Università degli Studi di Ferrara

DIPARTIMENTO DI ECONOMIA, ISTITUZIONI, TERRITORIO

Corso Ercole I D'Este n.44, 44100 Ferrara

Quaderni del Dipartimento

n.10/2002

April 2002

Non Monotonic Convergence in a Solow Model with Public Capital

Silvia Bertarelli – Roberto Censolo

Non monotonic convergence in a Solow model with public capital

Silvia Bertarelli[*] Roberto Censolo[†]

April 2002

Abstract

In this paper we propose a standard Solow model augmented with public capital in the production function. The model solution displays two steady states, an unstable poverty trap and an efficient stable equilibrium. As a result the model predicts both divergence and non monotonic convergence; that is a transition path characterized by increasing growth rates up to the point where traditional convergence behavior describes the successive evolution towards the stationary equilibrium. Using traditional cross section techniques the hypothesis of non monotonic conditional convergence is tested against a large sample of coutries, obtaining favorable evidence.

[*]Dipartimento di Scienze Economiche,Strada Maggiore, 45, 40125 Bologna. E-Mail bertarel@spbo.unibo.it

[†]Dipartimento di Economia Istituzioni Territorio, Corso Ercole I d'Este, 44; 44100 Ferrara, Italy. Tel. +39 (0532) 291955, Fax +39 (0532) 202102 . E-mail: censolo@spbo.unibo.it

1 Introduction

The traditional Solow [27] growth model has provided an exceptionally fertile framework to address the two central questions of growth: "why are rich and poor economies characterized by vast differences in per capita income?" and its dynamic implication "does the income gap between the poor and the rich tend to vanish, to persist, or to increase over time?". The availability since the mid 1980s of internationally comparable data for a large number of countries[1] has made the above question the challenge of empirical research, starting an outstanding run between evidence, theory, and methodological issues.

This research has centered around the "convergence debate". That is, the neoclassical growth theory prediction that less developed economies should grow faster than richer ones, as far as they approach the same steady state. As it is well known in the convergence literature, considering post World War II data, samples of rich countries (such as OECD, but also regions and prefectures within a given country)[2] have displayed marked convergence, whereas no convergence or even a slight divergence emerges from larger samples of countries.

To the purpose of locating our contribution within the huge variety of papers aimed at addressing this puzzling evidence, we survey the existing literature along two alternative hypotesis on technology (see figure 1)[3].

On the one side, the persistence over time of output per worker differences motivates an endogenous growth approach. This relies on increasing returns to scale and spillover or external effects between sectoral and national level (Romer [24], Romer [25], Lucas [19], King and Rebelo [18], Rebelo [23], Grossman and Helpman [13]). As a consequence of the departure from the diminishing returns to capital assumption, different economies may perpetually diverge or not converge depending on their starting point conditions, even if they share identical structure (population growth, institutional settings, time preferences, market structure, etc.).

From the left side of figure 1 depart two competing explanations of non-convergence based on the same traditional neoclassical specification of the

[1] The most update version of this international data collection refers to Summer and Heston [28].

[2] See for example Barro and Sala-i-Martin [5], Sala-i-Martin [26].

[3] More exhaustive surveys can be found in Barro and Sala-i-Martin [5], Quah [21], Sala-i-Martin [26], Galor [11], Capolupo [7], De la Fuente [10].

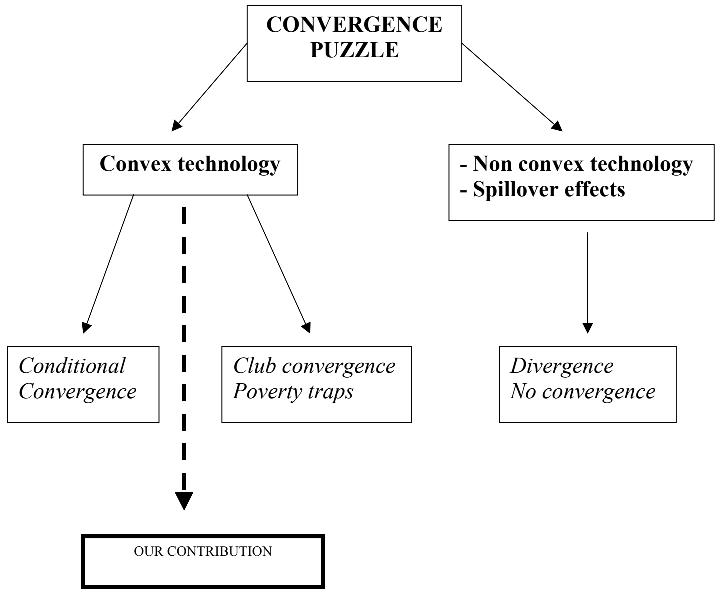

Figure 1:

production function: "conditional convergence" and "club convergence".

The conditional convergence approach emerges from neoclassical growth models displaying a unique non-trivial stable steady state (Ramsey [22], Solow [27], Cass [9]), which implies that the growth rate of per capita income declines as the economy approaches to its long run equilibrium. Therefore, different countries should converge to one another in their levels of output per capita as long as they are characterized by the same structure, independently of their initial conditions.

Several contributions have tested the above prediction, employing cross country regressions motivated on the ground of the neoclassical one sector growth theory. Barro [3], Mankiw, Romer, and Weil [20], Barro and Sala-i-Martin [5], Sala-i-Martin [26] report supportive evidence of conditional convergence considering large samples of countries[4].

However, various papers offer arguments and evidence that make questionable the conditional convergence hypothesis. Benhabib and Gali [6] re-

[4]Alternatively, a panel estimation approach can be employed. See for example Islam [15] [16], Arellano and Bond [1], and Caselli *et al.* [8]. For a debate between cross section vs panel estimation see Goddard and Wilson [12].

port evidence against convergence, testing the correlation between growth rates and capital/output ratios, considering two successive subsamples of ten years and a set of countries including the less developed economies Adopting a theoretical setting similar to Mankiw, Romer, and Weil [20], Jones [17] empirically analyzes the steady state distribution of per capita income, reporting that "holding differences in technology levels constant, the world income distribution will be characterized by additional divergence at the bottom and convergence and overtaking at the top" (pp. 147-148).

These criticisms to the conditional convergence hypothesis point in favor of an alternative exlanation of nonconvergence, based on the concept of "club convergence". Retaining the standard neoclassical assumption about technology, an overlapping generation setting is able to generate multiple locally stable steady states (Galor[11]). As a result, different countries starting their transition with different stocks of production factors, may end up in different steady states [5].

The economic theory of the present paper is based on a variation of the augmented Solow model by Mankiw, Romer, and Weil [20], that generates multiple steady states. For this reason, in figure 1 our contribution find place between "conditional convergence" and "club convergence". In particular, we consider a production function with traditional properties, with private capital, public capital and labor as factors of production. Public capital is provided by the goverment through non distortionary taxation. Assuming the public sector objective to be the maximization of per capita consumption in the long run, the model solution displays two steady state levels of capital per effective worker. The Pareto optimum steady state is stable, whereas the Pareto inferior one is unstable. As a consequence, the model predicts both divergence for low levels of initial private capital, and non monotonic conditional convergence for levels of capital between the two steady state values. The non monotonic convergence result predicts that the transition towards the non trivial steady state is characterized by increasing growth rates in per capita income (I-convergence) up to the point where the traditional convergence behavior (II-convergence) eventually brings the economy to its stationary equilibrium. Therefore, the model enables a new interpretation of the significance level of the β-convergence parameter estimated in "old regressions". In particular, evidence of β-divergence might be evidence

[5] See Azariadis [2] for an extensive survey, as well as for survey criticism to the conditional convergence hypothesis.

of conditional I-convergence; whereas, not statistically significant estimates of the speed of convergence might suggest that the sample of countries lies between I-convergence and II-convergence.

To test the above prediction we consider a large sample of countries ordered by their initial real per capita income. Then we run standard conditional convergence regressions, splitting the country sample into successive sub-samples and moving from the poorest economies to the richest ones. Employing this procedure we find favorable evidence to the hypothesis of non monotonic convergence.

The remainder of the paper is organized as follows. In section 2 we present the model and derive the steady state solutions. Section 3 offers a new quantitative interpretation of the Mankiw, Romer and Weil [20] regressions aimed at explaining the cross country per capita income differences. In section 4, we formally introduce our non monotonic conditional convergence hypothesis, the estimation procedure and the empirical findings. Section 5 summarizes the main results obtained.

2 The model

We assume the aggregate output $Y(t)$ to be generated by a constant returns to scale technology which employs private capital $K(t)$, labour $L(t)$, and public capital $H(t)$

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} [A(t) L(t)]^{1-\alpha-\beta} \tag{1}$$
$$0 < \alpha < 1, 0 < \beta < 1, \alpha + \beta < 1$$

where $A(t)$ represents labor-augmenting technical progress. $A(t)$ and $L(t)$ are assumed to grow at the exogenous rates x and n:

$$\begin{aligned} A(t) &= A(0) e^{xt} \\ L(t) &= L(0) e^{nt} \end{aligned}$$

In our context, public capital should be interpreted as an aggregate good including all those factors of production that are not autonomously provided by the private sector, typically infrastructures, but also the fraction of human capital which accumulates through public spending on education.

Total output is divided between consumption $C(t)$, private investment $I(t)$, and public investment $G(t)$:

$$Y(t) = C(t) + I(t) + G(t)$$

Given δ_H the rate of depreciation of public capital, $H(t)$ evolves according to

$$\dot{H}(t) = G(t) - \delta_H H(t) = g(t) H(t) - \delta_H H(t) \tag{2}$$

where $g(t) H(t) = G(t)$ represents gross public investment and $g(t)$ is the discretionary rate of public capital accumulation settled by the government.

We assume that the government runs a balanced budget, financing its expenditures through lump sum taxation. Therefore, gross private expenditure devoted to the accumulation of $K(t)$ is a fraction s of total disposable income $(Y(t) - G(t))$. In addition, private capital depreciates at rate δ_K. Thus

$$\dot{K}(t) = I(t) - \delta_K K(t) = s[Y(t) - G(t)] - \delta_K K(t) \tag{3}$$

We define with lower-case letters quantities per effective unit of labor $[k = K/AL, h = H/AL, y = Y/AL]$. In addition, we assume that private capital and public capital depreciate at the same rate $\delta_K = \delta_G = \delta$. Then equations (1), (2), and (3) in intensive form are the following

$$y(t) = k(t)^{\alpha} h(t)^{\beta} \tag{4}$$

$$\dot{h}(t) = [g(t) - (n + \delta + x)] h(t) \tag{5}$$

$$\dot{k}(t) = sy(t) - sg(t) h(t) - (n + \delta + x) k(t) \tag{6}$$

Equations (4), (5), and (6) illustrate how the government influences the private sector accumulation process of capital. On the one side, current public investment discourages capital accumulation, because taxation reduces the amount of total saving that can be devoted to new investments. On the other hand, taxation is employed to create additional factors of production, which enhance the productivity of private capital, and the ability of the private sector to generate additional saving in the future to support new investment projects.

To characterize the steady state values for $k(t)$ and $h(t)$ we set $\dot{k}(t) = 0$ and $\dot{h}(t) = 0$. From equation (5) we obtain the long run growth rate g^* consistent with a constant level of public capital per effective worker

$$\dot{h}(t) = 0 \rightarrow g^*(t) = n + \delta + x \tag{7}$$

Replacing $g(t)$ in (6) with its steady state level $g^*(t)$ we get

$$\dot{k}(t) = 0 \rightarrow sk(t)^{\alpha} h^{*\beta} - s(n + \delta + x) h^* - (n + \delta + x) k(t) = 0 \tag{8}$$

with h^* defining the steady state value of $h(t)$.

To derive from (8) an explicit solution for the steady state value of $k(t)$ we have to define a specific government policy rule as to the long run level of public capital provided to the economy. We assume that the government chooses the level of public capital, which maximizes per capita consumption in the steady state. Therefore, the stationary level of public capital must be the solution to the following static optimization problem:

$$\begin{aligned} &\max c = (1 - s)\left[f(k, h) - (n + x + \delta) h\right] \\ s.t. \quad &sf(k, h) - s(n + x + \delta) h - (n + x + \delta) k = 0 \end{aligned} \tag{9}$$

where c indicates per capita consumption. Necessary conditions imply that $\frac{\partial f(k,h)}{\partial h} \equiv f_h = (n + \delta + x)$. Substituting back into $\dot{k} = 0$ condition, we obtain the optimal steady state values for k , h, and y:

$$k^* = \left[\frac{\beta^{\beta}\left[s(1 - \beta)\right]^{1-\beta}}{(n + x + \delta)}\right]^{\frac{1}{1-\alpha-\beta}} \tag{10}$$

$$h^* = \left[\frac{\beta^{1-\alpha}\left[s(1 - \beta)\right]^{\alpha}}{(n + x + \delta)}\right]^{\frac{1}{1-\alpha-\beta}} \tag{11}$$

$$y^* = \left[\frac{\beta^\beta \left[s\left(1-\beta\right)\right]^\alpha}{\left(n+x+\delta\right)^{\alpha+\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \tag{12}$$

Moreover, our model implies that the government objective at pursuing the highest per capita consumption level is equivalent to maximize the steady state stock of private capital. Totally differentiating equation (8), we get:

$$sf_k \frac{dk\left(h\right)}{dh} + sf_h - \left(n+x+\delta\right)s - \left(n+x+\delta\right)\frac{dk\left(h\right)}{dh} = 0$$

which can be set equal to zero, as we are interested in pairs of k and h that satisfy the $\dot{k} = 0$ condition. Solving for $\frac{dk(h)}{dh}$ we get the same $f_h = (n + \delta + x)$ condition as before[6]. Therefore, the long run government objective of maximizing per capita consumption is achieved incentivating the private sector to invest additional resources as to reach the highest level of per capita capital as possible. To pursue this goal the government accumulates public capital up to the point where the last unit of h displays "crowding-in" and "crowding-out" effects on $\dot{k}(t)$ that exactly offset each other.

The above solution k^* represents the unique stable steady-state of the model-economy. However, equation (8) is also solved for an alternative value of private capital $k' < k^*$ which determines a second (unstable) equilibrium (see figure 2). We are not able to derive an exact solution for k'.

However, we can obtain an approximated value applying the Newton iterative procedure. It can be shown that (see Appendix A)

$$\lim_{\substack{\alpha\to 0 \\ and/or\beta\to 0}} k'\left(k^*\right) = \beta^{\frac{1}{\alpha}} k^* \tag{13}$$

We then assume (13) as the first step of the approximation procedure. Assuming α and β to be less than 0.4, the second step of the Newton procedure suffices to provide a good approximation for k':

[6] To see the equivalence between the max c and the max k problem in a more immediate way, derive $f(k,h)$ from the $\dot{k} = 0$ condition and substitute into the maximand in equation (9): the max c problem translates into a max $\left[(1-s)\left(n+\delta+x\right)/s\right]k$ problem.

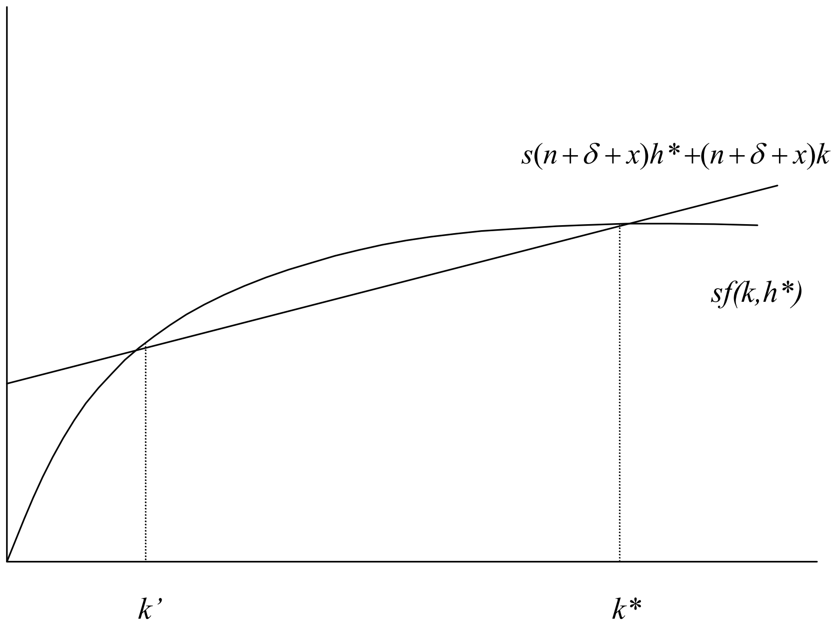

Figure 2:

$$k' \simeq \frac{\alpha\beta}{\alpha\beta^{\frac{\alpha-1}{\alpha}} - (1-\beta)} k^* \tag{14}$$

where the multiplicative coefficient is less than 1.

Substituting back into equation (4) and considering y^* in equation (12) it is easy to show that y' is smaller than y^* and is given by the following expression

$$y' \simeq \left(\frac{\alpha\beta}{\alpha\beta^{\frac{\alpha-1}{\alpha}} - (1-\beta)} \right)^{\alpha} y^* \tag{15}$$

3 New interpretation of old regressions

Our theoretical setting allows a direct comparison with the empirical findings reported in Mankiw, Romer and Weil [20] (MRW henceforth) as to the ability of the textbook Solow model to explain cross country income differences. Using cross country data, MRW run the following regression[7]

$$\begin{aligned} \ln \left[\frac{Y}{L} \right] &= const + a \ln s + b \ln (n + \delta + x) + \epsilon \qquad (16) \\ \text{where } a &= \frac{\alpha}{1-\alpha} \; ; b = -\frac{\alpha}{1-\alpha} \; ; const = \ln A(0) \end{aligned}$$

Estimated coefficients enter (16) with the right signs and are highly statistically significant. Moreover, the regression accounts for a large portion of cross-country differences in real output. However, the interpretation offered by MRW of the estimated coefficients in (16) - in terms of the implied capital share- may actually depend on the specific assumption about technology, which is adopted in the model. Indeed, assuming public capital as an exogenous factor of production, we obtain an empirical specification of the theoretical implications of saving and population growth on real output, which is observationally equivalent to (16), but underlies a different interpretation of the estimated coefficient.

[7] See MRW, table 1 p.414.

In the steady state, per effective capita output $\frac{Y(t)}{A(t)L(t)}$ is given by equation (12). Taking logs and simply considering output per capita we get

$$\ln\left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + xt + \frac{\beta}{1-\alpha-\beta}\ln\beta + \frac{\alpha}{1-\alpha-\beta}\ln(1-\beta) + \frac{\alpha}{1-\alpha-\beta}\ln s - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+\delta+x)$$

Assuming $t=0$ for simplicity the previous equation specifies as

$$\ln\left(\frac{Y}{L}\right) = const + a\ln s + b\ln(n+\delta+x) \tag{17}$$

$$\text{where } a = \frac{\alpha}{1-\alpha-\beta}\ ;\ b = -\frac{\alpha+\beta}{1-\alpha-\beta}\ ;$$

$$const = \ln A(0) + \frac{\beta}{1-\alpha-\beta}\ln\beta + \frac{\alpha}{1-\alpha-\beta}\ln(1-\beta)$$

Employing (17), we can perform an alternative interpretation of the original findings of MRW[8]. Results are reported in table 1.

Table 1 relates the estimated coefficients by MRW from the textbook Solow model perspective. The implied capital share in output appears excessively high in the Intermediate and in the Non.oil sample; a more grounded value of α is obtained within the OECD sample. The table 2 below reports the MRW results obtained with the extended human capital model. The α coefficients drop to half of their previous value, determining the share of physical capital in income equal to $\frac{1}{3}$ in the Non-oil and Intermediate sample. The 0.14 value obtained for the OECD sample seems too low.

The upper part of table 2 reports the interpretation of the MRW estimates obtained from (16) in the light of the coefficient restrictions implied by (17). Here β and α represent the share of public and private capital in total income respectively. The effect of public capital as an additional input in the production function is to lower significantly the value of α, which drops to 0.47 and to 0.43 for Non-oil and Intermediate, and reaches 0.28 in the OECD sample.

[8] MRW test equation (16) using Summer and Heston [28] annual data over the period 1965-1985. Specifically, MRW divide the whole country set into three sub-samples: Non oil, Intermediate, and OECD. For details see MRW pp. 412-413.

In comparison with the MRW estimates, these α values may be regarded as highly satisfactory: considering the Non.oil and Intermediate samples we obtain slightly higher values, but, on the other hand, our α values for the OECD country sample seems more realistic than the 0.14 value implied by the MRW estimates[9].

The ability of the imposed coefficient restrictions (17) to capture more realistic income distribution shares between capital and labor can be remarked by looking at the values of constants. In the regression (16), the constant reflects all those factors which are relevant to the aggregate production process and are not included among the arguments inside the production function; typically, technology, but also resource endowments, climate, and other political, social and environmental variables. The inclusione of human capital in MRW regression changes significantly the value of the constant, with respect to the standard Solow model regression. We obtain very similar figures when we calculate the aggregate factor $\ln A(0)$ by adjusting the estimated values with the restriction in (17).

Table 1 *Implied α: textbook Solow model*

[9]Moreover, differently from the results obtained by using equation (16), the MRW conditional convergence regression deliver implied α and β which are very close to our findings:

	Non-oil	Intermediate	OECD
α	0.48	0.44	0.38
β	0.23	0.23	0.23

Source: MRW [20], pp. 429, table VI

Sample :	Non-oil	Intermediate	OECD
const	5.48 (1.59)	5.36 (1.55)	7.97 (2.48)
lns	1.42 (0.14)	1.31 (0.17)	0.50 (0.43)
$ln(n+\delta+x)$	−1.97 (0.56)	−2.01 (0.53)	−0.76 (0.84)
MRW implied α	0.60 (0.02)	0.59 (0.02)	0.36 (0.15)

Source: Mankiw, Romer and Weil [20]

Table 2 *Implied α and β: public capital vs human capital*

	Non-oil	Intermediate	OECD
Public capital:			
const	6.69	6.72	8.54
Implied α	0.4781 (0.09)	0.4352 (0.08)	0.2841 (0.27)
Implied β	0.1852 (0.15)	0.2326 (0.13)	0.1477 (0.46)
MRW with human capital:			
const	6.89	7.81	8.63
Implied α	0.31 (0.04)	0.29 (0.05)	0.14 (0.15)
Implied β	0.28 (0.03)	0.30 (0.04)	0.37 (0.12)

4 Non monotonic conditional convergence

In neoclassical growth models such as Solow [27], a country's per capita growth rate tends to be negatively related to its starting level of per capita income. Therefore, poor and rich countries should converge in terms of per capita income levels after controlling for the determinants of the steady state (conditional convergence).

Our specification of technology, however, determines two steady state values of per capita income, of which one is unstable, whereas the other one proves to be stable and Pareto efficient. As a consequence, a country can display convergence or divergence depending on the value of the starting level of income per capita. In the convergence situation, however, the relationship between the growth rate of real per capita income and the starting level of real income is not monotone. Figure 3 gives a geometric interpretation of the result. The differerence between sy/k and $(n+\delta+x)(sh^*/k+1)$ measures the rate of growth of (private) capital. For values of k at the left of $k^{'}$, $(\dot{k}/k)$ is negative: the government spending aimed at maintaining a constant stock of public capital is excessive with respect to the size of the economy. As a consequence, private investment is crowded out, resulting in negative rates of capital accumulation.Between $k^{'}$ and k^* the economy displays non monotonic convergence: the rate of growth of capitale increases as the accumulation process moves towards k^*, reaching a maximun in $k = \mathbf{k}$ Moving on from $\mathbf{k}$, higher levels of capital are associated to decreasing growth rates of $(\dot{k}/k)$, providing the traditional convergence result.

We define the positive correlation between k and $(\dot{k}/k)$ for $k^{'} < k \leq \mathbf{k}$ as I-convergence (or convergence of the I type), and we refers to II-convergence (or convergence of the second type) to indicate the negative correlation between k and $(\dot{k}/k)$ for $\mathbf{k} < k \leq k^*$.

To test the non monotonic conditional convergence hypothesis we have to make an assumption about the public sector behavior in the neighborhood of k^*. A reasonable possibility might be to assume, that the government aims at reaching the efficient long run equilibrium in the shortest time as possible. Due to the non distortionary character of taxation, the above minimum time problem turns out to have no solutions (see the Appendix C for details). An important implication is that whatever fiscal policy rule is assumed in the neighborhood of the steady state, it turns out to be arbitrary. In the present continous time context, the only restriction required under our lump-sum financing assumption, is that in the last period, just before reaching the steady state, $\dot{h}$ must be non positive[10]. Therefore, in the present contest, to work out testable predictions of the model, we simply consider that $\dot{h} = 0$, with $h(t) = h^*$. This assumption corresponds, for example, to a policy rule aimed at accumulating the optimal stock of public capital as fast as possible, and then leaving the market mechanism autonomously reaching the optimal

[10] See the Appendix C for a formal proof.

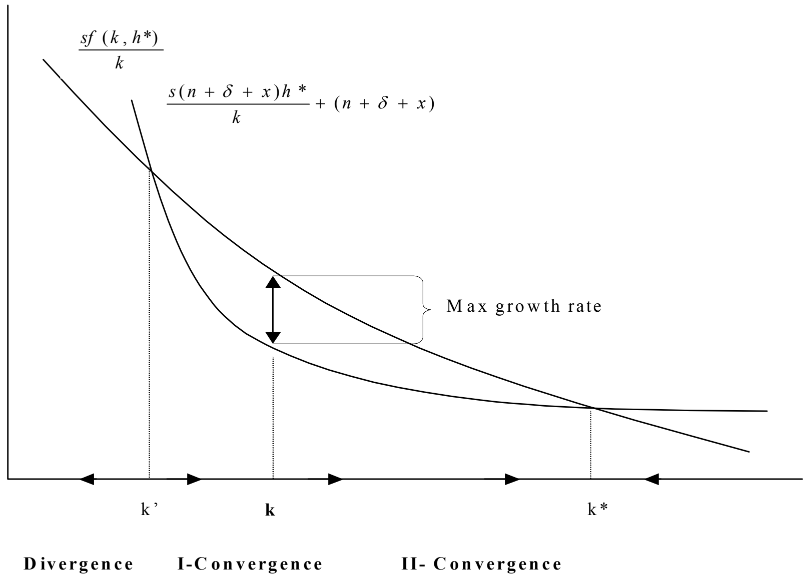

Figure 3:

long run position[11].

Let y^* and y' be the steady state levels of income per effective worker given by equation (12) and (15), and let $y(t)$ be the actual value at time t. Approximating around the steady states, at the left of y^* and at the right of y' respectively, the speeds of convergence are given by

$$\frac{d\ln y\left(t\right)}{dt}=\lambda\left[\ln y^{*}-\ln y\left(t\right)\right] \tag{18}$$

$$\frac{d\ln y\left(t\right)}{dt}=\lambda\left[\ln y\left(t\right)-\ln y'\right] \tag{19}$$

where

[11] In the discrete time context, the $\dot{h}=0$ assumption in the neighbourhood of the steady state is perfectly plausible. Indeed, to reach k^* in period T , it must verified that $(h_T-h_{T-1})=0$, which implies that $h_{T-1}=h^*$.

$$\lambda = (n + x + \delta) \frac{1 - \alpha - \beta}{1 - \beta}$$

We can obtain quantitative predictions about the speed of I-convergence and II-convergence towards the steady state, following the MRW procedure. For technical details see Appendix B.

At the left neighbour of the stable steady state y^* we get

$$\ln\left[\frac{Y(t)}{L(t)}\right] - \ln\left[\frac{Y(0)}{L(0)}\right] = xt + a_1 \ln A(0) + a_1 const_1 - a_1 \ln\left[\frac{Y(0)}{L(0)}\right] + \\ + a_1 \frac{\alpha}{1-\alpha-\beta} \ln s - a_1 \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+\delta+x) \quad (20)$$

while at the right neighbour of the unstable steady state y' we have

$$\ln\left[\frac{Y(t)}{L(t)}\right] - \ln\left[\frac{Y(0)}{L(0)}\right] = xt - a_2 \ln A(0) + a_2 const_2 + a_2 \ln\left[\frac{Y(0)}{L(0)}\right] + \\ + a_2 \frac{\alpha}{1-\alpha-\beta} \ln s - a_2 \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+\delta+x) \quad (21)$$

with the followin parameters restrictions on coefficients

$const_1 = \frac{\beta \ln \beta + \alpha \ln(1-\beta)}{1-\alpha-\beta} < 0$

$a_1 = \left(1 - e^{-\lambda t}\right) > 0$

$const_2 = \left\{\frac{\beta \ln \beta + \alpha \ln(1-\beta)}{1-\alpha-\beta} + \alpha \ln\left[\frac{\alpha\beta}{\alpha\beta^{\frac{\alpha-1}{\alpha}} - (1-\beta)}\right]\right\} < 0$

$a_2 = \left(e^{\lambda t} - 1\right) > 0$

Coefficients associated to $\ln s$ and $\ln(n+\delta+x)$ in equations (20) and (21) have the same sign. However, the effect of the starting level of real income per capita is positive and negative provided that the cross section sample displays I-convergence and II-convergence respectively. In other words, the same regression explaining real per capita income growth rate with $\ln s$, $\ln(n+\delta+x)$, and $\ln[Y(0)/L(0)]$ as regressors indicates I-convergence when the coefficient of $\ln[Y(0)/L(0)]$ is positive, and II-convergence, as the coefficient of $\ln[Y(0)/L(0)]$ is negative

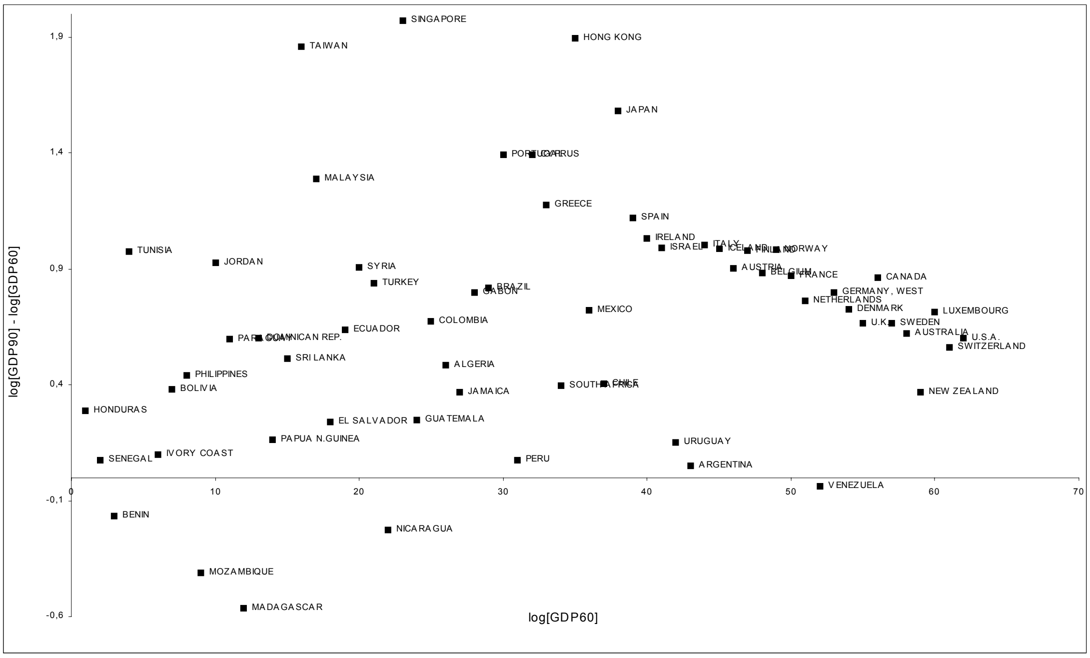

Figure 4:

Within this framework, an ambiguous sign and a non significance result can be interpreted with the presence of countries in the sample located both in I and II convergence regions.

To find empirical evidence we employ data from the Penn World Table (Mark 5.6 a) over the period 1960-1990, which include: real GDP per capita, real investment share of GDP, and population. Data on per capita GDP growth rate, calculated as the log difference of per capita real GDP in 1990 and per capita real GDP in 1960, and per capita real GDP level in 1960 are plotted in figure 4 for the entire sample (86 countries).

To test the non monotonic convergence assumption, we decided to split the sample into smaller groups of countries sharing more homogenous starting conditions. The basic idea is that, after controlling for differences in saving and population growth rates, in 1960 the poorest countries were more likely to be in the I-convergence situation, whereas the richest ones were probably experiencing their transition path within a II-convergence situation. To this purpose, we have sorted all countries with reference to real per capita GDP

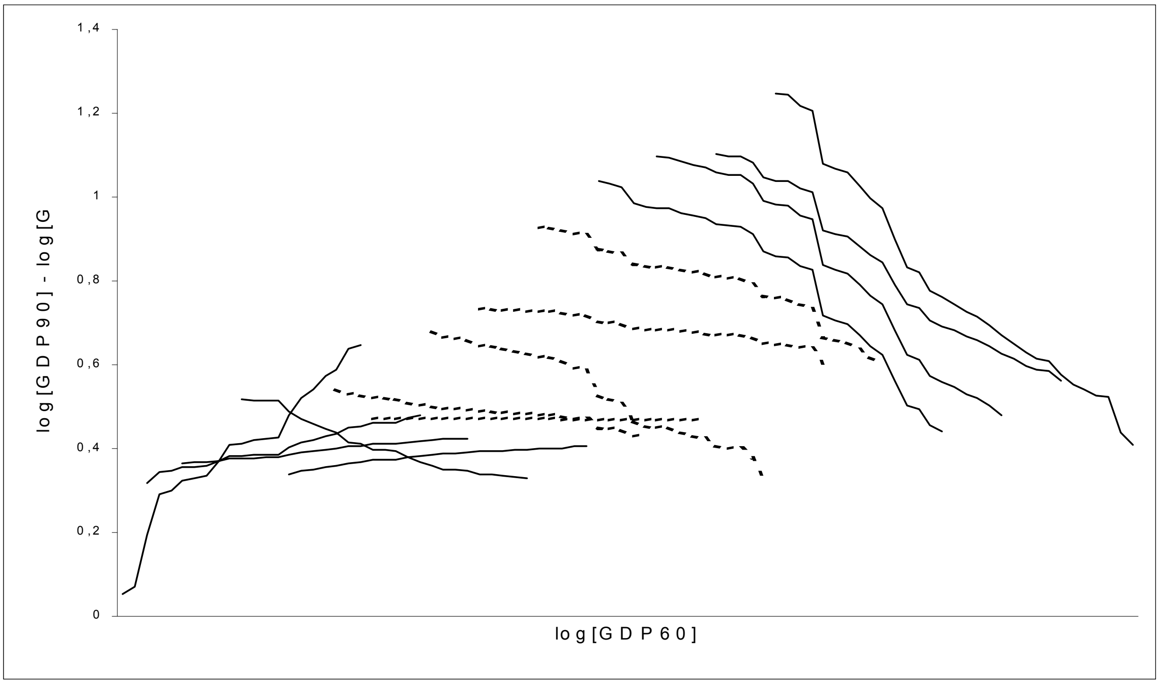

Figure 5:

in 1960, from the lowest to the highest level. Then we have considered groups of 30 countries, changing the sample by moving of 5 items (dropping the first 5 poorer and taking the next 5 richer countries). The moving samples show I-convergence when per capita income is low, ambiguous or no correlation for middle income samples and II-convergence results for high-income countries confirming our theoretical findings. Results are reported in table 3 and shown in figure 5.

Table 3 *Testing conditional non monotonic convergence (std.err. in brakets)*

Sample	const	log(gdp60)	log(n+δ+x)	log(s)	Adj. R^2
rgdp60<930	-4.8080	0.6703	-0.2016	0.6407	0.5464
	(3.7926)	(.3606)	(1.2139)	(.1297)	
400<rgdp60<1050	2.7039	0.2007	-2.2909	0.4703	0.2903
	(3.5557)	(.4713)	(1.4308)	(.1549)	
540<rgdp60<1110	4.3937	0.0860	-2.8384	0.5372	0.3750
	(3.0577)	(.4471)	(1.3293)	(.1442)	
632<rgdp60<1180	10.1420	-0.3091	-4.4079	0.5961	0.4859
	(3.7684)	(.4153)	(1.4088)	(.1320)	
650<rgdp60<1300	8.2204	0.1132	-4.8840	0.5661	0.5239
	(4.5667)	(.5546)	(1.5543)	(.1196)	
810<rgdp60<1620	10.2795	-0.1727	-4.8998	0.5751	0,5711
	(4.3766)	(.4753)	(1.3791)	(.1129)	
930<rgdp60<1720	8.7649	-0.0025	-4.7383	0.5472	0.5910
	(4.1589)	(.4070)	(1.2603)	(.1072)	
1050<rgdp60<2050	6.6482	-0.5381	-1.9197	0.6416	0.5820
	(3.6766)	(.4231)	(.8448)	(.1187)	
1110<rgdp60<2850	3.1641	-0.1385	-1.5567	0.6008	0.5079
	(3.4123)	(.3771)	(.8220)	(.1356)	
1180<rgdp60<3500	3.6846	-0.3011	-1.3392	0.6945	0.4891
	(3.0087)	(.3069)	(.7473)	(.1624)	
1300<rgdp60<5200	2.5075	-0.4631	-0.7962	1.1394	0.5144
	(2.7479)	(.2156)	(.6565)	(.2350)	
1620<rgdp60<6100	1.9916	-0.4651	-0.7207	1.2601	0.5396
	(2.5600)	(.1820)	(.6469)	(.2577)	
1720<rgdp60<7250	1.4921	-0.3824	-0.6780	1.1816	0.4742
	(2.0856)	(.1362)	(.5107)	(.2946)	
rgdp60>2050	2.9713	-0.5358	-0.5465	1.0546	0.5771
	(1.6705)	(.1100)	(.4513)	(.2355)	
OECD sample	4.1910	-0.4336	-0.9244	0.6164	0.7416
	(1.1266)	(0.0599)	(0.3707)	(0.2025)	

Equations (20) and (21) imply a positive coefficient for $\ln s$ and negative for $ln(n+\delta+x)$. Moreover, these signs comes out to be verified for each sub-sample. However, the coefficient of $ln(n+\delta+x)$ should be higher (in absolute value) than the coefficient of $\ln s$; this is not verified for the first sample and for the richest four samples (countries with a real GDP grater than 1,300 $), implying a negative share for public capital (see table 4). This result is not new in the empirical literature of growth. See for example Islam [15].

Table 4 *implied α, β, λ for all sub-samples*

Sample	α	β	λ
rgdp60<930	.7348	-.5036	.0171
	(1.7959)	(2,1444)	(.0072)
400<rgdp60<1050	.1888	.7307	.0061
	(.2247)	(.2324)	(.0131)
540<rgdp60<1110	.1837	.7869	.0028
	(.1450)	(.1880)	(.0137)
632<rgdp60<1180	.1264	.8081	.0123
	(.0636)	(.1027)	(.0106)
650<rgdp60<1300	.1133	.8641	.0036
	(.0642)	(.1095)	(.0166)
810<rgdp60<1620	.1134	.8526	.0063
	(.0594)	(.0881)	(.0135)
930<rgdp60<1720	.1154	.8841	.0001
	(.0637)	(.0740)	(.0135)
1050<rgdp60<2050	.2611	.5200	.0257
	(.1699)	(.1629)	(.0092)
1110<rgdp60<2850	.3544	.5639	.0050
	(.3469)	(.2064)	(.0110)
1180<rgdp60<3500	.4234	.3931	.0119
	(.3489)	(.2490)	(.0079)
1300<rgdp60<5200	.9047	-.2725	.0207
	(.5804)	(.6490)	(.0049)
1620<rgdp60<6100	1.0626	-0.4549	.0209
	(2.5600)	(.1820)	(.6469)
1720<rgdp60<7250	1.1143	-.4749	.0161
	(.6651)	(.6705)	(.0033)
rgdp60>2050	.9745	-.4695	.0256
	(.5319)	(.5812)	(.0024)
OECD sample	.4539	.2268	.0189
	(.2677)	(.2434)	(.0014)

For a direct comparison with MRW findings we also estimated the convergence equation for the OECD group. In this case (see table 4), the implied physical capital share α, the human/public capital share β, and the convergence speed λ are very close to the MRW values, obtained in the restricted convergence regression including a measure of human capital among the explanatory variables[12].

Finally, we observe that the average speed of convergence is increasing for richer countries in terms of per capita 1960 real GDP. A country appears to move slowly away from the unstable steady state. In contrast when it approaches the Pareto superior stable steady state the dynamics gets faster.

5 Concluding remarks

In this paper we propose a standard Solow model augmented with public capital in the production function. Public capital is provided by the goverment through non distortionary taxation. Assuming the public sector objective to be the maximization of per capita consumption in the long run, the model solution displays two steady state levels of capital per effective worker. The Pareto optimum steady state is stable, whereas the Pareto inferior one is unstable. As a consequence, the model predicts both divergence for low levels of initial private capital, and non monotonic conditional convergence for levels of capital between the two steady states. The non monotonic convergence result predicts that the transition towards the non trivial steady state is characterized by increasing growth rates in per capita income (I-convergence) up to the point where the traditional convergence behavior (II-convergence) eventually brings the economy to the stationary equilibrium. Therefore, the model enables a new interpretation of the significance level of the β-convergence parameter estimated in MRW. In particular, findings of β-divergence might be evidence of conditional I-convergence; whereas, not statistically significant estimates of the speed of convergence might suggest that the sample of countries lies between I-convergence and II-convergence.

Moreover, our model specification allows a new interpretation of traditional growth regressions, testing the ability of saving and populazione growth to explain cross.country income differences, in terms of better figures of the implied capital share.

[12] In this case MRW [20] find the following result for the restricted regression: $\alpha = 0.38$ $\beta = 0.23$ $\lambda = 0.0206$.

APPENDIX A: Approximation of the Unstable Private Capital Steady State Solution

Let $k^{'}$ and k^{*} be the real roots of $\dot{k}=0$ with $h=h^{*}$. From the twin conditions

$$\begin{aligned} sf(k^{'},h^{*})-s(n+\delta+x)-(n+\delta+x)k^{'} &= 0 \\ sf(k^{*},h^{*})-s(n+\delta+x)-(n+\delta+x)k^{*} &= 0 \end{aligned}$$

we get:

$$\frac{sh^{*\beta}}{(n+\delta+x)}(k^{*\alpha}-k^{'\alpha})=(k^{*}-k^{'}) \tag{A1}$$

[A1] cannot be directly solved for k'. Anyway, after repeated numerical sulutions employing various sets of parameters values, we guess $k^{'}$ to be a linear function of k^{*}

$$k^{'}=\theta k^{*} \tag{A2}$$

Substituting [A2] into [A1], and considering the explicit values for h^{*} and k^{*}we obtain

$$g(\theta)=\theta^{\alpha}-\theta(1-\beta)-\beta=0 \tag{A3}$$

To obtain an approximation for the non trivial solution to [A3][13], we try to apply the Newton method. Therefore, we need to set an initial value for θ as the first step of iteration procedure. To this purpose, we try to guess the solution for limiting values of α and β. In particular, we proceed deriving $(k^{'},\, k^{*})$ in the following three cases: (i) $\beta\rightarrow 0$, (ii) $\alpha\rightarrow 0$, (iii) $\beta\rightarrow 0$ and $\alpha\rightarrow 0$[14].

Recall equation [8] and derive the $\dot{k}=0$ condition in each case. Consider first (i).

[13] It is easily cheked that $\theta=1$ represents the trivial solution to [A3].

[14] The remaining limiting case $(\alpha+\beta)\rightarrow 1$ is not relevant, because, depending on the values of $(n,\, s,\, x,\, \delta)$, $k^{*}$and $k^{'}$ can be both zero or infinity.

$$\lim_{\beta \to 0} sf(k, h^*) - s(n+\delta+x)h^* - k(n+\delta+x) = sk^\alpha - k(n+\delta+x) = 0 \quad \text{(A4)}$$

[A4] has the two solutions:

$$\begin{aligned} k^* &= \left(\frac{s}{(n+\delta+x)}\right)^{\frac{1}{1-\alpha}} \\ k' &= 0 \end{aligned}$$

Considering case (ii), as α goes zero, $sf(k, h^*)$ approaches to a constant ($sf(k, h^*) = [\beta/(n+\delta+x)]^{\frac{\beta}{1-\beta}}$), while h^* tends to $h^* = [\beta/(n+\delta+x)]^{\frac{1}{1-\beta}}$. Then, the resulting steady state capital values are

$$\begin{aligned} k^* &= s(1-\beta)\left(\frac{\beta^\beta}{(n+\delta+x)}\right)^{\frac{1}{1-\alpha}} \\ k' &= 0 \end{aligned}$$

Finally, considering case (iii), as α and β go to zero, $sf(k, h^*)$ approaches to s , while h^* tends to zero. Then, the resulting steady state capital values are

$$\begin{aligned} k^* &= \frac{s}{(n+\delta+x)} \\ k' &= 0 \end{aligned}$$

Our next step is to guess a solution to [A3], with the requirement that $\theta(\alpha, \beta) = 0$, for α and/or β approaching to zero. With the aid of numerical computations we find that $\theta(\alpha, \beta) = \beta^{\frac{1}{\alpha}}$ satisfies the requirements.

Then, we consider $\theta = \beta^{\frac{1}{\alpha}}$ as the first step of the Newton approximation procedure:

$$\begin{aligned} \text{step 0} &: \quad \theta_0 = \beta^{\frac{1}{\alpha}} \\ \text{step 1} &: \quad \theta_1 = \theta_0 - \frac{g(\theta_0)}{g'(\theta_1)} = \frac{\alpha\beta}{\alpha\beta^{\frac{\alpha-1}{\alpha}} - (1-\beta)} \end{aligned}$$

Given estimates of α and β lower than 0.4, the second step of the iteration provides a good approximation for the true θ solution, so that θ_1 can be used to obtain an (approximated) algebric expression for $k^{'}$.

APPENDIX B: The Determination of the Convergence-Divergence Equation

Equation [18] implies that:

$$\ln y(t) = (1 - e^{-\lambda t}) \ln y^* + e^{-\lambda t} \ln y(0) \tag{B1}$$

where $y(0)$ represents output per effective worker at some initial date. Similarly, equation [9] implies that

$$\ln y(t) = -(1 - e^{\lambda t}) \ln y^{'} + e^{\lambda t} \ln y(0) \tag{B2}$$

Subtracting $\ln y(0)$ from both sides and substituting for y^* and $y^{'}$ we get

$$\ln y(t) - \ln y(0) = a_i\ const_i + a_i y(0) + a_i \frac{\alpha}{1-\alpha-\beta} \ln s - a_i \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+\delta+x) \tag{B4}$$

where $i = 1$ refers to the stable steady state y^* and $i = 2$ to the unstable steady state $y^{'}$.Coefficients in [B3] take different values in the two cases:

$$\begin{aligned} const_1 &= \frac{\beta \ln \beta + \alpha \ln(1-\beta)}{1-\alpha-\beta} < 0 \\ a_1 &= (1 - e^{-\lambda t}) > 0 \\ const_2 &= \left\{ \frac{\beta \ln \beta + \alpha \ln(1-\beta)}{1-\alpha-\beta} + \alpha \ln \left[\frac{\alpha\beta}{\alpha\beta^{\frac{\alpha-1}{\alpha}} - (1-\beta)} \right] \right\} < 0 \\ a_2 &= -(1 - e^{\lambda t}) > 0 \end{aligned}$$

For estimation purpose it is useful toexpress the equations in terms of variables per capita instead of per effective worker. Taking into accounts of the condition $A(t) = A(0)e^{xt}$in the left neighbour of y^*we get:

$$\ln\left[\frac{Y(t)}{L(t)}\right]-\ln\left[\frac{Y(0)}{L(0)}\right] = xt-a_1\ln A(0)+a_1\,const_1-a_1\ln\left[\frac{Y(0)}{L(0)}\right]+$$
$$+a_1\frac{\alpha}{1-\alpha-\beta}\ln s-a_1\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+\delta+x)$$

and in the right neighbour of $y^{'}$:

$$\ln\left[\frac{Y(t)}{L(t)}\right]-\ln\left[\frac{Y(0)}{L(0)}\right] = xt-a_2\ln A(0)+a_2\,const_2+a_2\ln\left[\frac{Y(0)}{L(0)}\right]+$$
$$+a_2\frac{\alpha}{1-\alpha-\beta}\ln s-a_2\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+\delta+x)$$

APPENDIX C : The Minimum Time Problem

Assume the following Minimum Time (MT) problem (switched to a max problem):

$$\max\int_0^T -1\;dt \tag{C1}$$

subject to:

$$\begin{aligned}\dot{k} &= sf(k,h)-su-s(n+\delta+x)h-(n+\delta+x)k \\ \dot{h} &= u\end{aligned} \tag{C2}$$

and the following terminal conditions, with given initial values for k and h:

$$
\begin{aligned}
h(T) &= h^* && \text{(C3)} \\
\dot{k}(T) &= 0 \\
k(T) &= k^* \\
u(T) &= 0 \\
T \quad & free
\end{aligned}
$$

We set up the Hamiltonian:

$$
H(t) = -\lambda_0 + \lambda(t) \left[sf\left(k, h\right) - s(n + \delta + x)h - (n + \delta + x)k \right] + \left[\mu(t) - s\lambda(t) \right] u \tag{C4}
$$

where $\lambda(t)$ and $\mu(t)$ are the multipliers associated to the law of motions governing the evolution of k and h, and λ_0 is the auxilary variable (which can be either 0 or 1) associated to the maximand [C1].

The necessary conditions for an interior solution are:

$$
\begin{aligned}
\frac{\partial H}{\partial k} &= -\dot{\lambda} \implies \dot{\lambda} + \lambda(sf_k - (n + \delta + x)) = 0 && \text{(C4.1)} \\
\frac{\partial H}{\partial h} &= -\dot{\mu} \implies \dot{\mu} + s\lambda(f_h - (n + \delta + x)) = 0 && \text{(C4.2)} \\
\frac{\partial H}{\partial u} &= \mu - s\lambda = 0 && \text{(C4.3)}
\end{aligned}
$$

where f_k and f_h indicate the partial derivatives of $f(k, h)$ with respect to k and h respectively. Furthermore, because the terminal date T is free, and k and h are given in T, the following terminal condition must be met:

$$
\begin{aligned}
H(T) &= -\lambda_0 + \lambda(T) \left[sf\left(k^*, h^*\right) - s(n + \delta + x)h^* - (n + \delta + x)k^* \right] && \text{(C5)} \\
&\quad + \left[\mu(T) - s\lambda(T) \right] u(T) = 0
\end{aligned}
$$

If condition [C4.3] cannot be satisfied, then the solution is "bang-bang", that is the control u will be discontinuous. In particular:

$$
\begin{aligned}
\mu - s\lambda &> 0 \implies H \text{ is maximized setting } u = u_{\max} && \text{(C4.4)} \\
\mu - s\lambda &< 0 \implies H \text{ is maximized setting } u = u_{\min}
\end{aligned}
$$

with $u_{\max}$ and $u_{\min}$ discretionary upper and lower limits exogenously set by the goverment. We can now state several results, which can help to outline the shape of the solution .

Result 1

The objective [C1] does not matter in the solution, i.e. λ_0 *must be zero* .

Proof. Consider condition [5]. Because $u(T) = 0$, $k(T) = k^*$ and $h(T) = h^*$, this implies that $sf(k^*, h^*) - (n+\delta+x)k^* - s(n+\delta+x)h^* = \dot{k}(T) = 0$. Therefore, the transversality condition can only be satisfied if

$\lambda_0 = 0$. ■

The hamiltonian [C4] then reformulates as follows:

$$H(t) = \lambda(t)\left[sf\left(k,h\right) - s(n+\delta+x)h - (n+\delta+x)k\right] + \left[\mu(t) - s\lambda(t)\right]u \tag{C6}$$

Result 2

The Hamiltonian is constant and equal to zero for $0 \leq t \leq T$.

Proof. The problem is autonomous. Therefore $\partial H/\partial t = 0$. As $H(T) = 0$ at the terminal date, this implies that $H(t) = 0$ for $0 \leq t \leq T$. ■

These preliminary considerations leads to the following proposition:

Proposition 1 *Any interior solution cannot be sustained over the time interval* $[0, T]$.

Proof. Any interior solution must be driven by condition [C4.3]. Substituting [C4.3] into conditions [C4.2] and [C4.1], obtains the following implication:

$$\mu = s\lambda \implies sf_k = f_h \tag{i}$$

condition [i] implies that the ratio k/h is constant:

$$\frac{k(t)}{h(t)} = \frac{s\alpha}{\beta} \tag{ii}$$

As $\mu - s\lambda = 0$, the $H(t) = 0$ condition implies that $sf\left(k,h\right) - s(n+\delta+x)h - (n+\delta+x)k = 0$, or, equivalently, that:

$$\dot{k} + s\dot{h} = 0 \tag{iii}$$

From (iii) follows that $\dot{k}$ and $\dot{h}$ must have opposit sign, which contradicts condition [ii] ■

We now look for an admissible bang-bang solution. To this purpose, we prove the following propositions:

Proposition 2 *If* $u_{\min} < 0$ $(u_{\max} > 0)$ *then no bang-bang solution exists*

Proof. We first note that, if $u_{\min} < 0$, then from the $H = \lambda\dot{k} + \mu\dot{h} = 0$ condition follows that $\dot{k} > 0$ and (λ, μ) must have the same sign. From [4.1] we know that the sign of λ is enterely determined by $\lambda(0)$. Suppose $\lambda > 0$ (the reverse reasoning apply if $\lambda < 0$). Then, when $u_{\max} > 0$ applies, $\dot{k}$ must be negative. To see this, consider that $\mu - s\lambda > 0 \rightarrow u_{\max} > 0$, as a consequence $sf(k,h) - s(n+\delta+x)h - (n+\delta+x)k$ must be negative for $H = 0$, which eventually implies that $\dot{k} < 0$.

Second, consider the behaviour of f_h and sf_k over the transition path:

$$\begin{aligned} s\dot{f}_k &= sf_k\left[(\alpha-1)\frac{\dot{k}}{k} + \beta\frac{\dot{h}}{h}\right] \\ \dot{f}_h &= f_h\left[\alpha\frac{\dot{k}}{k} + (\beta-1)\frac{\dot{h}}{h}\right] \end{aligned}$$

These preliminary considerations provide the following restrictions to the bang-bang solution:

Rule A ($u_{\max}$): $\mu - s\lambda > 0$; $\dot{h} > 0$; $\dot{k} < 0$; $\dot{f}_h < 0$; $s\dot{f}_k > 0$

Rule B ($u_{\min}$): $\mu - s\lambda < 0$; $\dot{h} < 0$; $\dot{k} > 0$; $\dot{f}_h > 0$; $s\dot{f}_k < 0$

Furthermore, define $z = \mu - s\lambda$. Then, from [C4.1[and [C4.2[we get the following result:

$$sf_k \gtreqless f_h \Leftrightarrow \dot{z} \gtreqless 0$$

Given the above restrictions, we note that the A rule can lead to a switch (B rule) only if $sf_k < f_h$.Now suppose the contrary. Then, as $\dot{f}_h < 0$ and

$s\dot{f}_k > 0$, it is ever verified $\dot{z} > 0$. Thus, the A rule can never reach a switch if $sf_k > f_h$. It follows that, the A rule can be actually set only if the B rule leads to a switch with $\left\{\dot{h} > 0, \dot{k} < 0\right\}$ determining $z > 0$ and $sf_k < f_h$. Now, as the B rule is characterized by $z < 0$, to reach a switch the B rule $\left\{\dot{h} < 0, \dot{k} > 0\right\}$ must be accompanied by $\dot{z} > 0$. But, this implies that $sf_k > f_h$, so that in the switch point it will necessarily verified $sf_k \geq f_h$, a condition which starts up an explosive path, with $\dot{z} > 0$ forever, perpetually diverging the economy from the $f_h(T) > sf_k(T)$ terminal condition[15].

Therefore, as the A rule can never be implemented, a bang-bang solution with $u_{\min} < 0$ cannot exist. ■

Proposition 3 $u_{\min} = 0$ $(u_{\max} > 0)$ *cannot be a bang bang solution*

Proof. Proof is immediate. Because $\dot{h} = u_{\min} = 0$, under the B rule the hamiltonian reduces to

$$H(t) = \lambda(t)\left[sf(k,h) - s(n+\delta+x)h - (n+\delta+x)k\right]$$

which can be zero only if $\lambda(t) = 0$. This implies that $\lambda(t) = 0$ for any $0 \leq t \leq T$. Consider now the opposite rule. If $\dot{h} = u_{\max} > 0$, from the $H = \lambda\dot{k} + \mu\dot{h} = 0$ condition follows that μ must be zero. Thus, the A rule can never be set ■

Proposition 4 $u_{\min} > 0$ $(u_{\max} > 0)$ *cannot be a bang-bang solution*

Proof. We show this, by proving first that the B rule can never lead to a switch. From $u_{\min} > 0$, it follows that $\dot{k} > 0$. Then, from the $H = \lambda\dot{k} + \mu\dot{h} = 0$ condition follows that λ and μ must have opposite sign. Suppose for simplicity that $\lambda > 0$ $(\mu < 0)$ We know that the B rule sets as $\mu - s\lambda < 0$, and a switch occurs when $\mu - s\lambda = 0$. But, because $\mu < 0$, while $\lambda > 0$, a switch can occur only if μ turns positive. But with $\mu > 0$ the B rule cannot met the condition $H = 0$.

Now we complete the proof, by showing that the A rule can never lead to a switch. The A rule is characterized by $\dot{h} = u_{\max} > 0$; this policy rule can be associated to $\dot{k} \gtreqless 0$. Let consider each possible case.

[15] In the steady state, with $k(T) = k^*$ and $h(T) = h^*$, $sf_k(T) = \frac{\alpha}{1-\beta}(n+\delta+x)$, and $f_h(T) = (n+\delta+x)$. Given that $(\alpha+\beta) < 1$, it must be verified that $f_h(T) > sf_k(T)$.

(i) $\left(\dot{h} = u_{\max} > 0,\, \dot{k} > 0\right)$. With $\dot{k} > 0$ the A rule cannot be sustained, because the $H = \lambda \dot{k} + \mu \dot{h} = 0$ condition might be satisfied only if $\mu < 0$, which implies the B rule, as $\mu - s\lambda < 0$.

(ii) $\left(\dot{h} = u_{\max} > 0,\, \dot{k} = 0\right)$.With $\dot{k} = 0$, the $H = 0$ condition can be satisfied only if $\mu = 0$, which again implies the B rule.

(iii) $\left(\dot{h} = u_{\max} > 0,\, \dot{k} < 0\right)$.By proposition 2 we already know that, with $\dot{h} > 0$ and $\dot{k} < 0$ the A rule can never reach a switch ■

The above results summarizes into the following main proposition:

Proposition 5 *The MT problem [1], subject to [2] has non solution.*

Proof. Result [1] and propositions [1,2,3,4] show that the necessary conditions for the existence of a control $\mathbf{u}(t)$ (and associated evolution of $\mathbf{k}$) optimal for the above MT problem do not exist ; i.e. do not exist a constant λ_0 and continuous functions $\lambda(t)$ and $\mu(t)$ with $\{(\lambda_0,\, \lambda(t),\, \mu(t)) \neq (0, 0, 0)\}$ for all $t \in [0, T]$, such that $H(\mathbf{u}, \mathbf{k},\lambda, \mu) \geq H(u, \mathbf{k},\lambda, \mu)$. ■

References

[1] Arellano M. and S. Bond (1991) Some Test of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. *Review of Economic Studies*, 58, pp. 277-297.

[2] Azariadis C. (1996) The Economics of Poverty Traps Part I: Complete Markets. *Journal of Economic Growth*, 1, pp. 449-486.

[3] Barro R.J. (1991) Economic Growth in a Cross Section of Countries, *Quarterly Journal of Economics*, 106, pp. 407-443.

[4] Barro R.J. and X.X. Sala-i-Martin (1991) Convergence across states and regions, *Brookings Paper on Economic Activity*, 1, pp. 107-82.

[5] Barro R.J. and X.X. Sala-i-Martin (1995) *Economic Growth*, New York:McGraw-Hill.

[6] Benhabib J. and J. Gali (1995) On Growth and Indeterminacy: Some Theory and Evidence. *Carnegie Rochester Series in Public Policies*, 43, pp. 163-211.

[7] Capolupo R. (1998) Convergence in recent growth theories: a Survey, *Journal of Economic Studies*, 25, pp. 496-537.

[8] Caselli, F., G. Esquivel and F. Lefort (1996) Reopening the convergence debate: a new look at cross-country growth empirics. *Journal of Economic Growth*, 1, pp. 363-89.

[9] Cass D. (1965) Optimum Growth Theory in an Aggregative Model of Capital Accumulation, *Review of Economic Studies*, 23, pp. 233-240.

[10] De la Fuente A. (2000) Convergence across Countries and Regions: Theory and Empirics, CEPR Discussion Paper n. 2465 (May).

[11] Galor O. (1996) Convergence? Inferences from Theoretical Models. *Economic Journal*, 106, pp. 1056-1069.

[12] Goddard J. and J. Wilson (2001) Cross-sectional and panel estimation of convergence, *Economics Letters*, 70, pp. 327-333.

[13] Grossman, G.M. and E. Helpman (1991) Trade Knowledge Spillovers and Growth, *European Economic Review*, 35, pp. 517-26.

[14] Hamilton J.D. and J. Monteagudo (1998) The Augmented Solow Model and the Productivity Slowdown, *Journal of Monetary Economics*, 42, pp. 495-509.

[15] Islam N. (1995) Growth Empirics: A Panel Data Approach. *Quarterly Journal of Economics*; 110, pp. 1127-70.

[16] Islam, N. (1998) Growth Empirics: A Panel Data Approach - A Reply. *Quarterly Journal of Economics*; 113, pp. 325-29.

[17] Jones C. I. (1997) Convergence Revisited. *Journal of Economic Growth*, 2, pp. 131-153.

[18] King R.G. and S. Rebelo (1990) Public Policy and Economic Growth: Developing Neoclassical Implications, *Journal of Political Economy*, 98, pp. S126-S150.

[19] Lucas, R.E. Jr. (1988) On the Mechanics of Economic Development, *Journal of Monetary Economics*, 22, pp. 3-42.

[20] Mankiw, N.G., D. Romer and D.N. Weil (1992) A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics*, 107, pp. 407-437.

[21] Quah D. (1995) Empirics for Economic Growth and Convergence, CEPR Discussion Paper, N. 1140, March.

[22] Ramsey F.P. (1928) A Mathematical Theory of Saving. *Economic Journal*, 38, pp. 543-559.

[23] Rebelo S. (1991) Long-run Policy Analysis and Long-run Growth, *Journal of Political Economy*, 99, pp. 500-21.

[24] Romer P.M. (1986) Increasing Returns and Long-Run Growth, *Journal of Political Economy*, 94, pp. 1002-1037.

[25] Romer P. M. (1987) Growth Based on Increasing Returns Due to Specialization, *American Economic Review*, 77, pp. 56-62.

[26] Sala-i-Martin X.X. (1996) The Classical Approach to Convergence Analysis, *Economic Journal*, 106, pp. 1019-1036.

[27] Solow, R. (1956) A Contribution to the Theory of Economic Growth, *Quarterly Journal of Economics*, 70, pp. 65-94.

[28] Summers, R. and A. Heston (1991) The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988, *Quarterly Journal of Economics*, 106, pp. 327-368.