# Welfare in a Differentiated Oligopoly with Free Entry: a Cautionary Note*

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**Abstract**

We model a free entry equilibrium in a differentiated oligopoly where firms compete either in prices or in quantities. We prove that Cournot competition allows for a larger number of firms to survive in equilibrium. Hence, the conventional wisdom on industry output and social welfare may not hold. In fact, we prove that there exists a region of parameters where the aggregate output level as well as social surplus are larger under Cournot than Bertrand competition.

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# 1 Introduction

We model a free entry equilibrium in a differentiated oligopoly where single-product firms compete either in prices or in quantities. Entry requires a fixed cost which reflects the efforts of firms in product innovation activities.

In the existing literature on oligopoly markets, comparative welfare evaluations between price and quantity competition are usually carried out for a *given* market structure, that is, for a given number of firms (see Singh and Vives, 1984; Vives, 1985; Okuguchi, 1987, *inter alia*). All these contributions point out that, for a given number of firms (and product varieties), the harsher competition and the larger aggregate output characterising a Bertrand market *vis à vis* its Cournot counterpart imply that price competition is socially preferable to quantity competition, while obviously the opposite holds for firms.

Our aim is to highlight that such a comparison may be misleading to the extent that the number of firms in the long-run equilibrium is *endogenously* established by the entry process. In particular, we prove that Cournot competition allows for a larger number of firms to survive in equilibrium. Hence, the conventional wisdom on industry output and social welfare may not hold. In fact, we show that *there exists a region of parameters where the aggregate output level as well as social surplus are larger under Cournot than under Bertrand competition.*

While somewhat surprising, to our knowledge this result has been overlooked by the literature so far, possibly because the issue of product diversity under free entry has been mostly tackled in terms of monopolistic competition rather than oligopoly. As it is well known, with monopolistic competition the choice between setting prices and setting quantities is immaterial for firms (see, e.g. Dixit and Stiglitz, 1977; Ottaviano and Thisse, 1999).

The remainder of the paper is structured as follows. The model is laid out in section 2. The free entry equilibria under Cournot and Bertrand behaviour are derived in section 3. Section 4 provides a comparative evaluation of aggregate output and social welfare levels in the two settings. Concluding remarks are in section 5.

# 2 The setup

We consider a market where single-product firms sell differentiated products. There exists a fixed entry fee $F$, which can be thought of as the R&D cost of developing a certain variety. Firms enter the market until the individual profit becomes nil. Once they have entered the market, firms produce at the same constant marginal cost. Without further loss of generality, we normalise the marginal cost to zero. Let $n$ be the number of firms operating in the market. The demand structure is borrowed from Spence (1976). When firms compete in quantities, the inverse demand function for variety $i$ is:

$$p_i = a - bq_i - d \sum_{j \neq i} q_j \tag{1}$$

where $d \in [0, b]$ is the symmetric degree of substitutability between any pair of varieties. If $d = b$, products are completely homogeneous; if $d = 0$, products are completely independent and each firm becomes a monopolist. Under price competition, the corresponding direct demand function for variety $i$ is (Majerus, 1988):

$$q_i = \frac{1}{b + d(n-1)} \cdot \left\{ a - \frac{b + d(n-2)}{b - d} \cdot p_i + \frac{d}{b - d} \cdot \sum_{j \neq i} p_j \right\} . \tag{2}$$

In both settings, firm $i$'s objective function is $\pi_i^k = R_i^k - F = p_i q_i - F$, while consumer surplus is $CS^k = \left[\sum_{i=1}^{n} (A - p_i) q_i\right] /2$. Superscript $k = B, C$ indicates whether market competition takes place *à la* Bertrand or *à la* Cournot. Since profits are nil in the free entry equilibrium, consumer surplus alone is the appropriate measure for social welfare.

# 3 The free entry equilibrium

Here we investigate the long-run equilibria under the alternative assumptions of Cournot and Bertrand behaviour.

## 3.1 Cournot competition

The first order condition for firm $i$ is:

$$\frac{\partial \pi_i^C}{\partial q_i} = a - 2bq_i - d \sum_{j \neq i} q_j = 0. \tag{3}$$

From (3) we immediately derive the best reply function:

$$q_i = \frac{a - d\sum_{j \neq i} q_j}{2b} \, . \tag{4}$$

On the basis of ex ante symmetry across the population of firms, we introduce the following assumption:

$$\sum_{j \neq i} q_j = (n-1)q_i. \tag{5}$$

This allows us to drop, in the remainder, the indication of the identity of the firm. The individual output level in equilibrium is

$$q^C(n) = \frac{a}{2b + d(n-1)} \, , \tag{6}$$

to which the following profits are associated:

$$\pi^C(n) = \frac{a^2 b}{[2b + d(n-1)]^2} - F \, . \tag{7}$$

For future reference, observe that the equilibrium price is $p^C(n) = ab/\left[2b + d(n-1)\right]$ and industry output is $Q^C(n) = nq^C(n)$. Solving $\pi^C(n) = 0$ with respect to $n$, we obtain the number of firms operating in the industry in the free entry equilibrium:

$$n^C = 1 - \frac{2b}{d} + \frac{a}{d}\sqrt{\frac{b}{F}} \, , \tag{8}$$

as long as $n^C \geq 1$, i.e., $F \leq a^2/4b$. Notice that this inequality establishes that the fixed cost must not exceed the gross profits of a monopolist. Given that at the free entry equilibrium the industry profits are nil, we obtain

$$SW^C(n^C) = CS^C(n^C) = \frac{\left(a - \sqrt{bF}\right)\left[a\sqrt{b} - (2b-d)\sqrt{F}\right]}{2d\sqrt{b}} \tag{9}$$

which is everywhere positive in the admissible range of parameters.

## 3.2 Bertrand competition

To derive the equilibrium in the Bertrand setting, we proceed as in the previous subsection. The first order condition for firm $i$ is:

$$\frac{\partial \pi_i^B(n)}{\partial p_i} = \frac{a(b-d) - 2\left[b + d(n-2)\right]p_i + d\sum_{j \neq i} p_j}{(b-d)[b + d(n-1)]} = 0. \tag{10}$$

Using the symmetry assumption $\sum_{j\neq i} p_j = (n-1)p$, we derive the equilibrium price

$$p^B(n) = \frac{a(b-d)}{2(b-d)+d(n-1)} \ . \tag{11}$$

Plugging (11) into the profit function and simplifying, we get the individual equilibrium profits:

$$\pi^B(n) = \frac{a^2(b-d)\left[b+d(n-2)\right]}{\left[2(b-d)+d(n-1)\right]^2\left[b+d(n-1)\right]} - F \ . \tag{12}$$

Individual and industry output are, respectively,

$$q^B(n) = \frac{a\left[b+d(n-2)\right]}{(2(b-d)+d(n-1))(b+d(n-1))} \tag{13}$$

and $Q^B(n) = nq^B(n)$. Imposing that $\pi^B(n) = 0$, we derive the equilibrium number of firms, $n^B$, in the Bertrand setting.[1] Correspondingly, we get the equilibrium level of social welfare $SW^B(n^B) = CS^B(n^B)$.

# 4 A comparative assessment

Consider individual profits under Cournot and Bertrand competition, respectively, i.e., (7) and (12). It is easily verified that

- when $n = 1$, $\pi^C(1) = \pi^B(1) = \pi_M = a^2/(4b) - F$, where subscript $M$ stands for *monopoly*;
- $\lim_{n\to\infty} \pi^C(n) = \lim_{n\to\infty} \pi^B(n) = 0$;
- for all $n \in (1, \infty)$, the ratio

  $$\frac{R^B(n)}{R^C(n)} = \frac{(b-d)(b-2d+dn)(2b-d+dn)^2}{b(2b-3d+dn)^2(b-d+dn)}$$

  is always strictly lower than one. This, given the fixed cost $F$, establishes that quantity competition is always more profitable than price competition in this range of $n$;

[1] The equation $\pi^B = 0$ has three solutions in $n$, of which only one is real.

- for all $n \in [1, \infty)$, both $\pi^C(n)$ and $\pi^B(n)$ are everywhere decreasing in $n$.

Hence, the following result can be established:

**Lemma 1** *The number of firms in steady state is larger under Cournot competition, i.e.,* $n^C > n^B$.

**Proof.** This follows from the above discussion, joint with the assumption that the entry cost $F$ is invariant in $n$. ■

As to output levels, the following facts are well known:

- for all $n \in [1, \infty)$, $q^B(n) \geq q^C(n)$ and $Q^B(n) \geq Q^C(n)$, with strict equalities at $n = 1$;

- for all $n \in [1, \infty)$, both $Q^B(n)$ and $Q^C(n)$ are everywhere increasing in $n$.

**Proposition 1** *A priori, the sign of* $Q^C(n^C) - Q^B(n^B)$ *is ambiguous.*

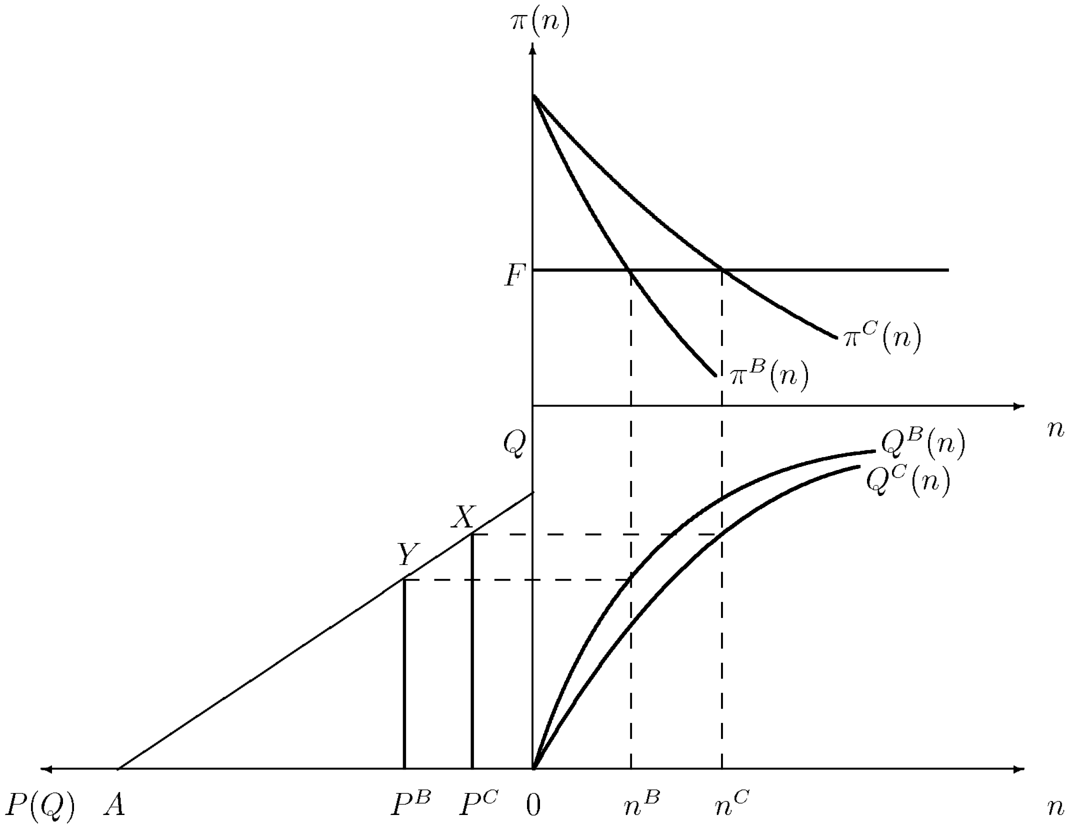
The above claim can be justified on the following grounds. For a given $n \in (1, \infty)$, we know that $Q^C(n) - Q^B(n) < 0$. However, on the basis of lemma 1, $n^C > n^B$. This, combined with the fact that aggregate output is increasing in the number of firms under both Cournot and Bertrand competition, implies that $Q^C(n^C) - Q^B(n^B)$ may have either sign.

The relevant implication of this argument is that an analogous ambiguity arises as to welfare levels, in that the sign of $CS^C(n^C) > CS^B(n^B)$ (and $SW^C(n^C) - SW^B(n^B)$) is necessarily the same as $Q^C(n^C) - Q^B(n^B)$. Hence,

**Lemma 2** *At the free entry equilibrium, a necessary and sufficient condition for* $CS^C(n^C) > CS^B(n^B)$ *is* $Q^C(n^C) > Q^B(n^B)$.

Provided that $Q^C(n) \leq Q^B(n)$ for all $n$, a situation where $SW^C(n^C) > SW^B(n^B)$ may obtain if $Q^C(n)$ is not too far below $Q^B(n)$. This is illustrated in figure 1.

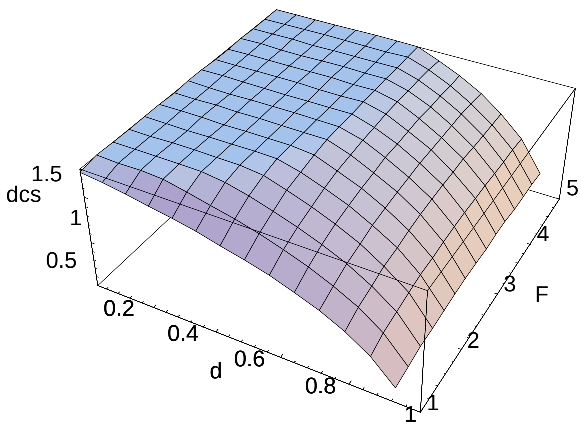
**Figure 1 :** The free entry equilibrium




The area of the trapezoid $P^BP^CXY$ measures $CS^C(n^C) - CS^B(n^B) > 0$, that is, the gain in consumer surplus (and welfare) produced by the larger product diversity in the Cournot free entry equilibrium, compared to the Bertrand market. An example is given in figure 2, where we consider $\{a = 10; b = 1\}$ and we plot the difference $CS^C(n^C) - CS^B(n^B)$ (or $SW^C(n^C) - SW^B(n^B)$) over $\{d \in [0.1, 1]; F \in [1, 5]\}$. The viability condition that at least one firm enters the market is met, in that $F < \pi_M = a^2/(4b) = 25$. It can also be verified that both $n^C$ and $n^B$ are strictly larger than one. Finally, in terms of the integer problem, it can also be checked that the difference between $n^C$ and $n^B$ is not irrelevant.[2]

[2]For example, in the case where $\{d = .1, F = 4\}$, we get $n^C - n^B = 1.18$.

Figure 2 : $dcs = CS^C(n^C) - CS^B(n^B)$




The intuition behind the result illustrated in figure 2 is as follows. The gain in consumer surplus can be traced back to the larger product variety characterising Cournot competition. To fix ideas, consider the case where product differentiation is relatively low, i.e., $d$ is close to $b$. In this case, the incentive to enter the market is significantly weaker when firms expect market competition to be *à la* Bertrand rather than *à la* Cournot. In the limit, as $d/b \rightarrow 1$, for any $F \in [0, \pi^C(n^C))$, the equilibrium market structure with Bertrand behaviour is at most a monopoly, while $n^C$ firms may enter if competition takes place in quantities. In this situation, we would compare the social welfare of a monopoly against that of a Cournot oligopoly, and this assessment is straightforward.

# 5 Concluding remarks

The conclusion has the same flavour of the point made by Norman and Thisse (1996), about the social desirability of soft rather than tough pricing regime. Dealing with price deregulation policies in a model of spatial competition, Norman and Thisse observe that fierce price competition among incumbent firms may deter the entry of new firms, and hence may be detrimental to consumer and social welfare, in that "the benefits of lower prices with a given number of firms may be more than offset by a reduction in the number of firms that can enter the market " (Norman and Thisse, 1996, p. 77).

The above analysis could be extended in several directions, in order to test the robustness of our results. First, an infinite time horizon could be explicitly introduced. In such a case, the profits of firm $i$ would be $\pi_i^k = R_i^k/\rho - F$, where $\rho$ is the discount factor which is assumed to be common to all firms. The entry process would stop when $R_i^k = \rho F$, and the above discussion would hold unmodified. A second extension could consider the possibility of knowledge spillovers from earlier entrants to the population of outside firms. This idea goes back at least to Arrow (1962) and has a relevant role in recent research (Romer, 1990; Grossman and Helpman, 1991). The externality in the R&D activity aimed at product innovation would turn the horizontal line in the north-east quadrant of figure 1 into a downward-sloping function of $n$, increasing thus the difference $n^C - n^B$. This, in turn, would make it more likely to obtain that social surplus is larger under quantity behaviour than under price behaviour.

# References


[1] Arrow, K.J. (1962), "The Economic Implications of Learning by Doing", *Review of Economic Studies*, **29**, 155-173.

[2] Dixit, A.K. and J.E. Stiglitz (1977) Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, **67**, 297-308.

[3] Grossman, G.M. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, Cambridge, MA, MIT Press.

[4] Majerus, D. (1988), "Price vs Quantity Competition in Oligopoly Supergames", *Economics Letters*, **27**, 293-297.

[5] Norman, G. and J.-F. Thisse (1996), "Product Variety and Welfare under Tough and Soft Pricing Regimes", *Economic Journal*, **106**, 76-91.

[6] Okuguchi, K. (1987), "Equilibrium Prices in the Bertrand and Cournot Oligopolies", *Journal of Economic Theory*, **42**, 128-139.

[7] Ottaviano, G.I.P. and J.-F. Thisse (1999), "Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity", Center for Operations Research and Econometrics, Louvain-la-Neuve, mimeo.

[8] Romer, P.M. (1990), "Endogenous Technological Change", *Journal of Political Economy*, **98**, S71-S102.

[9] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, **15**, 546-554.

[10] Spence, A.M. (1976), "Product Differentiation and Welfare", *American Economic Review*, **66**, 407-414.

[11] Vives, X. (1985), "Efficiency of Bertrand and Cournot Equilibria with Product Differentiation", *Journal of Economic Theory*, **36**, 166-175.