing the consumer product variety instead of the stock of intermediate factors of production. We do not assume any spillover effect, so the engine of growth relies completely

upon the accumulation of human capital.

We have employed this framework to analyze the effect of the composition of the consumption expenditure on the steady state labor market allocation of resources. Our conclusions are that an increase in the share allocated to the purchase of the innovative good, stimulates the R&D activity, and the overall stock of knowledge relative to the stock of human capital, and decreases the labor effort employed in the consumption goods industries.

Moreover, we have investigated the role of the composition of demand along the transition towards the steady state, modelling a peculiar effect of education on preferences. In particular, we have assumed that the accumulation of human capital increases the liking for the refined commodity. As a result, the individual is willing to allocate a higher fraction of his income resources to purchase the composite commodity. This effect on the relative expenditure share is not rationally percieved by the individual, as we consider the income reward the only motivation determinig the investment in education. But it is percieved by the market, as it changes the incentive to innovate as a consequence of the changing demand conditions. The transition is therefore characterized by a growth rate of innovation above the steady state level; i.e. as the economy moves towards more hedonistic pattern of consumption, the accumulation of knowledge tends to speed up with respect to the accumulation of human capital.

The model represents a first attempt, aimed at investigating the role of the composition of private consumption in a context of endogenous growth. It can be explicitely solved, and this encourages future research in at least two directions. First, to explore the possibility of extending the model to allow a richer interaction betwen transition and long run dynamics. That is, how the evolving preferences of individuals and the resulting changing composition of demand can influence the steady state growth rate of the economy. Second, if the composition of demand does matter as a structural feature of the economy, then the fiscal policy, with particular reference to public consumption, should be expected to affect the economy not only through the traditional channel of the level of expenditure, but also through the specific composition of public consumption choosen by the goverment.

APPENDIX

Given constant the real wage, from eqs. [F3, F4, F5] it follows that λ/μ is constant, which, in turn implies that $r = B_H$. Having r set to its steady state value and α constant, the time evolution of C, D and E, implied by the first order conditions [F1, F2], immediately follows the balanced growth path:

$$\gamma_C + \gamma_{P_C} = \gamma_D + \gamma_{P_D} = \gamma_E = B_H - \rho \tag{A1}$$

Consider the arbitrage condition $V_n r = \dot{V}_n + \pi$. Given $r = B_H$, $w = B_C$, $V_n = B_C/B_n$ it follows that $\dot{V}_n = 0$, which in turn implies that E/n is constant:

$$V_n r = \dot{V}_n + \pi\ ;\ \ r = B_H,\, V_n = B_C/B_n\ ,\ \pi = \frac{E}{n}\alpha(1-\epsilon)$$
$$\frac{B_C B_H}{B_n}\frac{1}{\alpha(1-\epsilon)} = \frac{E}{n} \tag{A2}$$

Eq. [A2] determines that the transitional growth rates of real consumption expenditure E and of the number of new varieties entering the market must be equal:

$$\gamma_E = \gamma_n \tag{A3}$$

Now, turn to the dynamic resource constraint $\dot{a} = ra + wuH - E$. Taking into account that $a = nV_n$, it can be written:

$$\dot{n} = B_H n + B_n uH - \frac{B_n}{B_C}E \tag{A4}$$

From Eq. [A3] $n = n(0)\ \exp[(B_H - \rho)t]$ and $E = E(0)\ \exp[(B_H - \rho)t]$. Then, eq. [A4] rewrites:

$$uH = \exp[(B_H - \rho)t]\left(\frac{E(0)}{B_C} - \frac{\rho n(0)}{B_n}\right) \tag{A5}$$

Given the motion of human capital $\dot{H} = B_H(1-u)H$, we obtain from Eq. [A5] the following first order differential equation:

$$\dot{H} - B_H H + \exp[(B_H - \rho)t]\left(\frac{E(0)}{B_C} - \frac{\rho n(0)}{B_n}\right) = 0$$

whose solution is:

$$H(t) = Cost\exp(B_H t) + \exp[(B_H - \rho)t]\left(\frac{E(0)}{\rho B_C} - \frac{n(0)}{B_n}\right) \tag{A6}$$

with $Cost$ defining the constant of integration. We substitute Eq. [A6] into the trasversality condition $\lim_{t\to\infty}\mu H = 0$, taking into account that $\mu(t) = \mu(0)\exp(-B_H t)$:

$$\lim_{t\to\infty}\exp(-B_H t)\left[Cost\ \exp(B_H t) + \exp[(B_H - \rho)t]\left(\frac{E(0)}{\rho B_C} - \frac{n(0)}{B_n}\right)\right] = 0$$
$$\lim_{t\to\infty}\left[Cost + \exp(-\rho t)]\left(\frac{E(0)}{\rho B_C} - \frac{n(0)}{B_n}\right)\right] = 0$$

Since $\rho > 0$, the second term inside the brackets converges to zero. Hence, the transversality condition requires the constant of integration to be zero. Therefore, Eq. [A6] implies that:

$$H(t) = \exp[(B_H - \rho)t]\left(\frac{E(0)}{\rho B_C} - \frac{n(0)}{B_n}\right)$$
$$\dot{H} = (B_H - \rho)H$$
$$\gamma_H = \gamma_n = \gamma_E$$

i.e. the model does not display any transitional dynamics.

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