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Profit and Decision Sharing in a Principal Agent Model

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Pro t and Decision Sharing in a Principal-Agent Model

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ABSTRACT. Workers participation may be related to economic factors, through pro t sharing, or to organizational aspects, through workers involvement to the decision making process of the rm. Aim of the paper is to investigate the different implications of workers participation, with respect to either economic or organizational factors.

The analysis is conducted in the standard PA model under moral hazard, as it represents the optimal incentive contract form for a wage contingent on output.

Results show that the inclusion of pro t sharing in the PA model under moral hazard is irrelevant in changing effort and welfare equilibrium levels. Instead, changes in the bargaining structure, having worker partecipation in the decision making process of the rm, originates a higher level of effort response, hence a welfare improvement.

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INTRODUCTION

Workers participation may be analyzed from both an economic and an organizational perspective. As far as the former is concerned, workers participation is characterized by remuneration being linked to some firm's measures of economic performance. Instead, workers involvement in the decision making process of the firm defines participation within the organizational perspective.

The main aim of the paper is to investigate the different implications of workers participation, as related either to economic or to organizational factors.

We emphasize what the consequences of the two forms of workers participation on the level of effort delivered by the workers and on the entrepreneur and workers welfare are.

The analysis will be conducted in the standard Principal Agent model under moral hazard, as it represents the optimal incentive contract form for a wage contingent on output. Whether this approach can give some insights about the relative importance between contingency on output and contingency on profits, about effort response to the wage schedule and about social welfare, will be investigated. Moreover, we are interested in analyzing the implications on the effort response and social welfare, when the bargaining structure of the Principal Agent approach changes.

Therefore we will investigate what happens to the equilibrium conditions of worker and entrepreneur when, within the P-A model, (i) a share of the principal profit is introduced into the agent revenue and when (ii) the bargain structure changes, allowing the agent to share decisions with the principal in a collusive setting.

In order to achieve our purpose, the paper is structured as follows.

Section 1 is devoted to the analysis of the equilibrium conditions of the principal and the agent when a share of the firm profit is introduced into the agent revenue. The benchmark of our analysis is represented by the standard Principal-Agent model under moral hazard with one principal and one agent. The consequences of allowing profit sharing in this approach are then analyzed.

Section 2 is dedicated to the PA equilibrium conditions analysis, when the bargaining structure changes, by allowing the agent to share the maximization problem with the principal. There, a collusive setting under moral hazard is considered. Whether in this new setting profit sharing has some role, will then be investigated.

In section 3 we will see how the incentive constraint loses meaning, when the agent participates to the decision process. As a consequence, how effort and welfare are affected in a collusive setting under complete information will be shown.

Concluding remarks close the paper.

1. THE PRINCIPAL-AGENT MODEL UNDER MORAL HAZARD

We start our analysis from the Principal Agent model under moral hazard as it represents the optimal incentive contract for a wage contingent on output. Our purpose is to analyze whether in this approach contingency on profits may have some relative importance with respect to contingency on output, on level of effort and welfare.

In order to achieve our purpose, we first solve the standard problem under no profit sharing, and then we introduce a share of the principal profit into the agent revenue, in every state of nature.

1.1. Assumptions and definitions in the P-A Model.

This section provides a tedious but necessary list of the most relevant definitions and assumptions for the Principal - Agent model that will be used throughout the paper.

Definition 1. Output is determined by the agent's effort level $e \in [e_0; e_1]$ and by a random variable $s \in S \equiv [0; 1]$ which defines the states of nature

$$y = y(e; s) \equiv y_s(e) \in [y_0; y_1]$$

Definition 2. Each effort level implies a distribution function of outcomes contingent on s

$$F[y_s; e]$$

Assumption 1.

$$F_e[y_s; e] < 0$$

The increases in effort reduce the probability of getting an output less than any specified level.

Assumption 2. Convex distribution function condition (CDFC)

$$F_{ee} \geq 0 \tag{1}$$

Again, the effort increase reduce the probability of getting an output less than any specified level, but does so at a decreasing rate.

Definition 3. The distribution $F[y_s; e'']$ stochastically dominates the distribution $F[y_s; e']$, if for $e'' > e'$

$$F[y_s; e''] \leq F[y_s; e']; \quad \forall y \in [y_0; y_1]$$

Definition 4. The probability density function contingent on e is:

$$\frac{\partial F[y_s; e]}{\partial y} \stackrel{def}{=} f(y_s; e)$$

Assumption 3. Monotone likelihood ratio condition (MLRC)

$$\frac{f_e(y_s; e)}{f(y_s; e)} \text{ is non-decreasing in } y \tag{2}$$

or:

$$\frac{\partial (f_e/f)}{\partial y_s} = \frac{f_{ey} f - f_e f_y}{f^2} \geq 0 \tag{3}$$

Corollary 1. MLRC implies $w'(y_s) \in [0; 1]$.

Proof. See the derivation of the first order conditions characterizing the optimal contract under incomplete information. ■

Definition 5. The agent's utility function contingent on the state of nature is:

$$U_s \stackrel{def}{=} u[w(y_s)] - c(e)$$

Therefore we assume that the agent's utility function is additively separable in the components w (wage or pay-off) and e.

Assumption 4. The utility function of the agent is concave in the pay-off:

$$u'(.) > 0; \; u''(.) \leq 0$$

the agent may be either risk-neutral or risk-averse.

Assumption 5. The disutility. function of the agent is convex in the effort:

$$c'(.) > 0; c''(.) \geq 0$$

Definition 6. The principal's profit function contingent on the state of nature is:

$$\pi_s \overset{def}{=} \pi[y_s - w(y_s)]$$

Assumption 6. The profit function of the principal is concave:

$$\pi'(.) > 0; \pi''(.) \leq 0$$

the principal may be either risk-neutral or risk-averse.

Definition 7. The expected utility function of the agent is:

$$\begin{aligned} EU &\overset{def}{=} \int_{y_0}^{y_1} u[w(y)]\, f(y;e)\, dy - c(e) = \\ &= u[w(y)]\, F[y;e]_{y_0}^{y_1} - \int_{y_0}^{y_1} u[w(y)]\, F[y;e]\, dy - c(e) = \\ &= u[w(y_1)] - \int_{y_0}^{y_1} u'[w(y)]\, w'(y)\, F[y;e]\, dy - c(e) \end{aligned}$$

Definition 8. The expected profit function of the principal is:

$$\begin{aligned} E\pi &\overset{def}{=} \int_{y_0}^{y_1} \pi[y - w(y)]\, f[y;e]\, dy = \pi[y - w(y)]\, F[y;e]_{y_0}^{y_1} + \\ &- \int_{y_0}^{y_1} \pi'[y - w(y)]\, [1 - w'(y)]\, F[y;e]\, dy = \\ &= \pi[y_1 - w(y_1)] - \int_{y_0}^{y_1} \pi'[y - w(y)]\, [1 - w'(y)]\, F[y;e]\, dy \end{aligned}$$

Proposition 1. Under the assumptions (1) and (2) the expected profit function and the expected utility function are concave in the effort.

Proof.

$$\frac{\partial E\pi}{\partial e} = -\int_{y_0}^{y_1} \pi'[y - w(y)]\, [1 - w'(y)]\, F_e[y;e]\, dy > 0 \tag{4}$$

because $F_e < 0$, and because MLRC implies $w'(y) \in (0;1)$

$$\frac{\partial^2 E\pi}{\partial e^2} = -\int_{y_0}^{y_1} \pi'[y - w(y)]\, [1 - w'(y)]\, F_{ee}[y;e]\, dy \leq 0 \tag{5}$$

because $F_{ee} \geq 0$.

$$\frac{\partial Eu}{\partial e} = -\int_{y_0}^{y_1} u'[w(y)]\, w'(y)\, F_e[y;e]\, dy > 0 \tag{6}$$

because $F_e < 0$, and because MLRC implies $w'(y) \in (0;1)$

$$\frac{\partial^2 Eu}{\partial e^2} = -\int_{y_0}^{y_1} u'[w(y)]\, w'(y)\, F_{ee}[y;e]\, dy \leq 0 \tag{7}$$

because $F_{ee} \geq 0$, and $c''(e) \geq 0$. ■

Definition 9. Define the "constant contract under complete information" the pair:

$$\left(\beta^{CI}; e^{CI}\right)$$

Definition 10. Define the "not revealing output level" (y_{ni}), that level of output for which the likelihood ratio is zero:

$$y_{ni}: \quad \frac{f_e(y_{ni};e)}{f}=0$$

and such that the agent's pay-off is constant and equal to the contract under complete information

Corollary 2. The function $f_e\left(y;e^{CI}\right)$ is such that:

$$\begin{array}{l} Y^- := \left\{y\in[y_0;y_{ni}) : f_e\left(y_s;e^{CI}\right)<0\right\} \\ Y^+ := \left\{y\in(y_0;y_{ni}] : f_e\left(y_s;e^{CI}\right)>0\right\} \end{array}$$

that is, given a signal of output less than y_{ni} every increase in the effort implies a decrease in the probability and vice versa.

1.2. The P-A. Model under "No profit-sharing".

The problem is the standard Principal-Agent model under moral hazard with one principal and one agent, that we show here just to set the framework and because it represents the benchmark of our analysis.

As usual, the principal maximizes her expected profit subject to the participation constraint and to the incentive compatibility constraint:

$$\max_{w;e} E\pi=\int_{y_0}^{y_1}\pi[y-w(y)]\,f(y;e)\,dy$$

$$s.t.:\quad \begin{cases} \int_{y_0}^{y_1}u[w(y)]\,f(y;e)\,dy-c(e)\geq U_R \\ \int_{y_0}^{y_1}u[w(y)]\,f_e(y;e)\,dy-c'(e)=0 \end{cases}$$

Notice that the incentive constraint imposes that the agent is maximizing his objective function with respect to e.

The Lagrangian is:

$$L=\int_{y_0}^{y_1}\pi[y-w(y)]\,f(y;e)\,dy+\lambda\left(\int_{y_0}^{y_1}u[w(y)]\,f(y;e)\,dy-c(e)-U_R\right)+$$
$$+\mu\left(\int_{y_0}^{y_1}u[w(y)]\,f_e(y;e)\,dy-c'(e)\right)$$

The problem becomes a system of four equations in four unknown variables $(w;e;\lambda;\mu)$. The four equations are the first order conditions with respect to w, e, λ and μ.

Since the FOCs with respect to λ and μ are just the participation and the incentive constraints, we concentrate our analysis only on the FOCs with respect to w and e.

FOC w.r.t. w.

$$\frac{\partial L}{\partial w}:\frac{\partial\pi[y-w(y)]}{\partial[y-w(y)]}\frac{\partial[y-w(y)]}{\partial w(y)}f(y;e)+\lambda\frac{\partial u[w(y)]}{\partial w(y)}f(y;e)+ \tag{8}$$
$$+\mu\frac{\partial u[w(y)]}{\partial w(y)}f_e(y;e)=0;\quad w>0$$

where $\frac{\partial[y-w(y)]}{\partial w(y)}=-1$.

Dividing through by $\frac{\partial u[w(y)]}{\partial w(y)}$ and $f(y;e)$, and under the condition of binding constraint, we get the standard P-A incentive contract:

$$\frac{\frac{\partial \pi}{\partial [y-w(y)]}}{\frac{\partial u}{\partial w(y)}} = \lambda + \mu \frac{f_e}{f} \tag{9}$$

Since the likelihood ratio, $\frac{f_e}{f}$, is non decreasing in output (see equation (2)), the contract is always increasing in the result in output obtained. In particular, if we denote by $\rho_P = -\pi''/\pi'$ the principal s measure of absolute risk-aversion, and by $\rho_A = -u''/u'$ the agent s measure of absolute risk-aversion, we get the following.

Proposition 2. The revenue of the agent, ceteris paribus, is increasing in the realization of output, the less he is risk- averse. Formally:

$$\frac{dw}{dy} = \frac{\rho_P + \mu \frac{u'}{\pi'} \frac{\partial (f_e/f)}{\partial y(s)}}{\rho_P + \rho_A}$$

Proof. By differentiating (8) w.r.t. y; and recalling that $\rho_P = -\pi''/\pi'$ and $\rho_A = -u''/u'$ ■

The above proposition states that the more the agent is risk-averse, the less she likes the dependence of his wage on output.

Let s now see what the optimal condition, with regards to effort, shows.

FOC w.r.t. e.

Since the dependence on effort is only in the probability, we get:

$$\frac{\partial L}{\partial e}: \int_{y_0}^{y_1} \pi[y - w(y)]\, f_e(y;e)\, dy + \lambda \left(\int_{y_0}^{y_1} u[w(y)]\, f_e(y;e)\, dy - c'(e) \right) + \\ + \mu \left(\int_{y_0}^{y_1} u[w(y)]\, f_{ee}(y;e)\, dy - c''(e) \right) = 0; \quad e > 0 \tag{10}$$

We immediately see that the second term is zero because of the incentive constraint, while the third term is negative for the incentive constraint to hold. Hence, finally:

$$\frac{\partial E\pi}{\partial e} = -\mu \left(\frac{\partial^2 Eu}{\partial e^2} - c''(e) \right) \geq 0$$

that means that the principal is never satisfied with the agent s effort. She always wants more.

Let s now see what kind of considerations can be made about welfare.

Welfare.

Since welfare is given by the sum of principal and agent expected values of the objective functions at the equilibrium point, we have to calculate these values when w, e, λ and μ are those of equilibrium. For what concerns agent s expected utility there are no problems, since by the participation constraint it is equal to the reservation utility. For what concerns principal s expected profit we proceed as follows.

Let s consider (8), integrating it with respect to the $\{w_s\}$ sequence

$$\int \frac{\partial \pi [y_s - w(s)]}{\partial [y_s - w(s)]} \frac{\partial [y_s - w(s)]}{\partial w(s)} f(y_s;e)\, dw(s) + \lambda \int \frac{\partial u[w(s)]}{\partial w(s)} f(y_s;e)\, dw(s) + \\ + \mu \int \frac{\partial u[w(s)]}{\partial w(s)} f_e(y_s;e)\, dw(s)$$

one obtains the primitive along the equilibrium sequence:

$$\pi[y_s - w^*(s)] = \left(\lambda^* + \mu^* \frac{f_e}{f}\right) u[w^*(s)]$$

By integrating it over the states of nature, we have

$$\int_{y_0}^{y_1} \pi[y_s - w^*(s)]\, f(y_s;e)\, ds = \lambda^* \int_{y_0}^{y_1} u[w^*(s)]\, f(y_s;e)\, ds + \mu^* \int_{y_0}^{y_1} u[w^*(s)]\, f_e(y_s;e)\, ds;$$

but, knowing that the expected value of the utility function, by the participation constraint, is equal to $U_R + c(e^*)$, we have:

$$E\pi[y_s - w^*(s)] = \lambda^*[U_R + c(e^*)] + \mu^* M(e^*)$$

where $M(e^*)$ has positive or negative sign depending on where the point $\hat{y}$ such that $f_e^*\left(\hat{y}_s; e^*\right) = 0$ is (remembering the following assumption:

$$\begin{cases} Y^- := \left\{ y \in [y_0; y_{n-1}) : f_e\left(y_s; e^{CI}\right) < 0 \right\} \\ Y^+ := \left\{ y \in (y_0; y_{n-1}] : f_e\left(y_s; e^{CI}\right) > 0 \right\} \end{cases}$$

Welfare, hence, results:

$$W(w^*; e^*) = U_R + \lambda^*[U_R + c(e^*)] + \mu^* M(e^*)$$

1.3. The P-A. Model under Profit-sharing.

We want now to analyze what happens to the level of effort delivered by the agent and to the principal and agent welfare when we introduce a share of the principal profit in the agent revenue.

What we are now assuming is that the principal offers to the agent a remuneration which includes the standard incentive contract and a share of the profits which remain after the contract has been paid.

When we introduce a share of the principal profit into the agent revenue, in every state of nature, the problem becomes:

$$\max_{w;e} E\pi(w; e \mid y) = \int_{y_0}^{y_1} \pi\left\{(1-\theta)\left[y - w^{PS}(y)\right]\right\} f[y;e]\, dy$$

$$\text{s.t.:} \begin{cases} \int_{y_0}^{y_1} u\left\{w^{PS}(y) + \theta\left[y - w^{PS}(y)\right]\right\} f[y;e]\, dy - c(e) \geq U_R \\ \int_{y_0}^{y_1} u\left\{w^{PS}(y) + \theta\left[y - w^{PS}(y)\right]\right\} f_e[y;e]\, dy - c'(e) = 0 \end{cases}$$

The Lagrangian is:

$$L = \int_{y_0}^{y_1} \pi\left[x^{PS}\right] f(y;e)\, dy + \lambda\left(\int_{y_0}^{y_1} u\left[z^{PS}\right] f(y;e)\, dy - c(e) - U_R\right) + \mu\left(\int_{y_0}^{y_1} u\left[z^{PS}\right] f_e(y;e)\, dy - c'(e)\right);$$

where:

$$x^{PS} = (1 - \theta)\left[y - w^{PS}(y)\right]$$

is the principal revenue under profit sharing, and

$$z^{PS} = w^{PS}(y) + \theta\left[y - w^{PS}(y)\right] = (1 - \theta)\, w^{PS}(y) + \theta y$$

is the agent revenue under profit sharing.

Notice that:

$$\frac{\partial x^{PS}}{\partial w^{PS}} = -(1 - \theta)$$

and:

$$\frac{\partial z^{PS}}{\partial w^{PS}} = (1 - \theta)$$

Following the structure of the previous section, we concentrate our analysis only on the FOCs with respect to w^{PS} and e.

FOC w.r.t. w^{PS}.

$$\frac{\partial L}{\partial w^{PS}} : \frac{\partial \pi}{\partial x^{PS}} \frac{\partial x^{PS}}{\partial w^{PS}} f(y; e) + \lambda \frac{\partial u}{\partial z^{PS}} \frac{\partial z^{PS}}{\partial w^{PS}} f(y; e) + \tag{11}$$
$$+ \mu \frac{\partial u}{\partial z^{PS}} \frac{\partial z^{PS}}{\partial w^{PS}} f_e(y_s; e) \quad 0 \qquad w^{PS} \geq 0$$

and since

$$\frac{\partial x^{PS}}{\partial w^{PS}} = -(1 - \theta); \qquad \frac{\partial z^{PS}}{\partial w^{PS}} = (1 - \theta);$$

we get:

$$\frac{\partial L}{\partial w^{PS}} : -\frac{\partial \pi}{\partial x^{PS}} (1 - \theta) f(y; e) + \lambda \frac{\partial u}{\partial z^{PS}} (1 - \theta) f(y; e) + \tag{12}$$
$$+ \mu \frac{\partial u}{\partial z^{PS}} (1 - \theta) f_e(y_s; e) \quad 0 \qquad w^{PS} \geq 0$$

The term $(1 - \theta)$ cancels out leading to:

$$\frac{\partial L}{\partial w^{PS}} : -\frac{\partial \pi}{\partial x^{PS}} + \frac{\partial u}{\partial z^{PS}} \left[\lambda + \mu \frac{f_e(y; e)}{f(y; e)}\right] \quad 0 \qquad w^{PS} \geq 0 \tag{13}$$

From equation (13), and under the condition of binding constraint, the PA incentive contract under profit sharing is:

$$\frac{\frac{\partial \pi}{\partial x^{PS}}}{\frac{\partial u}{\partial z^{PS}}} = \lambda + \mu \frac{f_e}{f}$$

Since the contract defining the solution for z^{PS} is the same as the contract defining w in the P-A model under no profit sharing, we are able to establish the first irrelevance result.

Proposition 3. In the PA model, profit sharing in every state of nature does not change the agent equilibrium revenue. Formally:

$$w = z^{PS} \equiv w^{PS} + \theta\left(y - w^{PS}\right)$$

Proof. Consider the equilibrium four FOC equations defining the under no profit sharing P-A equilibrium: $\{w^*, e^*, \lambda^*, \mu^*\}$. Consider now the equilibrium four FOC equations defining the under profit sharing P-A equilibrium $\left\{w^{PS*}, e^{PS*}, \lambda^{PS*}, \mu^{PS*}\right\}$. We first notice that z^{PS*} solves (13) as w^* solved (8). Hence the agent total revenue does not change ■

Hence, in the P-A model under profit sharing, the contingent wage varies according to keep $z^{PS} \equiv w^{PS} + \theta\left(y - w^{PS}\right) = w$.

Corollary 3. *The optimal level of share, θ^*, is uniquely determined by equation (13).*

Moreover, we get the following result.

Corollary 4. *The two revenue components, the contingent wage w^{PS} and the profit share $\theta\left(y - w^{PS}\right)$ are perfect substitutes.*

Proof. Let's rewrite the agent revenue $z^{PS} = w^{PS} + \theta\left(y - w^{PS}\right) = (1-\theta)\,w^{PS} + \theta y$. By totally differentiating the (12) w.r.t. the two arguments $(1-\theta)\,w^{PS}$ and θy, we obtain:

$$\frac{dw_s^{PS}}{d\left[\theta\left(y_s - w_s^{PS}\right)\right]} = -\frac{1}{1-\theta} \tag{14}$$

Infact:

$$\left[-\frac{\partial^2 \pi_s}{\partial (x^{PS})^2} f(y_s;e) + \lambda \frac{\partial^2 u_s}{\partial (z^{PS})^2} f(y_s;e) + \mu \frac{\partial^2 u_s}{\partial (z^{PS})^2} f_e(y_s;e)\right](1-\theta)\,dw_s^{PS} +$$
$$+\left[-\frac{\partial^2 \pi_s}{\partial (x^{PS})^2} + \lambda \frac{\partial^2 u_s}{\partial (z^{PS})^2} f(y_s;e) + \mu \frac{\partial^2 u_s}{\partial (z^{PS})^2} f_e(y_s;e)\right] d\left[\theta\left(y_s - w_s^{PS}\right)\right] = 0$$

■

The above irrelevance result is due to the homogeneity of the two sources of revenue, wage and share of profits, since w and y are measured by the same unit. Anyway, since the agent is maximizing over the contingent wage and not over the total revenue, the result is not expected at all.

Let's now analyze what the FOC with respect to e shows.

FOC w.r.t. e:

$$\frac{\partial L}{\partial e} = \int_{y_0}^{y_1} \pi\left[x^{PS}\right] f_e(y;e)\,dy + \lambda\left(\int_{y_0}^{y_1} u\left[z^{PS}\right] f_e(y;e)\,dy - c'(e)\right) +$$
$$+\mu\left(\int_{y_0}^{y_1} u\left[z^{PS}\right] f_{ee}(y;e)\,dy - c''(e)\right) \le 0; \qquad e \ge 0$$

Also in this case[1] the second term is zero because of the incentive constraint, and the third term is negative for the incentive constraint to hold. Hence:

$$\frac{\partial E\pi}{\partial e} = -\mu\left(\frac{\partial^2 Eu}{\partial e^2} - c''(e)\right) \ge 0$$

which is the same condition as in the P-A model under no profit sharing.

Moreover, we can prove that the level of effort chosen by the agent under profit sharing is the same as the one chosen under no profit sharing.

[1] See FOC w.r.t. e in the P-A model under no profit sharing.

Proposition 4. $e\left(z^{PS*}\right) \equiv e(w^*)$.

Proof. Since the agent revenue does not change as proved by Prop. 3, the agent maximizing choice of effort cannot change, leading to the same equilibrium probability distribution $f(y;e^*)$ ■

Hence, we get the following result.

Proposition 5. $\lambda^{PS*} \equiv \lambda^*,\ \mu^{PS*} \equiv \mu^*$

Proof. Given the same probability distribution, $f(y;e^*)$, and the same agent revenue, z^{PS}, the two constraints are the same, leading therefore to the same values for λ^* and μ^* ■

Moreover, we can prove that also the principal revenue remains the same.

Proposition 6. In the PA model, profit sharing doesn't change

(i) the principal equilibrium revenue in every state of nature, formally:

$$(1-\theta)\left[y^{PS}-w^{PS}\right] = (y-w)$$

(ii) the expected value of principal profits

$$(1-\theta)\int_y \pi\left[y^{PS}-w^{PS}\right] f(y;e^*)\,dy \equiv E\pi^{PS} = \tag{15}$$
$$= E\pi \equiv \int_y \pi(y-w)\, f(y;e^*)\,dy$$

Proof. By the proposition 3 we established that $w=(1-\theta)w^{PS}+\theta y^{PS}$. Hence:

$$w^{PS} = \frac{w}{(1-\theta)} - \frac{\theta}{1-\theta}y^{PS}$$

Substituting in the principal revenue

$$x^{PS} = (1-\theta)\left\{y^{PS} - \left[\frac{w}{(1-\theta)} - \frac{\theta}{1-\theta}y^{PS}\right]\right\}$$

$$\begin{aligned} x^{PS} &= (1-\theta)\left\{y^{PS} - \left[\tfrac{w}{(1-\theta)} - \tfrac{\theta}{1-\theta}y^{PS}\right]\right\} \\ &= \left\{y^{PS}-w\right\} \end{aligned}$$

but since the equilibrium effort has not changed, the density function of output is the same, hence $\left\{y^{PS}-w\right\} = \{y-w\}$. Being the equilibrium effort the same, the equilibrium density function is the same, then the expected value of profits does not change, leading to (15) above. ■

Let's now analyse welfare under profit sharing.

Welfare.
following the same reasoning of the no profit sharing case we get:

$$E\pi^{PS} = \lambda^*\left[U_R + c(e^{PS*})\right] + \mu^* M(e^{PS*});$$

where $M(e^{PS*})$ has positive or negative sign depending on where the point $\hat{y}$ such that $f_e^*(\hat{y}_s; e^*) = 0$ is. However, since equilibrium effort is the same as in the no profit sharing case, the term $M(e^{PS*})$ is the same as before. Hence:

$$
\begin{aligned}
W^{PS}\left[z^{PS}\left(w^{PS};\theta^{PS}\right);e^{PS*}\right] &= U_R + \lambda^*\left[U_R + c(e^{PS*})\right] + \mu^* M(e^{PS*}) = \\
&= U_R + \lambda^*[U_R + c(e^*)] + \mu^* M(e^*) = W(w^*;e^*)
\end{aligned}
$$

We are now able to assert that in the Principal Agent model the introduction of a share of the principal profits into the agent revenue has no effects neither on the level of effort delivered by the agent, nor on the welfare.

In the P-A model agent revenue contingent on output already represents the optimal incentive contract and the introduction of a share of profits is not able to add anything more.

Proposition 7. Irrelevance of profit sharing in the Principal Agent model.

Proof. Because we have proved that:

$$z^{PS} = w$$

$$e^{PS*} \equiv e^*$$

$$\lambda^{PS*} \equiv \lambda^*; \qquad \mu^{PS*} \equiv \mu^*$$

$$x^{PS} = \{y - w\}$$

$$W^{PS} = W$$

■

2. COLLUSION UNDER MORAL HAZARD

We are now interested in analysing whether there is a role for profit sharing in a different bargaining structure. We wonder whether, when the agent is allowed to share decisions with the principal, the introduction of a share of principal profit in his revenue may have some effects both on the level of effort and on the welfare.

The idea of having the two subjects both with decision power is here represented through a collusive setting, in which both the principal and the agent maximize a joint objective function, given by the sum of the expected profit function and the expected utility function.

It will be shown that in this framework the incentive constraint loses meaning. Anyway, this subject will be analyzed in the following section. In this section, by continuity with the previous framework, we keep the two constraints, verifying what happens when λ and μ are both positive. We again solve the standard problem before under no profit-sharing and then under profit-sharing.

2.1 Collusion under No Profit-Sharing.

When we allow the agent to collude with the principal, we have that both the participants maximize a joint objective function given by the sum of the expected profit function and the expected utility function. Hence, the new standard problem is defined as:

$$\max_{w,e} \int_{y_0}^{y_1} \pi[y - w(y)]\, f(y;e)\, dy + \int_{y_0}^{y_1} u[w(y)]\, f(y;e)\, dy - c(e)$$

$$s.t.: \quad \begin{cases} \int_{y_0}^{y_1} u[w(y)]\, f(y;e)\, dy - c(e) \geq U_R \\ \int_{y_0}^{y_1} u[w(y)]\, f_e(y;e)\, dy - c'(e) = 0 \end{cases}$$

and the Lagrangian is:

$$L = \int_{y_0}^{y_1} \pi[y - w(y)]\, f(y;e)\, dy + (1+\lambda)\left(\int_{y_0}^{y_1} u[w(y)]\, f(y;e)\, dy - c(e) - U_R\right) + \\ + \mu\left(\int_{y_0}^{y_1} u[w(y)]\, f_e(y;e)\, dy - c'(e)\right)$$

As usual we concentrate our analysis on the first order conditions with respect to w and e.

FOC w.r.t. w.

$$\frac{\partial L}{\partial w}: \frac{\partial \pi[y - w(y)]}{\partial [y - w(y)]}\frac{\partial [y - w(y)]}{\partial w(y)} f(y;e) + (1+\lambda)\frac{\partial u[w(y)]}{\partial w(y)} f(y;e) + \\ + \mu \frac{\partial u[w(y)]}{\partial w(y)} f_e(y;e) = 0; \quad w > 0 \tag{16}$$

where $\frac{\partial [y - w(y)]}{\partial w(y)} = -1$.

Dividing through by $\frac{\partial u[w(y)]}{\partial w(y)}$ and $f(y;e)$, and under the condition of binding constraint, we get the collusive incentive contract:

$$\frac{\frac{\partial \pi}{\partial [y - w(y)]}}{\frac{\partial u}{\partial w(y)}} = 1 + \lambda + \mu \frac{f_e}{f}$$

Since the likelihood ratio, $\frac{f_e}{f}$, is non decreasing in output (see equation (2)), also the collusive incentive contract is always increasing in the result in output obtained. As in the P-A contract, the more the agent is risk-averse, the less he likes the dependence of his wage on output.

Proposition 8. Given the assumptions of section 1.1., the revenue of the agent, ceteris paribus, is increasing in the realization of output, the less she is risk- averse. Formally:

$$\frac{dw}{dy} = \frac{\rho_P + \mu \frac{u'}{\pi'} \frac{\partial (f_e/f)}{\partial y(s)}}{\rho_P + \rho_A}$$

Proof. By differentiating (16) w.r.t. y; and recalling that $\rho_P = -\pi''/\pi'$ is the principal s measure of absolute risk-aversion, and $\rho_A = -u''/u'$ is the agent s measure of absolute risk-aversion ■

FOC w.r.t. e.

$$\frac{\partial L}{\partial e}: \int_{y_0}^{y_1} \pi[y - w(y)]\, f_e(y;e)\, dy + (1+\lambda)\left(\int_{y_0}^{y_1} u[w(y)]\, f_e(y;e)\, dy - c'(e)\right) + \\ + \mu\left(\int_{y_0}^{y_1} u[w(y)]\, f_{ee}(y;e)\, dy - c''(e)\right) = 0; \quad e > 0 \tag{17}$$

Once again the second term is zero because of the incentive constraint, while the third term is negative for the incentive constraint to hold. Hence:

$$\frac{\partial E\pi}{\partial e} = -\mu\left(\frac{\partial^2 Eu}{\partial e^2} - c''(e)\right) \geq 0$$

Remark 1. The only innovation coming from collusion, $(1+\lambda)$ instead of λ, is nullified by the incentive constraint, leaving the same first order condition with respect to effort.

The equality between the FOCs w.r.t. e in collusion and in the P-A model leads to the following irrelevance result of decision sharing.

Proposition 9. The collusion incentive contract is the same as the P-A incentive contract, because $(1+\lambda^*_{CL}) = \lambda^*_{PA}$.

Proof. The system of FOC is the same as in the PA model, apart from the term $(1+\lambda^*_{CL})$ instead of λ^*_{PA}. But the scalar 1 is added to the constant λ_{CL}; hence if all other conditions are the same, the equilibrium parameter discounts the number 1 ■

Hence follows that also the level of effort delivered by the agent is the same as in the P-A model.

Result 1. $e^*_{CL} = e^*_{PA}$.

Proof. Since agent revenue is the same as in the P-A model and the new term $(1+\lambda)$ is nullified, the solution of the equation (17) is the same of the equation (10) in the P-A model ■

Moreover we have another irrelevance result of decision sharing for what concerns welfare.

Welfare.

Let s consider (16). By integrating it with respect to the $\{w_s\}$ sequence and over the states of nature we get:

$$\begin{aligned}\int_{y_0}^{y_1} \pi[y_s - w^*(s)]\, f(y_s;e)\, ds &= (1+\lambda_{CL})\int_{y_0}^{y_1} u[w^*(s)]\, f(y_s;e)\, ds + \\ &\quad + \mu \int_{y_0}^{y_1} u[w^*(s)]\, f_e(y_s;e)\, ds\end{aligned}$$

but, we know that the expected value of the utility function, by the participation constraint is equal to $U_R + c(e^*)$; hence

$$E\pi[y_s - w^*(s)] = (1+\lambda^*_{CL})[U_R + c(e^*)] + \mu^* M(e^*)$$

where $M(e^*)$ has positive or negative sign depending on where the point $\hat{y}$ such that $f^*_e\left(\hat{y}_s; e^*\right) = 0$.

However, since equilibrium effort is the same as in the PA model, the term $M(e^*)$ is the same as before, and since we have proved that $(1+\lambda_{CL}) = \lambda_{PA}$, we can also assert that welfare in collusion is the same as in the P-A setting:

$$\begin{aligned} W_{CL} &= EU + E\pi = U_R + (1+\lambda^*_{CL})[U_R + c(e^*)] + \mu^* M(e^*) = \\ &= U_R + \lambda^*_{PA}[U_R + c(e^*)] + \mu^* M(e^*) = W \end{aligned}$$

The reason for irrelevance of decision sharing is due to the fact that the incentive constraint implies the agent to be already effort maximizer. Hence he cannot do better.

Remark 2. The problem is this kind of structure for the collusive contract. It is meaningless the fact that, although the agent is maximizing at the objective function level, he maximizes at the incentive constraint level too. This second step is shown to be irrelevant.

2.2. Collusion under Profit-Sharing.

We want now to analyse if profit sharing may have some relevance in the new contractual form.

When we introduce a share of the principal profit into the agent revenue, under collusion, the problem becomes:

$$\max_{w;e} \int_{y_0}^{y_1} \pi\left\{(1-\theta)\left[y - w^{PS}(y)\right]\right\} f[y;e]\,dy + \int_{y_0}^{y_1} u\left\{w^{PS}(y) + \theta\left[y - w^{PS}(y)\right]\right\} f[y;e]\,dy - c(e)$$

$$\text{s.t.} \quad \begin{cases} \int_{y_0}^{y_1} u\left\{w^{PS}(y) + \theta\left[y - w^{PS}(y)\right]\right\} f[y;e]\,dy - c(e) \geq U_R \\ \int_{y_0}^{y_1} u\left\{w^{PS}(y) + \theta\left[y - w^{PS}(y)\right]\right\} f_e[y;e]\,dy - c'(e) = 0 \end{cases}$$

The Lagrangian is:

$$L = \int_{y_0}^{y_1} \pi\left[x_{CL}^{PS}\right] f(y;e)\,dy + (1+\lambda)\left(\int_{y_0}^{y_1} u\left[z_{CL}^{PS}\right] f(y;e)\,dy - c(e) - U_R\right) + \\ + \mu\left(\int_{y_0}^{y_1} u\left[z_{CL}^{PS}\right] f_e(y;e)\,dy - c'(e)\right);$$

where:

$$x_{CL}^{PS} = (1-\theta)\left[y - w^{PS}(y)\right]$$

is the principal revenue in collusion under profit sharing, and

$$z_{CL}^{PS} = w^{PS}(y) + \theta\left[y - w^{PS}(y)\right] = (1-\theta)\,w^{PS}(y) + \theta y$$

is the agent revenue in collusion under profit sharing.

Notice that also under collusion:

$$\frac{\partial x_{CL}^{PS}}{\partial w^{PS}} = -(1-\theta)$$

and:

$$\frac{\partial z_{CL}^{PS}}{\partial w^{PS}} = (1-\theta)$$

FOC w.r.t. w^{PS}.

$$\frac{\partial L}{\partial w^{PS}} : \frac{\partial \pi}{\partial x_{CL}^{PS}} \frac{\partial x_{CL}^{PS}}{\partial w^{PS}} f(y;e) + (1+\lambda) \frac{\partial u}{\partial z_{CL}^{PS}} \frac{\partial z_{CL}^{PS}}{\partial w^{PS}} f(y;e) + \\ + \mu \frac{\partial u}{\partial z_{CL}^{PS}} \frac{\partial z_{CL}^{PS}}{\partial w^{PS}} f_e(y_s;e) \quad 0 \qquad w^{PS} \geq 0 \tag{18}$$

and since

$$\frac{\partial x_{CL}^{PS}}{\partial w^{PS}} = -(1-\theta); \qquad \frac{\partial z_{CL}^{PS}}{\partial w^{PS}} = (1-\theta);$$

we get:

$$\frac{\partial L}{\partial w^{PS}}: -\frac{\partial \pi}{\partial x_{CL}^{PS}}(1-\theta)f(y;e) + (1+\lambda)\frac{\partial u}{\partial z_{CL}^{PS}}(1-\theta)f(y;e) + \\ +\mu\frac{\partial u}{\partial z_{CL}^{PS}}(1-\theta)f_e(y_s;e) \leq 0 \qquad w^{PS} \geq 0 \tag{19}$$

Also under collusion the term $(1-\theta)$ cancels out leading to:

$$\frac{\partial L}{\partial w^{PS}}: -\frac{\partial \pi}{\partial x_{CL}^{PS}} + \frac{\partial u}{\partial z_{CL}^{PS}}\left(1+\lambda+\mu\frac{f_e(y;e)}{f(y;e)}\right) \leq 0 \qquad w^{PS} \geq 0 \tag{20}$$

From equation (20), and under the condition of binding constraint, the collusive incentive contract under profit sharing is:

$$\frac{\frac{\partial \pi}{\partial x_{CL}^{PS}}}{\frac{\partial u}{\partial z_{CL}^{PS}}} = 1+\lambda+\mu\frac{f_e}{f}$$

Also under collusion the contract defining the solution for z_{CL}^{PS} is the same as the contract defining w in the collusive setting under no profit sharing. Hence we are able to establish the following irrelevance result.

Proposition 10. Also under collusion, profit sharing in every state of nature doesn't change the agent equilibrium revenue. Formally:

$$w = z_{CL}^{PS} \equiv w^{PS} + \theta\left(y - w^{PS}\right)$$

Proof. See proof of proposition 3 in the PA model case ■

Hence, also in the collusive model under profit sharing, the contingent wage varies according to keep $z_{CL}^{PS} \equiv w^{PS} + \theta\left(y - w^{PS}\right) = w$.

Corollary 5. The two revenue components, the contingent wage w_{CL}^{PS} and the profit share $\theta\left(y - w_{CL}^{PS}\right)$ are perfect substitutes.

Proof. See proof of corollary 4 in the P-A model case ■

Let's now analyse what the FOC with respect to e shows.

FOC w.r.t. e.

$$\frac{\partial L}{\partial e} = \int_{y_0}^{y_1}\pi\left[x_{CL}^{PS}\right]f_e(y;e)\,dy + (1+\lambda)\left(\int_{y_0}^{y_1}u\left[z_{CL}^{PS}\right]f_e(y;e)\,dy - c'(e)\right) + \\ +\mu\left(\int_{y_0}^{y_1}u\left[z_{CL}^{PS}\right]f_{ee}(y;e)\,dy - c''(e)\right) \leq 0; \qquad e \geq 0$$

Where the second term is zero because of the incentive constraint, and the third term is negative for the incentive constraint to hold. Hence:

$$\frac{\partial E\pi}{\partial e} = -\mu\left(\frac{\partial^2 Eu}{\partial e^2} - c''(e)\right) \geq 0$$

which is the same condition as in the collusive model under no profit sharing.

Moreover we can prove that also in the collusive setting the level of effort chosen by the agent under profit sharing is the same as the one chosen under no profit sharing.

Proposition 11. Also under collusion $e\left(z_{CL}^{PS*}\right) \equiv e(w^*)$:

Proof. See proof of Proposition 4 in the P-A setting ■

Hence, we get the following result.

Proposition 12. Also under collusion $\lambda^{PS*} \equiv \lambda^*$, $\mu^{PS*} \equiv \mu^*$

Proof. See proof of Proposition 5 in the P-A setting ■

Moreover, by the same reasoning as in the case of P-A model we can prove that also in the collusive setting the principal revenue under profit sharing remains the same as the one under no profit sharing.

Proposition 13. In the PA model, profit sharing in every state of nature doesn't change the principal equilibrium revenue. Formally:

$$\left[y^{PS} - z_{CL}^{PS}\right] = (y - w)$$

Proof. See proof of Proposition 7 in the P-A model ■

As a consequence we have that also in the collusive setting the expected value of profits under profit sharing is the same as in the collusive setting under no profit sharing.

Proposition 14. Since the equilibrium density function is the same, because the equilibrium effort is the same, the expected value of profits does not change

$$\begin{aligned}(1-\theta)\int_y \pi\left[y^{PS} - w^{PS}\right] f(y;e^*)\,dy \quad &\equiv \quad E\pi^{PS} = \\ &= \quad E\pi \equiv \int_y \pi(y-w)\, f(y;e^*)\,dy\end{aligned}$$

Proof. See proof of Proposition 7 in the P-A model ■

Welfare.

we prove that also under collusion the welfare position under profit sharing is coincident with the no profit sharing case.

By the same reasoning as in the no profit sharing case, we get:

$$E\pi^{PS} = (1+\lambda^*)\left[U_R + c(e^{PS*})\right] + \mu^* M(e^{PS*});$$

where $M(e^{PS*})$ has positive or negative sign depending on where the point $\hat{y}$ such that $f_e^*\left(\hat{y}_s; e^*\right) = 0$ is. However, since equilibrium effort is the same as in the no profit sharing case, the term $M(e^{PS*})$ is the same as before. Hence:

$$\begin{aligned} W_{CL}^{PS} &= U_R + (1+\lambda^*)\left[U_R + c(e^{PS*})\right] + \mu^* M(e^{PS*}) = \\ &= U_R + (1+\lambda^*)\left[U_R + c(e^*)\right] + \mu^* M(e^*) = W_{CL}\end{aligned}$$

Hence, also in the collusive model the introduction of a share of the principal profits into the agent revenue has no effects neither on the level of effort, nor on the welfare.

Proposition 15. Irrelevance of profit sharing in the Collusive model.

Proof. Because we have proved that:

$$z^{PS} = w$$

$$e^{PS*} \equiv e^{*}$$

$$\lambda^{PS*} \equiv \lambda^{*}; \qquad \mu^{PS*} \equiv \mu^{*}$$

$$x_{CL}^{PS} = \{y - w\}$$

$$W_{CL}^{PS} = W_{CL}$$

■

Moreover, since in the previous section we have proved the irrelevance of decision sharing in the P-A model under incomplete information, we can assert that, under incomplete information, the collusive equilibrium under profit sharing coincides with the P-A equilibrium under no profit sharing.

3. COLLUSION UNDER COMPLETE INFORMATION

In the previous section we have seen that the reason for irrelevance of decision sharing is connected to the presence of the incentive constraint.

But, in the collusive case the presence of the incentive constraint implies that the agent maximizes his utility function twice. Both at the final stage of the game (in the incentive compatibility constraint), when he chooses the optimal level of effort, and at the first stage of the game, when, together with the principal, he maximizes the collusive objective function, anticipating his third stage behavior, which is a nonsense!

Hence, when contract conditions change and both the principal and the agent maximize a joint objective function the incentive constraint looses meaning.

The basic idea is that if the agent is colluding in decision we are implicitly saying that he is relaxing information. That is, in collusion the principal faces the moral hazard problem by means of an organizational structure which eliminates the conflict of interests with the agent, reconducting the question to a situation of complete information.

Therefore, in order to analyze whether the change in bargaining power toward decision sharing sorts some effects on level of effort and welfare, we now consider the collusive model without the incentive constraint.

Since we have seen that the presence of profit sharing doesn't change the equilibrium conditions of principal and agent revenue, our analysis will be conducted in the no profit-sharing context.

In collusion with no incentive constraint, the problem is defined as:

$$\max_{w;e} \int_{y_0}^{y_1} \pi[y - w(y)]\, f(y; e)\, dy + \int_{y_0}^{y_1} u[w(y)]\, f(y; e)\, dy - c(e)$$

$$s.t. \quad \int_{y_0}^{y_1} u[w(y)]\, f(y; e)\, dy - c(e) \geq U_R$$

and the Lagrangian is:

$$L = \int_{y_0}^{y_1} \pi[y - w(y)]\, f(y;e)\, dy + (1+\lambda)\left(\int_{y_0}^{y_1} u[w(y)]\, f(y;e)\, dy - c(e) - U_R\right)$$

The problem becomes a system of three equations in three unknown variables (w, e, λ). The three equations are the first order conditions with respect to w, e and λ.

As usual we concentrate our analysis on the first order conditions with respect to w and e.

FOC w.r.t. w.

$$\frac{\partial L}{\partial w} : \frac{\partial \pi[y - w(y)]}{\partial [y - w(y)]} \frac{\partial [y - w(y)]}{\partial w(y)} f(y;e) + (1+\lambda) \frac{\partial u[w(y)]}{\partial w(y)} f(y;e) = 0; \quad w > 0 \qquad (21)$$

where $\frac{\partial [y - w(y)]}{\partial w(y)} = -1$.

Dividing through by $\frac{\partial u[w(y)]}{\partial w(y)}$, and under the condition of binding constraint, we get the collusive contract under complete information:

$$\frac{\frac{\partial \pi}{\partial [y - w(y)]}}{\frac{\partial u}{\partial w(y)}} = 1 + \lambda$$

Moreover, we get the following.

Result 2. In collusion under complete information risk sharing is as follows

$$\frac{dw}{dy} = \frac{\rho_P}{\rho_P + \rho_A}$$

where this result is expectedly equal to PA under complete information.

Proof. By differentiating (21) w.r.t. y, and recalling that $\rho_P = -\pi''/\pi'$ is the principal's measure of absolute risk-aversion, and $\rho_A = -u''/u'$ is the agent's measure of absolute risk-aversion. ■

By the above result and by the collusive contract, we obtain that in case of risk neutral principal[2], and risk averse agent, his wage is not contingent on y, and depends only on the equilibrium level of effort. Instead, in case of risk averse principal and risk averse agent, his wage is increasing in the realization of output obtained, the less he is risk averse.

Let's now see what the optimal condition with regards to effort shows.

FOC w.r.t. e.

$$\frac{\partial L}{\partial e} : \int_{y_0}^{y_1} \pi[y - w(y)]\, f_e(y;e)\, dy + (1+\lambda)\left(\int_{y_0}^{y_1} u[w(y)]\, f_e(y;e)\, dy - c'(e)\right) = 0; \quad e > 0$$

that can also be written as:

$$\frac{\partial L}{\partial e} : \frac{\partial E\pi}{\partial e} + (1+\lambda)\left(\frac{\partial Eu}{\partial e} - c'(e)\right) = 0; \quad e > 0$$

Proposition 16. Under collusion and complete information the optimal level of effort increases.

[2] when $\rho_P = -\pi''/\pi' = 0$.

Proof. By Def. 8 of the expected profit function, the FOC w.r.t. e can be rewritten as

$$\frac{\partial L}{\partial e} : -\int_{y_0}^{y_1} \pi'[y_s - w(y_s)][1 - w'(y_s)] F_e[y_s; e]\, dy = -(1+\lambda)\left(\frac{\partial Eu}{\partial e} - c'(e)\right)$$

since, for equation (4) $\frac{\partial E\pi}{\partial e} = -\int_{y_0}^{y_1} \pi'[y_s - w(y_s)][1 - w'(y_s)] F_e[y_s; e]\, dy > 0$, it implies that

$$\Rightarrow (1+\lambda)\left(\frac{\partial Eu}{\partial e} - c'(e)\right) < 0!$$

which means that the agent is beyond his optimal level of effort, in the descending slope of his utility function. ■

Therefore, under collusion, the principal gains more effort than the one obtained in a standard P-A contract, hence his welfare improves.

Since, for the participation constraint, the agent's welfare cannot worsen (his utility cannot be less than his reservation utility), we can assert that the change in bargaining power sorts a welfare improvement.

Hence, when collusion is analysed in a more appropriate setting, decision sharing sorts positive effects either on the level of effort and on the level of welfare.

4. CONCLUDING REMARKS

This paper achieves three main conclusions.

First, the inclusion of profit sharing in the Principal Agent model under moral hazard is irrelevant in determining effort and welfare equilibrium levels. The results show that wage and share of profits are perfect substitutes, so that the increase in the amount of contingent profit implies a decrease of the same amount in the contingent wage, leaving unchanged the equilibrium conditions of worker and entrepreneur revenues. The resulting irrelevancy is due to the homogeneity of the two sources of agent revenue, wage and share of profits.

Secondly, collusion under moral hazard is also irrelevant in determining effort and welfare equilibrium levels. In this case the irrelevancy stems from the incentive constraint playing no role in a collusive setting. In fact, the agent can not maximize twice, both at the joint objective function level, and at the incentive constraint level.

Building up on the afore mentioned irrelevancy we move to the third conclusion. Ruling out the incentive constraint from the collusive model leads to relevant implications in terms of effort and welfare. We prove that in this case worker offers a higher level of effort, if compared with the one offered in the standard P-A problem. This implies that the entrepreneur is better off, as her expected profit rises, and the agent is at least as well off, since his utility cannot be lower than his reservation level (for the participation constraint). It is so possible to assert that the change in contract conditions, towards worker participation to the decision making process of the firm, originates a welfare improvement.

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