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PHYSICAL CAPITAL INVESTMENT IN HUMAN CAPITAL ACCUMULATION IN THE CONTEXT OF A MODEL OF GROWTH *A'LA LUCAS*

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ABSTRACT

This paper deals with endogenous growth and human capital accumulation. We present a review of previous studies, particularly stressing the distinction between rival and non-rival uses of human capital. Our own model in section 2 is based on Lucas 1988 model of human capital, and it modifies the way human capital is accumulated, by using a Cobb-Douglas production function, in which physical investments in education are added to the usual arguments considered by the literature. The framework is a dynamic intertemporal maximization in which agents can allocate time between working, leisure and accumulation of human capital. If the fraction of capital devoted to education is made function of time, we obtain three results. First: increasing investment in human capital has a double positive effect on growth, whereas taking away resources from education to physical capital has a double negative effect. Second: the elasticity of these increases depends crucially on the internal productivity of human capital, that is, how much human capital stock today is important for the production of human capital tomorrow; this is in line with those studies who denies the conditional convergence predicted by exogenous growth models. Third: in steady state, human capital can grow faster than physical capital in competitive equilibrium, if the magnitude of the external effect is offset by the effect caused by the increase of the investment in education (which in turn depends crucially on the elasticity).

JEL Classification: O40, I20
Keywords: endogenous growth, human capital, investment in education.

This paper was firstly presented at the Ferrara workshop in memory of Prof.Antonio D'Atri, on the 9th October 2003, at the Faculty of Economics of the University of Ferrara. And to Antonio D'Atri, whose intellectual strength and dedication to young students is still in our eyes and our hearts, this paper is dedicated to.

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INTRODUCTION

In the last two decades, there has been a particular emphasis on the fundamental role of human capital formation for economic growth; particularly, the role of universities has been pointed out as crucial for the production and development of skills, technological innovations, human capital. This aspect has gained further relevance, since the strenghtening of globalization processes: the way forward for western developed economies, in fact, seems to be to focus on high-value-added production processes, leaving emerging developing countries free to fully exploit the comparative advantage in more traditional and unskilled-labour intensive sectors like texile, clothing, shoes, basic mechanic, agricolture and so on. This resulting new international division of labour on global scale leaves industrialized economies with the need to massively invest in reserch and development, so that generation of new ideas can lead to creation of new business, bring about substantial gains in productivity and ultimately drive the dynamics of economic growth.

In this paper we would like to stress an aspect which, although often mentioned, has failed to be explicitly analysed within the traditional literature: the physical investment in human capital formation. It is in fact our belief that the production of skills, whose crucial role in growth and development has long been established, does not come spontaneously. Indeed, it is a complex process that needs to be continuosly sustained and enhanced by physical investment: that could take the form of infrastructure and services, research grants and more generally investment in the quality of teaching, universities and research. How much resources are annualy devoted to production of human capital represents a crucial policy variable, whose general equilibrium effects have to be taken care of whenever we tackle the problem of growth.

The model here presented claims to be a very first step in the complex analysis of the conditions under which an economy chooses to allocate its capital stock between the production of physical capital and the accumulation of human capital; we aim at determining how growth is affected by two factors: human capital (through the enhancement of quality of labour) and physical capital investment into the enhancement of human capital itself[1]. The two above effects are not considered separately: the quantity of human capital which apparently depends exclusively on the time-allocation choices of agents, it is indeed positively affected by investment in education, for example because the expectation of a quality-improvement brought about by the increased

[1] Being the education system largely public (especially in Europe) this might easily be considered a policy variable directly controlled by governments (European institutions, national and local governments). Nevertheless, the role played by private investment in education (such as training expenditure by firms) should not be forgotten, although its empirical measurament is not easy.

expenditure in the sector may attract more students. Moreover, the increased knowledge stock ultimately requires an augmented stock of human capital capable to manage it within the production processes; it follows a perspective increase in demand for human capital, and thus an increase in expected earnings for advanced skills. This latter effect, in particular, is likely to affect the time-allocation choice of agents.

The model that we built is therefore an "augmented" Lucas' model, with the component of leisure, added to work and education as a possible allocation of time and the explicit consideration of investment in education[2]. Physical capital in order to produce human capital is indeed not a new idea: Rebelo (1991) used it in a very influential paper, in order to analyse the effect of taxation on the rate of growth.

Section 1 presents a review of previous studies. We distinguish between a general discussion on rival and not-rival sources of endogenous growth (subsection 1.1) and a survey on how economic literature has analysed and measured the effect of human capital and university expenditure on economic growth (subsection 1.2). Section 2 presents our model, obtained by enriching the Lucas'1988 model of human capital and economic growth, by the addition of more realistic hypothesis on human capital formation. Section 3 presents some empirical testing, although at this stage strongly undermined by lack of data. Concluding observations follow.

Section 1: Human capital and economic growth in the economic literature

1.1 Human capital and the distinction between rival and not-rival knowledge as sources of growth.

The problem of growth was first tackled in the 50's by two famous paper by Swan (1956) and Solow (1956), who marked the beginning of neo-classical models of growth. It is not always remembered that they used an aggregate analyitical framework built up by Clark in "*The Distribution of Wealth"* (1899), a central paper in the history of economic thought since it represented one of the most completed systematic departure from the classical theory of distribution, giving birth to the neo-classical one: each factor of production receives a quota of national income proportional to its contribution, in turn measured by its marginal productivity. However, the central conclusion of early neo-classical model of growth was that the eventual stable steady-state growth rate of aggregate output could be described as the sum of the growth rate of population (or employment) and the rate of labor-augmenting technological progress. Therefore, the

[2] As we will specify later, the inclusion of leisure is due to Robert Solow (2000).

growth rate of output per (employed) person was given by the rate of technological progress, which was taken to be exogenous in the model.

In the Solow model, therefore, growth has been explained with "material" factors and by technological progress. The latter is taken as a "residual", that is a factor which is not explained by accumulable factors of production ; research interest therefore focused on possible components of such residual and, in particular, on ***human capital*** and ***not-embodied knowledge***[3]. There is widespread consensus, on a theoretical ground, on the fundamental role played by these factors in the generation of sustained growth, but the exact way it acts on it has been frequently questioned; we find it particularly useful to stress the differences between the two. Human capital consists of the acquired abilities, skills and knowledge of individual workers and thus is rival and excludable just like the conventional economic goods; for example, the fact that an engineer's full effort is being devoted to one specific activity precludes the simultaneous use of his skills in some other activity. In this interpretation, human capital has been considered a factor improving the quality of labour, therefore directly entering in the production function (Mankiw, Romer, Weil, 1992). On the other hand, a new idea is certainly non-rival (although its degree of excludability can vary considerably[4]), since its use in one activity does not certainly preclude its use in another activity at the same time but in a different place. In this view, human capital does not indeed disappear, but has rather been seen as a factor that allows the introduction (Romer 1990) and absorption of technological progress or not-embodied knowledge, as we defined it (Nelson, Phelps, 1966).

The attention on these fundamental issues regarded both empirical and theoretical side. On the theoretical one, a turning point was determined by the theory of endogenous growth, attempting to endogenize what Solow had been taken as exogenous; human capital is indeed the crucial factor of these models. Models of endogenous growth can be distinguished in two broad streams: those who use human capital as a productive factor (thereby believing that it contributes to production by enhancing the quality, and thus the productivity, of labour) and those who, sometimes in addition, also consider human capital through the role played in the generation of new technology (in this view, human capital has also the indirect function of allowing a quicker generation of non-rival technological progress).

Lucas' model (Lucas, 1988) belongs to the first stream. The novelty in his view consists in the endogenization of human capital choice, and in the consideration of externalities generated by the accumulation of human capital and (in his more sophisticated model) by the learning process;

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[3] By "not-embodied knowledge" we basically mean codified ideas that drive technological progress, although not directly embodied in human beings.

[4] For instance, a new mathematical formula is surely not excludable at all, whereas a new satellite technology can be highly excludable.

these are the factors at the origin of the growth. In Romer's model (Romer, 1990) endogenizations of growth is based on the choice of individuals between R&D sector and productive sector. Therefore Romer belongs to the second "stream", as in his view human capital may influence production by determining the capacity of nations to innovate new technologies suited to domestic production.

It is however our belief that, beyond the crucial distinction above outlined, human capital in the sense of improved skills of individuals has to be considered the true engine of growth. In fact, generation of new ideas and technological progress is only made possible by an enhanced quality of human capital within the economy; in a sense, human capital is a necessary (although not certainly sufficient) condition for the generation of not-embodied knowledge. It is also worthwile to stress the peculiar role of universities in this context: they are either the place where higher educated people are formed (that is, human capital is produced) and the place where basic research is conducted and diffused (that is, abstract and not-directly-embodied knowledge is created and spread).

1.2 Empirical survey

Theoretical models of growth present a very high degree of abstraction; nevertheless it is possible to submit them to an empirical analysis. Therefore empirical literature on growth is abundant. The following is not a complete review of empirical studies on the growth theory, but we will focus mainly on "knowledge variables", that are mainly human capital and public expenditure in research, mainly citing those studies that may be considered as classics and remain at the core of the debate.

Thirtysix years after his famous paper, Mankiw, Romer and Weil (1992) decided "to take Solow seriously", testing his model with an OLS cross-section, considering three samples, the largest including 98 countries. The most important result they found is that differences in saving and population growth actually accounted for a large fraction of the cross-country variation of per-capita income, which is exactly what Solow model predicted. In this "basic" and original model human capital is not considered; nevertheless, being its contribution widely recognised, it may be considered an omitted variable. Therefore MRW augmented Solow model, by considering human capital as a multiplicative term in the aggregate production function, namely considering it as a production factor just as well as by labour and capital. The underlying hypotheses were that one unit of consumption can be transformed costlessy entirely into either one unit of physical capital or one unit of human capital and that human capital depreciated at the same rate of physical capital. In this model per-capita income depended on population growth and accumulation of physical and human capital, both depending on the quotas of savings devoted to each of them. The estimation results showed that human capital was a significant variable and that the fit of these regressions compared to the those deriving from the "basic" model was significantly improved. Moreover, MRW also find that Solow's prediction regarding conditional convergence is confirmed by their estimation and that the fit of the estimation itself is improved as soon as human capital is included. They consider this result a denial of the endogenous growth models prediction according to which, because of non-decreasing returns to the set of reproducible factors, countries need not to converge in income per capita even if they have the same preferences and technology.

Notwithstanding the good results, MRW estimates rises several questions. First of all, calculation of human capital accumulation is a very difficult issue and every reliable proxy is largely non accurate. In their estimation MRW use, as a proxy for the rate of human capital accumulation, the percentage of the working-age population that is in the secondary school. As the authors themselves underline, not only primary and higher education are ignored but, most importantly on a "theoretical" side, a very large part of investment in education takes the form of forgone labour earnings which present the additional difficulty of variation with the level of human capital. Moreover, productivity in terms of income is different with different kind of education (an engineer is on average more "productive" than a philosopher). To these problems it must be added the difficult comparability of quality of education across different countries.

While MRW estimated the level of per capita income as predicted by the Solow model at the steady state, Benhabib and Spiegel (1994) considered the starting point of the Solow-augmented model, that is the production function; they estimated the log differences of the Cobb-Douglas technology including human capital as a productive factor. In their results human capital always enters not significantly and almost always with a negative coefficient and this result is robust to a series of alternative specifications, also including ancillary variables, like political instability, the relative size of middle class in a country, etc. Therefore BS propose an alternative framework, based on the concepts developed by Nelson and Phelps (1966) and Romer (1990), already recalled in section 1.1. Human capital should be considered not as an additional input, rather education enhances the ability of a country to develop its own technological innovation (through human capital devoted to R&D, as in Romer's framework) and the ability to develop and implement technologies developed elsewhere at an exogenous rate (as Nelson and Phelps claim).

In this framework technological advances no longer depend on the growth but on the stock of human capital. Therefore the relevant variable in their estimates is the overall level of the log of human capital over the period. Including a catching-up term, that is a term considering initial income level (as country with a lower level of income are supposed to grow faster), human capital

enters significantly with the right positive sign. It is interesting to notice that without the catching-up term human capital is not significant and it has the wrong (negative) sign. This result is interpreted by the authors as evidence that the catching-up (and therefore conditional convergence) is a significant element in growth and that the importance of human capital consists mainly in helping to close the gap with advanced technologies. Therefore the role of human capital is to promote the adoption of foreign technologies rather than developing internal ones.
In BS' estimates the proxy for human capital stock is more complicated than MRW's one. Human capital stock is properly the average years of schooling in the labour force. But only enrolment ratio is generally available, therefore average years of schooling are obtained as a weighted average of past enrolment ratio in primary, secondary and tertiary education.

An important consideration follows from the framework implemented by BS: as the effect of human capital on growth acts through introduction and adoption of technology, those components of human capital more suited to treat technology gain more relevance: therefore scientific and technological education is more important than other forms. This is the way Lodde (1999) explains the Italian regional paradox: southern regions present higher average years of education but their economic performances are worse. One possible explanation is that in northern regions relative and absolute values of scientific educated people are higher. This different distributions of human capital reflect the different economy's structure: in southern Italy industry is weaker than in the North and the weight of public administration is higher. Because of unbalances between demand and supply of high educated people, many technological educated people work in public administration, where they may utilize their innovation potential only to a scarce extent. Consideration of quality of human capital, in the sense of quality of schools and kind of education is surely an important topic, perhaps not yet completely analysed.

We stated that engines of endogenous growth may be divided in human capital and not-embodied knowledge and that universities are the main source of both. Nevertheless, the effect on growth of public expenditure in education and research is another point not thoroughly examined , on a theoretical and empirical ground. Public expenditure in research affects technological advances and, through this, growth. On an empirical ground, the direct linkage has been analysed, considering the effect of public research on technological innovation, usually proxied with patents.
Furman, Porter, Stern (2002), who explicitly refer to Romer's model, found that the level of patents is positively affected by the share of R&D performed by universities. They also found that the level of abstract scientific knowledge, proxied with indicators of publications, has a positive impact on patenting. Sassu and Lodde (2002) distinguishing among private, university and other public

expenditure in research, found no effect of university research on patents, but a positive effect of other government expenditure in research. They also attempted to consider the quality of human capital, including in their regression the fraction of scientific engineers on total personnel in research, finding a positive, significant sign, even though very low, for this variable.

The approach of the knowledge production function (Griliches,1979) considered the relation between knowledge output (patents or innovation counts) and knowledge inputs (industry R&D and university research). The theoretical framework is based on the concept of local knowledge spillovers: academic research was considered to have a real effect on innovation on a local basis. The concept of knowledge spillovers is taken from the new theory of growth, but in this approach spatial interactions, therefore exchanges of tacit knowledge through personal relationships, networks of innovators, are particularly stressed. In this view therefore human capital is knowledge embodied in persons and fully exploited through personal relationships. In a cross-section across U.S. states, the impact of university research was calculated by Jaffe (1989) in his seminal paper as positive and significant, but this effect was not unanimously confirmed by other empirical studies, as Anselin, Varga and Acs (1999) report.

Concluding this review, we noted that the basic theoretical distinction between human capital as source of rival and not rival knowledge is reflected in empirical studies, which however do not offer well-defined evidence on the issue. Nevertheless, there is widespread consensus in all of them on the positive and significant effect of human capital accumulation on the dynamic of economic growth; more controversial is the effect of universities expenditure, although certainly much more work seems to be needed in that direction.

Section 2: A modification of Lucas framework: the explicit consideration of physical investment in human capital formation

2.1 Description of. the model

Maybe the most important contribution to the literature investigating the connection between human capital accumulation and economic growth comes from Robert Lucas[5], which actually opened the way for human capital macroeconomic models. Here we present (and modify) the Solow version of Lucas-model[6], which in turn entails a modification of the original work of Lucas insofar as it adds leisure in the choice of the allocation of time by consumers (while Lucas only considered studying time and working time). It is to be said that Solow includes this modification in order to

[5] Lucas (1988)

[6] Solow (2000)

prove that in this case the very same result of endogeneity of growth rate, achieved in the original version of the model, becomes highly doubt: in this paper we do not enter the issue, since as we will show, the problem arises in a later stage of the analysis with respect to the result that both Lucas and Solow derive and that we want to modify, and totally indipendently from it. Nevertheless, the contradiction highlighted by Robert Solow remains one of the most interesting issues in the field.

The general structure of Lucas'1988 model is like that of the standard neo-classical model in the "optimizing" version. Thus, the path of the economy is obtained by maximizing a utility integral:

$$\int e^{-\rho t} N(t) \left(c(t)^{1-\sigma} + al(t)^{1-\sigma}\right)/1-\sigma \qquad \text{with a : constant} \qquad [2.1]$$

where:
N : population
c(t): consumption per-capite
l(t): leisure time per-capite

In the original Lucas paper, the second argument of the individual utility function $[al(t)^{1-\sigma}]$ does not appear, since he assumers that any time that is not spent working is spent accumulating human capital; Solow introduces a little bit more of realism, by assuming also the presence of leisure time. The two constraints in the Lucas-Solow model regards the way the two state variables (human capital and physical capital) are accumulated:

$$N(t)c(t) + \overset{\circ}{K}(t) = K(t)^{\beta}\left[u(t)H(t)\right]^{1-\beta}\overline{H}(t)^{\gamma} \qquad [2.2]$$

It simply states that the aggregate consumption $N(t)c(t)$ plus investment $\overset{\circ}{K}(t)$ must equal total output, produced by a Cobb-Douglas production function whose input are physical capital stock $K(t)$ and human capital contribution $\left[u(t)H(t)\right]^{1-\beta}\overline{H}^{\gamma}$. In turns, the latter is made by the stock of human capital $H(t)$ weighted by the amount of time people spend working $u(t)$ and by an external effect $\overline{H}(t)$: if other people have accumulated human capital, the individual will be in fact more productive for any given amount of human capital that he has been accumulating. The bar over H indicates that this quantity is to be regarded as given for each individual maximizer, while for the

social planner H would have the exponent *(1-β-γ)* because he would internalize the externality[7]. Since we will be looking at the competitive equilibrium and not at the cooperative one, the external effect will be treated as a parameter at each stage.
The second constraint regards the accumlation of human capital:

$$\overset{\circ}{H} = \delta\, H(t)\, [1\text{-}l(t)\text{-}u(t)] \qquad \text{with } \delta \text{ constant} \qquad [2.3]$$

The above differential equations shows that the relative growth of human capital depends exclusively on the existing stock H and on the time people spend studying (the residual of working time and leisure time). So, according to Lucas, an economy accumulates human capital exclusively on the basis of how much human capital it has already accumulated and on the basis of time committed to acquisition of new skills and knowledge. Besides, from a mathematical point of view, it is evident that equation [3.3] is homogenous of degree two, and therefore exhibits very strong increasing returns to scale to his two inputs.

This paper is about an initial attempt to overcome the unrealism connected with equation [3.3]. We retain all the underlying hypothesis of Lucas model, but modifying as follows the differential equation governing the dynamics of human capital:

$$\overset{\circ}{H} = [\delta\, H(t)\, (1\text{-}l(t)\text{-}u(t))]^{\phi}\, [(1\text{-}\omega)\, K(t)]^{1\text{-}\phi} \qquad [2.3\text{ bis}]$$

In words, now the accumulation of human capital is governed by a Cobb-Douglas production function, whose arguments are augmented by the term *[(1-ω) K(t)],* which represent the fraction of the stock of physical capital devoted to the production of human capital. Consequently, the constraint expressing the accumulation of physical capital is modified as follows:

$$N(t)c(t) + \overset{\circ}{K}(t) = [\omega K(t)]^{\beta}\, \big[u(t)H(t)\big]^{1\text{-}\beta}\, \overline{H}(t)^{\gamma} \qquad [2.2\text{ bis}]$$

Where *[ωK(t)]* represent the fraction of stock of physical capital devoted to the accumulation of physical capital itself.

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[7] Note that the individual is facing constant return to scale (β + 1 - β) whereas the social planner would be looking at increasing returns to scale (β + 1 - β +γ).

In other words, now the physical capital has two alternative uses: either to take part in the production of physical output (according to equation 2.2 bis) or in the production of human capital (equation 2.3 bis). Note that the latter use still concurs into the production of physical output, since H is itself an argument of equation 2.2 bis.
The above modification seems plausible mainly because it is our belief that in modern economies the production and accumulation of human capital do not depend only on time and stock of knowledge, but also on the physical investment that an economy choose to devote to the university and research system; periodic statistics on the percentage of GDP invested in education by the main industrialized countries and political debates about them strongly support that view.

Therefore the path of the economy is obtained by consumers maximizing [2.1] over time subject to [2.2 bis] and [2.3 bis]. The current valued Hamiltionian is thus:

$$V: N(t)\ \left(c(t)^{1-\sigma} + al(t)^{1-\sigma}\right)/1-\sigma + \quad p(t)\left\{[\omega\ K(t)]^{\beta}\left[u(t)H(t)\right]^{1-\beta}\ \overline{H}(t)^{\gamma} - N(t)c(t)\right\}+$$

$$+q(t)\left\{\ [\delta\ H(t)\ (1\text{-}l(t)\text{-}u(t))]^{\phi}\ [(1\text{-}\omega)\ K(t)]^{1-\phi}\right\} \qquad [2.4]$$

Costate variables *p(t)* and *q(t)* represent respectively the shadow price for physical and human.
Now we can do the optimization with respect to *u(t)*, *l(t)* and *c(t)*: ouput has to be allocated between consumption and investment, and time has to be allocated among employment, leisure and studying (that is, accumulation of human capital). First order conditions are:

$$c(t)^{-\sigma} = p(t) \qquad [2.5]$$

$$N(t)al(t)^{-\sigma} = q(t)\ \delta\ H(t)\ \phi\left[(1\text{-}\omega)\ K(t)\right]^{1-\phi}\left[\delta\ H(t)\ (1\text{-}\ l(t)\text{-}\ u(t))\right]^{\phi-1} \qquad [2.6]$$

$$p(t)\ (1\text{-}\beta)\left[\omega K(t)\right]^{\beta} u(t)^{-\beta}\ H(t)^{1-\beta}\ \overline{H}^{\gamma} = q(t)\ \delta H(t)\ \phi\left[(1\text{-}\omega)K(t)\right]^{1-\phi}\left[\delta\ H(t)\ (1\text{-}\ l(t)\text{-}\ u(t))\right]^{\phi-1} \qquad [2.7]$$

There is an economic meaning for these conditions. Equation [2.5] says that, since output can be allocated either to consumption or to investment, the marginal utility of consumption $\left[c(t)^{-\sigma}\right]$ must be equal at each instant to the value of the marginal utility of net investment, that is, the shadow price *p(t)*. Since time can be allocated between leisure and work, or leisure and studying, or between work and studying, equations [2.6] and [2.7] take care of equating the marginal utilities of each of these three ways to spend time.

The dynamic maximization includes also the two costate equations for *p(t)* and *q(t)*:

$$\overset{\circ}{p}(t) = \rho p - dV/dK \qquad [2.8]$$

$$\overset{\circ}{q}(t) = \rho q - dV/dH \qquad [2.9]$$

Since in the course of calculations we will be need only the first one, let us develop it:

$$\overset{\circ}{p} = \rho p - \left\{ p(t)\omega^{\beta}\beta K(t)^{\beta-1} \left[u(t)H(t)\right]^{1-\beta} \overline{H}^{\gamma} + q(t)(1-\phi)\left[\delta H(1\text{-}l(t)\text{-}u(t))\right]^{\phi}\left[(1-\omega)K(t)\right]^{-\phi} \right\} \qquad [2.10]$$

Optimization is completed by the transversality conditions, which ensure that as time goes to infinite the value of human and physical capital eventually goes to zero (otherwise there would be incentive to postpone consumption forever):

$$\lim e^{-\rho t}\, p(t)K(t) = \lim e^{-\rho t}\, q(t)H(t) = 0 \qquad [2.11]$$

Let us now differentiate [2.5] with respect to time, to get:

$$-\sigma\, c^{-\sigma-1}\, \overset{\circ}{c} = \overset{\circ}{p}$$

Dividing through by p(t) $\left(= c(t)^{-\sigma}\right)$:

$$\overset{\circ}{p}/p = -\sigma\, \overset{\circ}{c}/c$$

Let us define $\overset{\circ}{c}/c$ (the rate of growth of consumption per-capite) by χ:

$$\overset{\circ}{p}/p = -\sigma\chi \qquad [2.12]$$

Now we divide both sides of [3.10] by *p*, in order to obtain an another expression for $\overset{\circ}{p}/p$:

$$\overset{\circ}{p}/p = \rho - \omega^{\beta}\beta\, K(t)^{\beta-1} \left[u(t)\, H(t)\right]^{1-\beta} \overline{H}^{\gamma} - q/p \times (1-\phi)\left[\delta H\,(1\text{-}l(t)\text{-}u(t))\right]^{\phi}\left[(1-\omega)\, K(t)\right]^{-\phi} \qquad [2.13]$$

Now we can set right-hand sides of [2.12] and [2.13] equal to each other:

$$\rho - \omega^{\beta}\beta K(t)^{\beta-1}\left[u(t)H(t)\right]^{1-\beta}\ \overline{H}^{\gamma} - q/p \times (1-\phi)\left[\delta H\,(1-l(t)-u(t))\right]^{\phi}\left[(1-\omega)\,K(t)\right]^{-\phi} = -\,\sigma\,\chi$$

Rearranging:

$$(\rho + \sigma\,\chi)\,/\beta = \omega^{\beta}\beta K(t)^{\beta-1}\left[u(t)H(t)\right]^{1-\beta}\overline{H}^{\gamma} + q/p\beta \times (1-\phi)\left[\delta H(1-l(t)-u(t))\right]^{\phi}\left[(1-\omega)K(t)\right]^{-\phi} \qquad [2.14]$$

In steady state, left-hand-side will be constant, since ρ and σ are parameters of the utility functions, β is a technological parameter of the production function, and obvioulsy the rate of growth of consumption χ in steady state will be constant. Thus, also the right-hand-side of [2.14] will be a constant, and therefore we are legittimated to set its (logarithmic) derivative equal to zero. To clarify the exposition, let us first write the logarithmic version of RHS of [2.14] and then we will proceed to differentiation:

$$\beta \ln \omega + (\beta-1)\ln K(t) + (1-\beta)\ln u(t) + (1-\beta)\ln H(t) + \gamma \ln \overline{H} + \ln q(t) - (\ln p(t) + \ln \beta) + \ln(1-\phi) + \phi \ln \delta$$
$$+\phi \ln H(t) + \phi \ln\left(1-l(t)-u(t)\right) - \phi \ln\left[(1-\omega)\,K(t)\right] \qquad [2.15]$$

We can now distinguish two cases, according to the policy consideration we are doing:

CASE I: ω is treated as constant (the fraction of capital devoted to production/education is exogenously fixed). Differentiation of [2.15] leads to:

$$(\beta-1)\,\overset{\circ}{K}/K + (1-\beta)\,\overset{\circ}{u}/u + (1-\beta+\gamma)\,\overset{\circ}{H}/H + \overset{\circ}{q} - \overset{\circ}{p} + \phi\,\overset{\circ}{H}/H - \phi\,\overset{\circ}{K}/K = 0 \qquad [2.16]$$

since obviously the derivative of constant terms are equal to zero and thus dropped.

Define:

$\overset{\circ}{K}/K$: rate of growth of physical capital = ε

$\overset{\circ}{H}/H$: rate of growth of human capital = ν

Furthermore, we assume that $\overset{\circ}{q} - \overset{\circ}{p}$ is equal to zero. This assumption may appear arbitrary, but actually it only says that the shadow price in utility terms of investment in human capital slighlty approximates the investment in physical capital in steady state (that is, taking away one unit of output from consumption brings approximately the same benefits in utility terms if that unit is devoted to production of physical capital or, alternatively, human capital).

Finally, note that $\overset{\circ}{u}/u$ is equal to zero in steady state (the amount of time people devote to working does not grow).

With the above notations and simplifications, [2.16] becomes:

$(1-\beta)\,\varepsilon + \phi\varepsilon = (1-\beta)\nu + \gamma\nu + \phi\nu$

$(1-\beta+\phi)\,\varepsilon = (1-\beta+\gamma+\phi)\nu$

$$\nu = \left[(1-\beta+\phi)/(1-\beta+\phi+\gamma)\right]\varepsilon \qquad [2.17]$$

This is exactly the result obtained by Lucas in his paper, depurated of course by the parameter ϕ, which we included in this version, following the modification of the law of accumulation of human capital, here tranformed into a Cobb-Douglas production function whose parameter is just ϕ. [2.17] says that human capital grows less than physical capital in the presence of externality ($\gamma > 0$), since the coefficient is less than one. If there were not externalities in the accumulation of human capital, that is, if a social planner could internalize them, physical capital and human capital would grow at the same rate. But the more interesting case in no doubt the second one.

CASE II : ω is a function of time (the fraction of physical capital devoted to production or education is not constant, and it can be interpreted as a choice variable by the policy maker)

In this case we define:

$\overset{\circ}{\omega}/\omega$ = rate of growth of the fraction of capital devoted to physical accumulation: α

$(1-\overset{\circ}{\omega})/(1-\omega)$ = rate of growth of the fraction of capital devoted to human capital production: τ

Differentiation of [2.15] in this case leads to:

$\beta\alpha + (\beta-1)\,\varepsilon + \gamma\nu + (1-\beta)\nu + \phi\,\nu - \phi\tau - \phi\varepsilon = 0$

Rearraging:

$$\nu = (1-\beta+\phi)\,/\,(1-\beta+\phi+\gamma)\,\varepsilon - \beta\alpha + \phi\,\tau \qquad [2.18]$$

2.2: Analysis of the model

2.2.1. Effects of increases in the amount of resources devoted to education.

Equation [2.18] represents the fundamental equation of this model. It expresses the rate of growth of human capital as a function of the rate of growth of investment in education and investment in physical capital. The assumption about the existence of a separation between a quota of capital devoted to formation of human capital [(1-ω)(t)K] and a quota to output [ω(t)K][8] makes it clear that if the rate of growth of the former (τ) is positive then rate of growth of the latter (α) must be negative, and viceversa. Therefore we distinguish two cases:

a) If ω raises, so the economy chooses to devote more resources to the production of output rather than investing in education; that means that α is positive, and thus τ is negative. Equation [2.18] shows that we have a double negative effect on the rate of growth of human capital: in fact, both the term [- βα] and [+ ϕτ] are negative.
b) Conversely, if (1-ω) raises, so the economy chooses to devote more resources to education rather than physical capital, τ is positive, and thus α negative. Therefore we have a double positive effect on the rate of growth of human capital and thus we are legittimated to say, even though a formal analysis needs still to be developed, on the overall path of growth of the economy.

A very important result is that, in steady state, the rate of growth of human capital can be greater than the rate of growth of physical capital even in presence of an external effect (that is, in competitive equilibrium): this will be the case if the magnitude of the external effect [γ] which makes the coefficient less than one, is more than offset by the increase of τ, which in turn depends on the elasticity ϕ.

2.2.2 Elasticities

Elasticites in this model are economically highly significant:

c) In presence of an increase of investment in education, human capital's increase will be larger if ϕ is larger: that is, recalling the role of parameter ϕ in the production function for human capital, if the importance of human capital stock itself in the production of human capital is large. In other words, if the **internal productivity of human capital (**that is, how much the existing stock of *H* today is relevant for the production of *H* tomorrow) is large (parameter ϕ large), the response of human capital growth following an increase in investment in education will be larger. This strongly suggests how important is the quality of human capital in the process of growth: given two countries with different productivity of H, *ceteris paribus*, the effect of the same relative increase in investment in education will be larger in the country where human capital is more productive. This result, following the tradition of almost all endogenous growth models, confirms that the conditional convergence predicted by exogenous growth models no longer holds: a country which starts

[8] Note that here we retain the assumption that quotas are function of time.

off with poor resources may as well not catch up with more developed economies. In other words, growth gaps are not necessarily doomed to be filled.

d) Similarly, the elasticity for an increase in physical investment is exactly the technological parameter β, that is the "importance" of capital in the production function of output. If ω is raised, human capital growth will decrease exactly by the amount indicated by the relative importance of physical capital in the production of output.

Note that, as mentioned above, the two elasticities always work together, since a positive sign of α is always coupled with a negative sign of τ and vice-versa. So effects a) and b) needs to be considered jointly.

Section 3: Econometric analysis

In this section we attempt an early econometric test of the central equation of our model, equation (2.18).We must stress that this is far from being a satisfactory empirical analysis, mainly because of the lack of full data and the small the number of observations. Particularly, it has proven to be particularly difficult to find data regarding the destination of capital, which is the crucial variable of this equation. Therefore we did not follow the usual procedure widely used in the empirical literature, that is, a cross-section estimate between countries, computing the rate of growth in a wide period. We rather chose to focus on Italy, using a time-series with annual observations from 1980 to 1999.

The most appropriate proxy for human capital we found is the percentage of labour force who attained *at least* high-school diploma. We have net capital in different branches of economy, including education, at 1995 prices. Therefore it was possible to calculate the quota of capital devoted to education and its rate of growth. The complement to one of this quota is of course the quota of capital not-devoted to education. Obviously, the rate of growth of the quota of capital devoted to education is negatively perfectly correlated with the quota of capital not-devoted to education; thus we dropped this last variable from the estimated equations.

While the time series of rate of growth of human capital (expressed, as usual, as the first log difference) is stationary, according to the augmented Dickey-Fuller test, time series of the rate of growth of capital and of the rate of growth of capital quota devoted to human capital are not stationary at 5%. Therefore it was necessary to differentiate the equation 2.13 in logs, obtaining therefore the second difference of logs, that is the acceleration of the rate of growth of the variables. These time series are significantly stationary (at 5%, except the acceleration of the rate of growth of capital, which is stationary at 10%). We then regressed the acceleration of the rate of growth of human capital on the acceleration of the rate of growth of capital and the acceleration of the rate of growth of the capital quota devoted to education (positive signs for both variables are expected).

We estimated an ARDL model, and the best result was found to be the following:

$$d2.logHK = 0.01 + -0.5\ d2.L.logHK + 12.29\ L.d2.logK + 9.19\ L4.d2.\log(1-\omega)$$
$$(0.52) \quad (-2.77) \qquad\qquad (1.51) \qquad\qquad (2.10)$$

Number of observations: 20

Adj. R-square: 0.6355

F (3, 10): 8.55

Prob>F: 0.0041

t-statistics are in brackets.

The one-year lag of the dependent variable is significant at 5%, while the variables of the model (acceleration of the rate of growth of capital and acceleration of the rate of growth of capital quota devoted to education) give best estimates respectively with a lag of one and of four years. Their signs are those expected, while the sign of the acceleration of the rate of growth of capital is not significant at 5%.

The model is not affected by heteroskedasticity, autocorrelation and residual are white-noise, according to the usual tests at 1% significance.

The different lags may be conceptually justified by the different ways physical capital and capital devoted to education are linked with the accumulation of human capital. The link between human capital and physical capital is direct: accumulation of human capital determines growth of income, which is divided between consumption and investment; investment is the accumulation of physical capital. Accumulation of human capital is in turn influenced, according to our hypothesis, by the physical capital devoted to education; it is possible to think that this last effect requires more time than the first one[9].

[9] It would be to estimate directly our hypothesis, regarding the accumulation of human capital (equation 2.3 bis), but it was not possible because of problems of non-stationarity of series.

Because of the limits of analysis in one only country and the shortness of this time series, this should be considered only an attempt of empirical analysis. Larger data, regarding a panel of countries and/or longer time series are required.

CONCLUSIONS

The above analysis presents some weak points. It is clearly still an early step in the complex derivations of a full-equipped endogenous theory, capable of expressing the steady-state rate of growth of output and consumption per capita as function of preferences parameter and investment in education. Furthermore, the the moment we consider ω and $1-\omega$ as functions of time, and thus we open the possibility for them to be variables chosen by agents (particularly the policy maker), it would be definitely more appropriate to make them the result of an intertemporal maximization problem by agents, rather than just considering them function of time.

Nevertheless, the analysis presents some novelties. It shows that if we modify the existing literature by having a more realistic accumulation of human capital, depending not only on the existing stock and on the amount of time people spend studying, but also on the physical investment in education, we obtain very sensible results::

- if we allow for the destination-quotas to be function of time, we find that a positive rate of growth of the amount of resources devoted to education rather than production of physical ouput has a double positive effect on the rate of growth of human capital, and therefore, on the economy. On the other hand, taking away resources from education sector causes a double negative effect. This strongly support investment in human capital formation in modern economies even in times of economic recessions.
- moreover, those effects on the rate of growth of human capital depends crucially on technological parameter, among which ϕ, the relative importance of human capital today in the production of human capital tomorrow (we called it "internal productivity" of human capital). The larger the internal productivity of human capital, the larger will be the response of growth following an increase in investment of education. Put it in another way, the increase in investment in education will be more productive, the larger is the internal productivity of human capital. This result is in line with those studies that adverse the conditional convergence among countries predicted by exogenous growth models of the

'50s and '60s: if a country has a poor stock of human capital, it may never fill the gap with more advanced economies.

- an important result is also that, with the above modification of the analysis, now in steady state human capital can grow faster than physical capital also in competitive equilibrium, if the magnitude of the external effect is more than offset by the increase in human capital growth caused by the increase in investment in education (which also depends on the internal productivity of human capital).

An early and "shy" empirical analysis seems to confirm the hypothesis of a role played by physical investment in human capital in the process of growth; surely much more work is needed in that direction.

An interesting attempt for further research, other than overcoming the limitations already outlined, would be to incorporate also non-rival technological progress in the analysis, whose accumulation may be made dependent by human capital stock (it is the engineer who discovers new inventions!) and by investment in human capital formation (the engineer can do it only if he can dispose of adequate means!).

References

Anselin, L., Varga, A., Acs, Z. (1997), *Local Geographic Spillovers Between University Research and High Technology Innovations,* Journal of Urban Economics, 42, 422-448, 1997.

Benhabib, J., Spiegel, M.M. (1994), *The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data*, Journal of Monetary Economics, 34, 143-173.

Furman, J.L., Porter, M.E., Stern, S. (2002), *The Determinants of National Innovative Capacity,* Research Policy, 31, 597-627.

Griliches, Z. (1979), *Issues in Assessing the Contribution of R&D to Economic Growth,* Bell Journal of Economics, 10, 92-116.

Jaffe, A.B. (1989), *Real Effects of Academic Research,* American Economic Review, 79, 957-970.

Jones, C. (1998), *Introduction to Economic Growth*, New York, W.Norton.

Lodde, S. (1999), *Education Growth: Some Disaggregate Evidence From the Italian Regions*, CRENoS Contributi di ricerca, 99/11.

Lucas, R.E. (1988), *On the Mechanics of Economic Development,* Journal of Monetary Economics 22, 3-42.

Mankiw, N.G., Romer, D., Weil, D.E. (1992), *A Contribution to the Empirics of Economic Growth,* Quarterly Journal of Economics, 107 (2), 408-37.

Rebelo S. (1991), *Long-Run Policy Analysis and Long-Run Growth"* Journal of Political Economy, vol.99 no.31 pp. 500-520

Romer, P.M. (1990), *Endogenous Technological Change*, Journal of Political Economy, 98 (5), 71-102.

Sassu, A., Lodde, S. (2002), *Ricerca e innovazione: un'analisi comparata,* L'industria, 4, 597-627.

Solow, R. (1956), *A Contribution to the Theory of Economic Growth,* Quarterly Journal of Economics, LXX, 65-94.

Solow, R. (2000), *Growth Theory. An exposition* – Second Edition – Oxford University Press.